Comparison of different fuzzy control laws of an inverted pendulum in real time

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Abstract

A control law approach, named SSF (Simultaneous Stabilization for Fuzzy models) (Vermeiren, 1998; Vermeiren et al., 1999), is investigated on the well-known inverted pendulum system. The synthesis of this control law uses a TSK fuzzy model, as for the classical PDC (Parallel Distributed Compensation) control law (Tanaka et al., 1996) and allows to obtain sufficient conditions of stability. The goal of this paper is to show the effectiveness of this approach, in comparison with a PDC law. Two other additional control laws are also provided, a linear feedback law and a fuzzy control law based on an output feedback.

keywords

Inverted pendulum, Fuzzy control, TSK Fuzzy model, Fuzzy observer.

Introduction

The purpose of the presented work is to use different control laws on an inverted pendulum, using a Takagi Sugeno modelisation of the system. Using this kind of fuzzy models allows to obtain some stability results of the closed loop. In this context, the following control laws have been investigated : PDC (Parallel Distributed Compensation) approach (Tanaka et al., 1998), SSF (Simultaneous Stabilization for Fuzzy models) approach (Vermeiren, 1998; Vermeiren et al., 1999). Two additional control laws are also provided, a linear feedback law and a classical fuzzy approach with output feedback.

These approaches have been applied to a real inverted pendulum device which allows a complete 360° rotation, and then from any initial conditions, to obtain an "around zero" control and cart position control purposes.

Lots of control methods have been tested on the inverted pendulum device. Among these different approaches we can denote : neural networks approach (Anderson, 1989), H^{∞} control approach (Van Der Linden and Lambrechts, 1993), linear control approach (Lin et al., 1996), momentum methods for PID calculus approach (Jacob, 1995) , non linear control approach based on energy control (Astrom and Furuta, 1996; Wei et al., 1995).

Let us quote some remarks about this well-known application. In most works concerned with the fuzzy control of an inverted pendulum based on a TSK model, either they are concerned with the SIMO case with restricted area for the angle $\theta(t) \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (Ma et al., 1998), or with the SISO case, i.e. without considerations of the cart position, $\theta(t) \in [-\pi, \pi]$ (Wang et al., 1996; Leung et al., 1996).

In the work presented, a mixed control law is used. A first non linear law corresponding to a swinging sequence (Wei et al., 1995), allowing to take into account high angles $|\theta(t)| > \frac{\pi}{2}$ and maximal position of the cart, is first used, then for a class of acceptable angles, one of the four laws given before is used.

Modelisation of The Inverted Pendulum System

The motion equations of the inverted pendulum device are :

$$\ddot{X}(t) = \frac{F(t) + f\dot{X}(t) + m\left(\frac{g}{2}\sin\left(2\theta(t)\right) - L\dot{\theta}^2(t)\sin\left(\theta(t)\right)\right)}{M + m\sin^2\left(\theta(t)\right)} \tag{1}$$

$$\ddot{\theta}(t) = \frac{g(M+m)\sin\left(\theta(t)\right) + F(t)\cos\left(\theta(t)\right) - f\dot{X}(t)\cos\left(\theta(t)\right) - mL\dot{\theta}^2(t)\sin\left(\theta(t)\right)\cos\left(\theta(t)\right)}{L\left(M + m\sin^2\left(\theta(t)\right)\right)} \tag{2}$$

with :

m:	mass of pendulum (0.025 kg)	L:	half a arm length (0.1 m)
M:	mass of cart (20 kg)	f:	dry friction (150 Nms/rad)
G:	power gain (67)	$u(t)$:	control force (N))
$\theta(t)$:	angle of pendulum (rad))	$X(t)$:	cart position (m)

A TSK fuzzy model with two rules :

$$\left\{\begin{array}{l} x(t+1) = \sum_{i=1}^{2} h_i\left(z(t)\right) A_i x(t) + B_i u(t) \\ y(t) = \sum_{i=1}^{2} h_i\left(z(t)\right) C_i x(t) \end{array}\right. \tag{3}$$

which represents the dynamics of the nonlinear system can be obtained with the following approximations :

- $\dot{\theta}^2(t)\sin\left(\theta(t)\right)$ is neglected, let us quote that if we want to use a model with rules taking into account $\dot{\theta}(t)$ then we will not have a separation principle available since, $\dot{\theta}(t)$ is not a measured variable (Ma et al., 1998);
- $m\sin^2\left(\theta(t)\right)$ is neglected with regard to M
- $\forall\theta(t) \in [-\theta_0\theta_0]\,, \left|\frac{1-\cos(\theta(t))}{1-\cos(\theta_0)} - \frac{\theta_0\cdot\theta(t)-\theta_0\cdot\sin(\theta(t))}{(\theta(t))\cdot(\theta_0-\sin(\theta_0))}\right| < 0.024$

With the state vector : $x(t) = \left[\theta(t)\ X(t)\ \dot{X}(t)\ \dot{\theta}(t)\right]^T$ the matrices are :

$$A_1 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ \frac{mg}{M} & 0 & \frac{-f}{M} & 0 \\ \frac{g}{L}\left(1+\frac{m}{M}\right) & 0 & \frac{-f}{ML} & 0 \end{bmatrix} B_1 = \begin{bmatrix} 0 \\ 0 \\ \frac{G}{M} \\ \frac{G}{ML} \end{bmatrix} A_2 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ \frac{mg\sin(\theta_0)\cos(\theta_0)}{M\theta_0} & 0 & \frac{-f}{M} & 0 \\ \frac{g}{L}\left(1+\frac{m}{M}\frac{\sin(\theta_0)}{\theta_0}\right) & 0 & \frac{-f\cos(\theta_0)}{ML} & 0 \end{bmatrix}$$

$$B_2 = \begin{bmatrix} 0 \\ 0 \\ \frac{G}{M} \\ \frac{G\cos(\theta_0)}{ML} \end{bmatrix} C_1 = C_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

When $\theta(t) = \pm\pi/2$, this model remains uncontrollable.

A discrete model of the TSK fuzzy model, with a 3 ms sampling time and for $\theta_0 = 1.2\,rad$, is given by :

$$A_{d1} = \begin{bmatrix} 1 & 0 & 0 & 0.003 \\ 0 & 1 & 0.003 & 0 \\ 0 & 0 & 0.9778 & 0 \\ 0.2947 & 0 & -0.2225 & 1 \end{bmatrix} \; A_{d1} = \begin{bmatrix} 1 & 0 & 0 & 0.003 \\ 0 & 1 & 0.003 & 0 \\ 0 & 0 & 0.9778 & 0 \\ 0.2643 & 0 & -0.1550 & 1 \end{bmatrix} \; B_{d1} = \begin{bmatrix} 0 \\ 0 \\ 0.0099 \\ 0.0994 \end{bmatrix}$$

$$B_{d2} = \begin{bmatrix} 0 \\ 0 \\ 0.099 \\ 0.0692 \end{bmatrix} \; C_{d1} = C_{d2} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

This mechanical device has a dead band non linearity caused by dry friction, which has not been taken into account in the inverted pendulum model. The oscillations around the equilibrium point $\theta_0 = 0\,rad$, viewed in most of the figures, are caused by this dead band.

A swinging sequence is used to allow any initial conditions on the angle (Wei et al., 1995). The different proposed control laws use this same swinging sequence to bring the angle to acceptable values before using the "around zero" control law.

When this condition is reached (acceptable angles), a control law is applied to stabilize the inverted pendulum around its unstable position. Different approaches are provided in the following sections.

PDC Fuzzy Controller and fuzzy observer

This part presents both PDC controller and fuzzy observer design based on Tanaka's work with separation principle of (Ma et al., 1998) as the premises variables are measurable. We have the following expressions :

Fuzzy observer :

$$\begin{cases} \hat{x}(t+1) = \sum_{i=1}^{2} h_i\left(\hat{z}(t)\right) A_i \hat{x}(t) + B_i u(t) + K_i\left(y(t) - \hat{y}(t)\right) \\ \hat{y}(t) = \sum_{i=1}^{2} h_i\left(\hat{z}(t)\right) C_i \hat{x}(t) \end{cases} \tag{4}$$

where F_i and K_i are respectively the control gain and observer gain matrices, $\hat{x}(t)$ is the predicted state vector, $y(t)$ and $\hat{y}(t)$ are respectively the fuzzy model and the observer output.

The dynamic of the whole closed loop system can be written :

$$x_a(t+1) = \sum_{i=1}^{r} \sum_{j=1}^{r} h_i\left(z(t)\right) h_j\left(z(t)\right) \begin{bmatrix} A_i - B_i F_j & B_i F_j \\ 0 & A_i - K_i C_j \end{bmatrix} x_a(t) \text{ with : } x_a(t) = \begin{bmatrix} x(t) \\ x(t) - \hat{x}(t) \end{bmatrix}$$

The F_i control gains and K_i observer gains can be obtained separately according to (Ma et al., 1998). In our case, the observer is computed by quadratic minimization. With the matrices :

$$Q_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 100 & 0 \\ 0 & 0 & 0 & 100 \end{bmatrix} \text{ and } R_0 = \begin{bmatrix} 0.001 & 0 \\ 0 & 0.001 \end{bmatrix} \text{ we obtain } K_{d1} = \begin{bmatrix} -0.0196 & 1.0930 \\ 1.0088 & -0.0169 \\ 3.2322 & -5.5763 \\ -6.8834 & 32.0953 \end{bmatrix} \text{ and}$$

$$K_{d2} = \begin{bmatrix} -0.0172 & 1.0749 \\ 1.0099 & -0.0152 \\ 3.5801 & -4.9947 \\ -6.0272 & 25.8077 \end{bmatrix}$$

The computed fuzzy controller is based on LMIs theorems given in (Tanaka et al., 1998). A result without added constraint is : $F_1 = [198.8803\ -26.5146\ -34.8360\ 6.6977]$, $F_2 = [208.9475\ -27.3146\ -32.0839\ 8.4961]$.

SSF Fuzzy Controller

This kind of control law is based on ideas found in (Petersen, 1987) and its synthesis is done in continuous time (Vermeiren, 1998; Vermeiren et al., 1999) and sufficient conditions of stability can be obtained. So we have to adapt it to discrete time for real time computation. This is obvious for the control law, but the use of a fuzzy observer raises the discretization problem of a non linear observer. In order to avoid this calculation, we approximate the cart and bar speeds at first order. In addition, in order to avoid big control values, a average filter is placed on the control output.

The control law is defined by : $u_{SSF}(x) = g_1(x) + g_2(x)$ with

$$g_1(x) = \min_{i \in (1,2)} \left(0, \frac{x^T Q_i x - \alpha \left\| x^2 \right\|}{2 B_i^T P x} - F_i x \right) \; for\, i\, such\, that\, B_i^T P x > 0$$

$$g_2(x) = \max_{i \in (1,2)} \left(0, \frac{x^T Q_i x - \alpha \left\| x^2 \right\|}{2 B_i^T P x} - F_i x \right) \; for\, i\, such\, that\, B_i^T P x < 0$$

Based on a LMIs formulation a result without added constraint is :

$F_1 = [96.1466 \; - 6.9250 \; - 23.7508 \; 12.2392]$,$F_2 = [89.0633 \; - 6.4152 \; - 21.4771 \; 11.3100]$,

$$Q_1 = \begin{bmatrix} 30280 & -2274 & -7192 & 3553 \\ -2274 & 173 & 541 & -267 \\ -7192 & 541 & 1712 & 844 \\ 3553 & -267 & 844 & 417 \end{bmatrix}, \; Q_2 = \begin{bmatrix} 13413 & -1051 & -3463 & 1579 \\ -1051 & 84 & 273 & 124 \\ -3463 & 263 & 903 & 408 \\ 1579 & 124 & 408 & 186 \end{bmatrix} \text{ and}$$

$$P_c = \begin{bmatrix} 517.07 & -39.94 & -125.96 & 60.795 \\ -39.94 & 7.715 & 11.07 & -4.725 \\ -125.96 & 11.07 & 33.569 & -14.96 \\ 60.795 & -4.725 & -14.96 & 7.677 \end{bmatrix}$$

Linear Quadratic Control

The weighting matrices have been chosen after several simulations to assure a good convergence of the state vector prediction error (in particular for the speed). When switching from the swinging control to the "around zero" control, we use first order approximation for the speed to avoid zero initial conditions on the speed observer. In order not to perturb the control law, we have to compute the speed values in the best way.

By minimizing the criteria we obtain the following weighting matrices based on the "around zero" linearized model, i.e. matrices (A_{d1}, B_{d1}, C_{d1}):

$$Q_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 100 & 0 \\ 0 & 0 & 0 & 100 \end{bmatrix} \text{ and } R_0 = \begin{bmatrix} 0.001 & 0 \\ 0 & 0.001 \end{bmatrix} \text{ is } K_d = \begin{bmatrix} -0.0196 & 1.0930 \\ 1.0088 & -0.0169 \\ 3.2322 & -5.5763 \\ -6.8834 & 32.0953 \end{bmatrix}$$

For the control law computation, matrices Q_0 and R_0 are chosen to conserve a great deal between control quality and control values. The following matrices are obtained by minimizing the criteria $J = \sum_t \left(y(t)^T Q_d y(t) + r_d u(t)^2 \right)$

$$D_d = \begin{bmatrix} 1 & 0 \\ 0 & 50 \end{bmatrix}, \; r_d = 0.001 \text{ and } F = [198.8803 \; - 26.5146 \; - 34.8360 \; 6.6977]$$

For these three control laws, stability results in simulation, on the TSK fuzzy model can be derived. They show that the linear control law is always the less robust, to initial conditions and to parametric variations (mass, pendulum length, ...). For the two other control laws, PDC and SSF, the performances are nearly the same according to initial conditions, SSF law being more efficient to parametric variations (Vermeiren, 1998). These results are confirmed in the next sections by real time trials.

Low Cardinality Rule Based Fuzzy Controller

A very interesting kind of fuzzy controller is given in (Vidolov, 1993), and was applied with success, in simulation, on an inverted pendulum. This controller uses a rule base of low cardinality given by :

$$\begin{cases} R_1 : If \left[\begin{array}{c} \theta(t) \text{ is positive and } \Delta\theta(t) \text{ is positive} \\ \text{OR } X(t) \text{ is positive and } X\Delta(t) \text{ is positive} \end{array} \right] & \text{then the control value is positive} \\ R_2 : If \left[\begin{array}{c} \theta(t) \text{ is negative and } \Delta\theta(t) \text{ is negative} \\ \text{OR } X(t) \text{ is negative and } X\Delta(t) \text{ is negative} \end{array} \right] & \text{then the control value is negative} \end{cases}$$

Membership functions use a tan formulation, and reasoning method uses a product triangular norm as AND operator and a co-norm triangular as OR operator. With : $\alpha_1(t) = f_{\theta P}(\theta(t)) \cdot f_{\Delta\theta P}(\Delta\theta(t))$; $\alpha_2(t) = f_{\theta N}(\theta(t)) \cdot f_{\Delta\theta N}(\Delta\theta(t))$; $\beta_1(t) = f_{XP}(X(t)) \cdot f_{\Delta XP}(\Delta X(t))$; $\beta_2(t) = f_{XN}(X(t)) \cdot f_{\Delta XN}(\Delta X(t))$ numerical output values are given by :

$$u(t) = \left\{ G \left[\min\left(1, \alpha_1(t) + \lambda\beta_1(t)\right)\right]^{q^{-1}} - \left[\min\left(1, \alpha_2(t) + \lambda\beta_2(t)\right)\right]^{q^{-1}} \right\}$$

with G a normalization gain and λ a coupling parameter.

We can see the achievement of the coupling parameter λ, the q parameter and the 5 scaling factors a, b, c, d and G as an optimization problem in $\mathbb{R}$. To avoid tuning these parameters by trial and error, an optimization method based on simulated annealing is used. Among these different stochastic methods, enhanced simulated annealing with heating cycle (Bonnemoy and Hamma, 1991) is a good one for dealing with optimization problems with lots of local optima. The main purpose of this method, applied in our case to the inverted pendulum, is the minimization of both angle and position errors, including either a view on the power of the control value. In order to avoid a multi-criteria method, a global performance criteria is chosen. In addition it takes into account several initial conditions. A criteria J is defined as follow :

$$i \in \{1, \ldots, 3\}, \; J = \max\left(J_i\left(\theta_i\right)\right) \text{ and } \theta_i = \frac{\pi}{8} \cdot i \text{ with : } \max\left(J_i\left(\theta_i\right)\right) = \sum_t \left(t \left[\alpha\theta^2(t) + \beta\left(X_c - X(t)\right)^2\right] + \delta\left(u(t) - u(t-1)\right)^2 \right)$$

It corresponds, for each sub-criteria, to the ITSE criteria with an added weight on the control value. With $\alpha = 0.7$, $\delta = 0.05$ which indicates clearly that the error angle is preferred, the results are the following : $a = 0.2568$ $b = 4.0176$ $c = 0.1523$ $d = 0.2250$ $q = 0.8847$ $\lambda = 0.5393$ and $G = 12.2610$.

Comparison Between Different Control Methods

It's really difficult to make an unbiased comparison between different control laws. These ones come from very different areas and can't be synthesized in the same way. We should also take into account the development time for each of these methods. In the state feedback case (PDC or SSF), we must note that robustness constraints can be added easily with LMI's, for output feedback laws by using an optimization on the fuzzy controller ...

Nevertheless, starting from the same optimized pendulum model, the development time for all the methods is nearly the same (about one day without taking into account unavoidable trials and errors and computing implementation of the algorithms). In that context, we can use several criteria among the classical ISE, ITSE, IAE,... to make a quantitative comparison.

Some figures illustrate the different results obtained, figure 1 shows a result made in the nominal case for a linear law u_{lin} with initial conditions $x(0) = [0.4\ 0\ 0\ 0]^T$. All curves are shown on 9s duration. We can notice that the dead band effects are the tiny oscillations of the bar and the cart around zero.

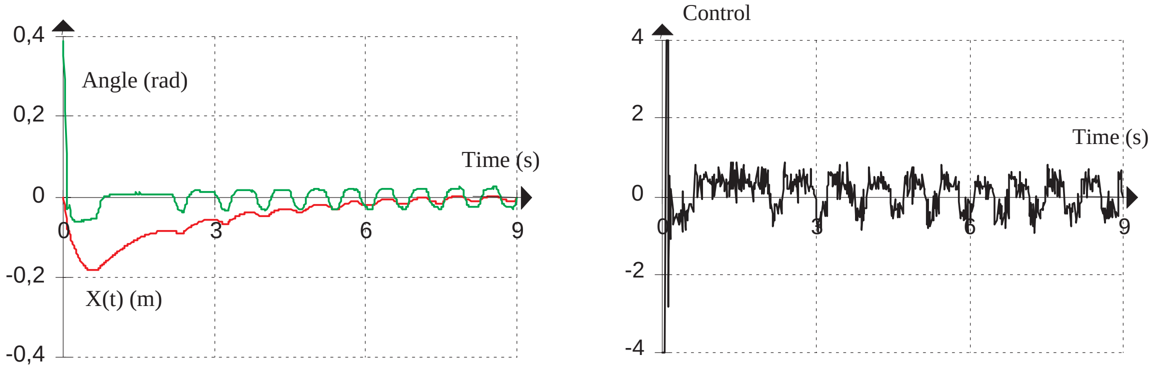

Figure 1: Law u_{lin} with $x(0) = [0.4\ 0\ 0\ 0]^T$

To illustrate the swinging sequence which allows to bring up the pendulum from its stable position, a result with a law u_{PDC} with initial conditions $x(0) = [0.4\ 0\ 0\ 0]^T$ is shown figure 2.

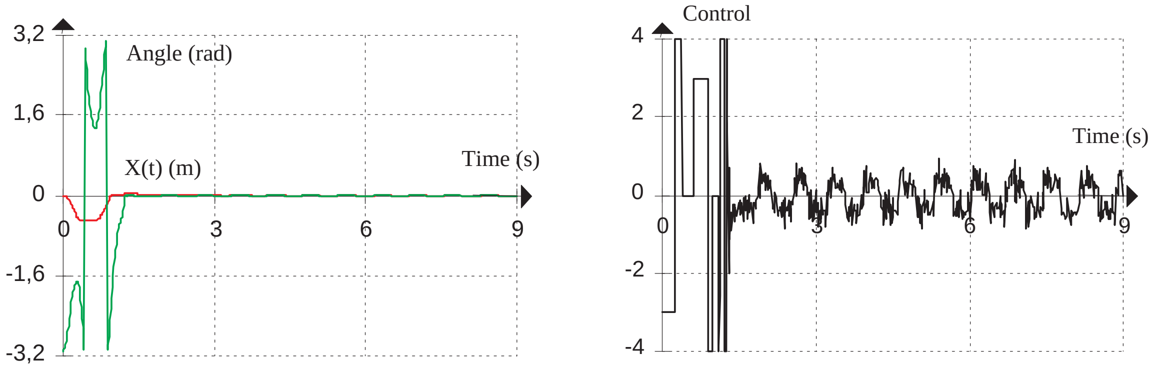

Figure 2: Law u_{PDC} with $x(0) = [\pi\ 0\ 0\ 0]^T$

Some tests of robustness are made using three different bars of different length and weight. The trials made with these three bars are given in tables 1 and 2. In the first table, only robustness is taken into account according to initial conditions around $0\,rad$, there is no swinging sequence.

	u_{lin}	u_{PDC}	u_{SSF}	u_{RF}
$\begin{cases} m = 0.025\,kg \\ L = 0.1\,m \end{cases}$	$[-0.8; 0.8]$	$[-0.8; 0.85]$	$[-0.8; 0.85]$	$[-0.8; 0.85]$
$\begin{cases} m = 0.5\,kg \\ L = 0.1\,m \end{cases}$	$[-0.75; 0.8]$	$[-0.75; 0.85]$	$[-0.8; 0.85]$	$[-0.8; 0.85]$
$\begin{cases} m = 0.2\,kg \\ L = 0.2\,m \end{cases}$	$[-0.65; 0.7]$	$[-0.7; 0.75]$	$[-0.7; 0.75]$	$[-0.7; 0.7]$

Table 1: Stability Domain according to initial conditions around $0\,rad$

The interval dissymetry can be explained by the cart conveyor belt which has a different length according to the tracking direction. As initial conditions are concerned, we can notice that for the three fuzzy control laws, the differences are not significant, only u_{lin} seems to be less effective.

To illustrate the different behaviors of these laws, the three following criteria have been computed for a $0.7\,rad$ initial condition. This value can be applied to the three different bars and the four control laws :

$$J_{ISE}\left(\theta(t)\right) = \sum_t \theta^2(t) \; J_{ISE}\left(X(t)\right) = \sum_t \left(X(t) - X_c\right)^2 \; J_{ITAE}\left(\theta(t)\right) = \sum_t t|\theta(t)| \; J_{ITAE}\left(X(t)\right) = \sum_t t|X(t) - X_c|$$

Stabilization time : t_{stab}, $\forall t > t_{stab}$ $|u(t)|$, $|\theta(t)|$, $|X(t) - X_c|$ centered on 0 and limited. t_{stab} corresponds to the time when the control value becomes centered around the dead band. Table 2 sums up these results.

	u_{lin}	u_{PDC}	u_{SSF}	u_{RF}
$\begin{cases} m = 0.025\,kg \\ L = 0.1\,m \end{cases}$	$J_{ISE}\left(\theta(t)\right) = 3.4$ $J_{ISE}\left(X(t)\right) = 34$ $J_{ITAE}\left(\theta(t)\right) = 11$ $J_{ITAE}\left(X(t)\right) = 29$ $t_{stab} = 7.5s$	$J_{ISE}\left(\theta(t)\right) = 3.8$ $J_{ISE}\left(X(t)\right) = 42$ $J_{ITAE}\left(\theta(t)\right) = 11.5$ $J_{ITAE}\left(X(t)\right) = 32$ $t_{stab} = 8s$	$J_{ISE}\left(\theta(t)\right) = 3.2$ $J_{ISE}\left(X(t)\right) = 35$ $J_{ITAE}\left(\theta(t)\right) = 3.9$ $J_{ITAE}\left(X(t)\right) = 62$ $t_{stab} = 9s$	$J_{ISE}\left(\theta(t)\right) = 8.6$ $J_{ISE}\left(X(t)\right) = 6$ $J_{ITAE}\left(\theta(t)\right) = 26$ $J_{ITAE}\left(X(t)\right) = 26$ $t_{stab} = 4.5s$
$\begin{cases} m = 0.5\,kg \\ L = 0.1\,m \end{cases}$	$J_{ISE}\left(\theta(t)\right) = 3$ $J_{ISE}\left(X(t)\right) = 26$ $J_{ITAE}\left(\theta(t)\right) = 10$ $J_{ITAE}\left(X(t)\right) = 31$ $t_{stab} = 8s$	$J_{ISE}\left(\theta(t)\right) = 2.9$ $J_{ISE}\left(X(t)\right) = 26$ $J_{ITAE}\left(\theta(t)\right) = 10$ $J_{ITAE}\left(X(t)\right) = 29$ $t_{stab} = 8s$	$J_{ISE}\left(\theta(t)\right) = 3.9$ $J_{ISE}\left(X(t)\right) = 38$ $J_{ITAE}\left(\theta(t)\right) = 4$ $J_{ITAE}\left(X(t)\right) = 64$ $t_{stab} = 9s$	$J_{ISE}\left(\theta(t)\right) = 7.5$ $J_{ISE}\left(X(t)\right) = 4.1$ $J_{ITAE}\left(\theta(t)\right) = 13$ $J_{ITAE}\left(X(t)\right) = 13$ $t_{stab} = 7.5s$
$\begin{cases} m = 0.2\,kg \\ L = 0.2\,m \end{cases}$	$J_{ISE}\left(\theta(t)\right) = 8$ $J_{ISE}\left(X(t)\right) = 47$ $J_{ITAE}\left(\theta(t)\right) = 16$ $J_{ITAE}\left(X(t)\right) = 36$ $t_{stab} = 8s$	$J_{ISE}\left(\theta(t)\right) = 5$ $J_{ISE}\left(X(t)\right) = 54$ $J_{ITAE}\left(\theta(t)\right) = 11$ $J_{ITAE}\left(X(t)\right) = 27$ $t_{stab} = 5s$	$J_{ISE}\left(\theta(t)\right) = 4.8$ $J_{ISE}\left(X(t)\right) = 80$ $J_{ITAE}\left(\theta(t)\right) = 2$ $J_{ITAE}\left(X(t)\right) = 84$ $t_{stab} = 9s$	$J_{ISE}\left(\theta(t)\right) = 27$ $J_{ISE}\left(X(t)\right) = 30$ $J_{ITAE}\left(\theta(t)\right) = 61$ $J_{ITAE}\left(X(t)\right) = 34$ $t_{stab} = ?s$

Table 2: Computation of the three criteria for a $0.7\,rad$ angle initial condition

The results are quite different. This shows that if the control values have good robustness performance according to similar initial conditions, table 2, the pendulum behavior is different.

According to the results, we notice that the robustness of the laws with regard to the bar mass is very good. On the contrary the robustness of the laws with regard to bar length is quite poor (4th line table 2). To illustrate this remark we focus on these curves in figures 3 to 6 for a $0.7 rad$ angle initial condition. We can notice that the strategy of the four laws are different. u_{PDC} and u_{SSF} laws give more importance to the angle than to the position, figure 4 et 5, and the u_{RF} law operates differently, figure 6. It tries to conserve a great deal between angle and position, but the resulting control values becomes unacceptable when the bar parameters are modified.

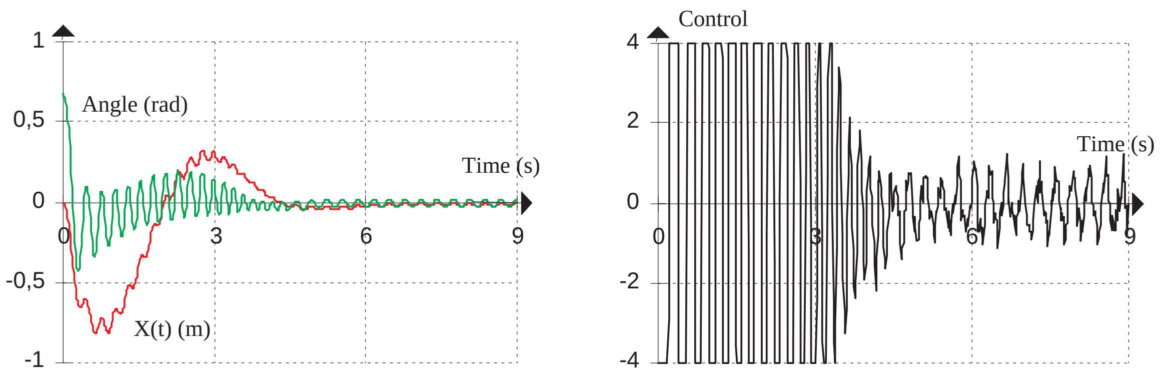

Figure 3: Law u_{lin} with $\begin{cases} m = 0.2\,kg \\ L = 0.2\,m \end{cases}$ et $x(0) = [0.7\;0\;0\;0]^T$

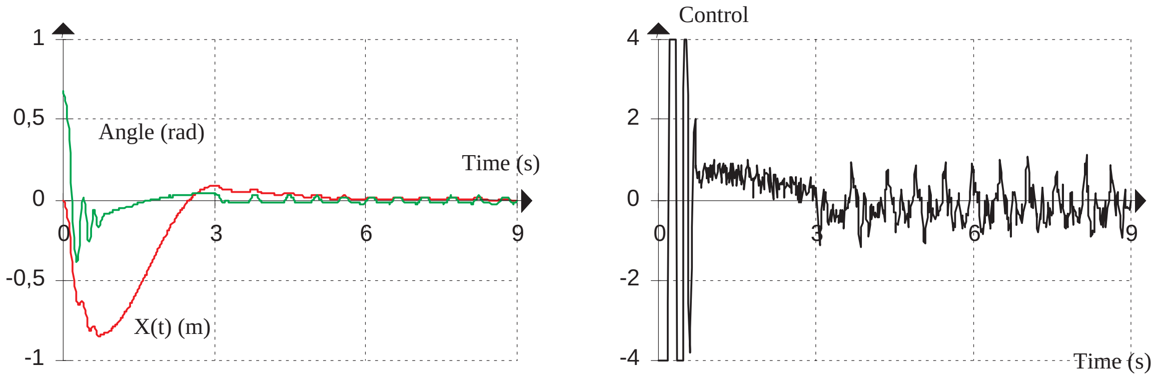

Figure 4: Law u_{PDC} with $\left\{\begin{array}{l} m = 0.2\,kg \\ L = 0.2\,m \end{array}\right.$ et $x(0) = [0.7\ 0\ 0\ 0]^{T\ T}$

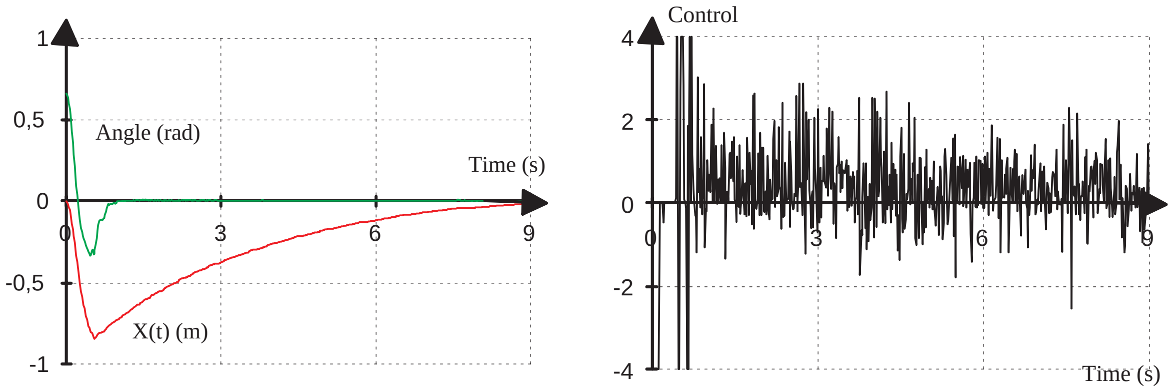

Figure 5: Law u_{SSF} with $\left\{\begin{array}{l} m = 0.2\,kg \\ L = 0.2\,m \end{array}\right.$ et $x(0) = [0.7\ 0\ 0\ 0]^{T}$

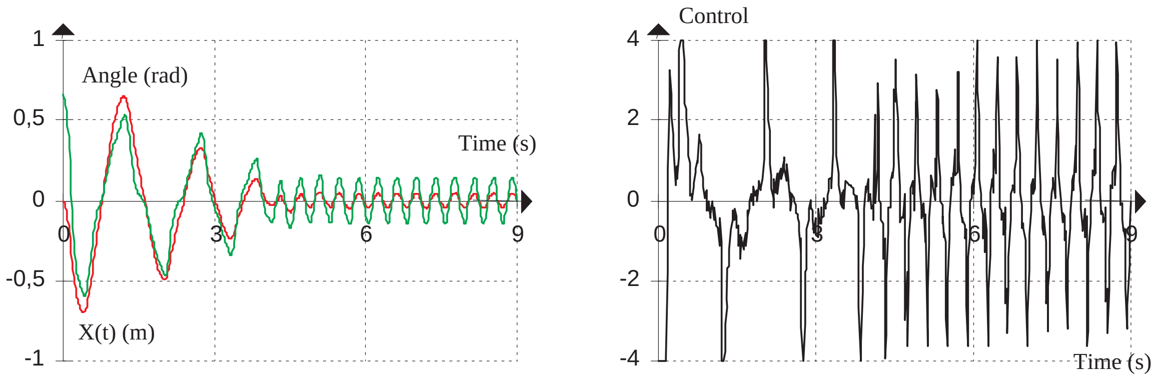

Figure 6: Law u_{RF} with $\left\{\begin{array}{l} m = 0.2\,kg \\ L = 0.2\,m \end{array}\right.$ et $x(0) = [0.7\ 0\ 0\ 0]^{T}$

In a second part, as the robustness with any initial conditions is concerned, in the trials we use the swinging sequence with the three different bars, table 3.

	u_{lin}	u_{PDC}	u_{SSF}	u_{RF}
$\left\{\begin{array}{l} m = 0.025\,kg \\ L = 0.1\,m \end{array}\right.$	$[-\pi; -2.9]$ $[2.9; \pi]$	All	All	All
$\left\{\begin{array}{l} m = 0.5\,kg \\ L = 0.1\,m \end{array}\right.$	$[-\pi; -2.9]$ $[2.9; \pi]$	All	All	All
$\left\{\begin{array}{l} m = 0.02\,kg \\ L = 0.2\,m \end{array}\right.$	Impossible	$[-\pi; -2.95]$ $[3.1; \pi]$	$[-\pi; -2.8]$ $[2.9; \pi]$	$[-\pi; -2.9]$ $[2.9; \pi]$

Table 3: Stability Domains according to initial conditions around $\pi\,rad$

For the mass variation, the results join those obtained in simulation which pointed out the robustness of the u_{PDC} and u_{SSF} laws according to this parameter. We notice that u_{lin} has worse performances and u_{SSF} law raises up.

We have to notice that u_{RF} has a good behavior, quite equal to the two fuzzy feedback laws results. Nevertheless these laws are preferred because their computations are based on a fuzzy model of the process.

Finally to show the robustness according to an external disturbances (a hit on the bar), a u_{SSF} law result is given in figure 7.

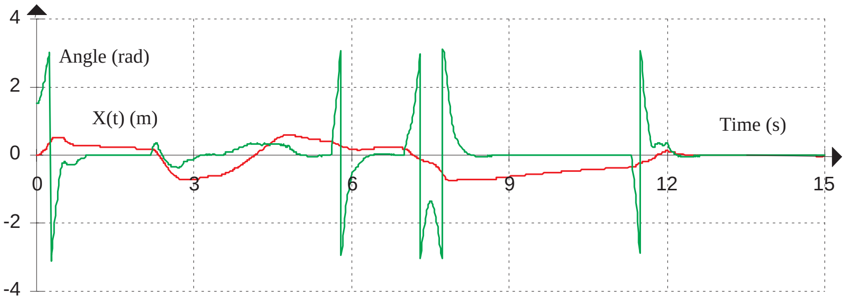

Figure 7: Law u_{SSF} with external disturbances

The initial condition is $x(0) = \left[\frac{\pi}{2}\ 0\ 0\ 0\right]^T$, we hit the bar at time 2s, and after it is continually pushed during 1s which explain the "flat" behavior of the angle response between time 4s and 5s. More violent hits which constrains the bar to make a complete rotation before stabilization, are applied to the bar at 5.5s, 7s and 11s.

Conclusion

This application compared different control laws on an experimental real device. This study made on the inverted pendulum has raised up the efficiency of the SSF approach which allowed to control angle and position on the process with nominal parameters for any initials conditions respectively on the intervals $[-\pi\,rad, \pi\,rad]$ and $[-0.8\,m, 0.8\,m]$. Nevertheless the PDC type fuzzy controllers and low cardinality rule based fuzzy controllers give good results with regard to initial conditions. Finally, both in simulation and on real process, the best behavior is obtained by the SSF law according to disturbances on the bar and change ofthe pendulum parameters.

The interest of TSK fuzzy model was to obtain with LMI synthesis approaches sufficient conditions for the stability closed loop model, for PDC and SSF laws. Nevertheless the pendulum problem with initial conditions greater than $\frac{\pi}{2}$ and without swinging sequence has to be solved.

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