Stability Analysis of a Speed Fuzzy Control System for DC Drives

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ABSTRACT: This paper presents a stability analysis of a fuzzy speed control systems for dc drives. The fuzzy controllers for any kind of rule base, inference method and defuzzification methods have nonlinear characteristics. We present some properties of the rule base and of the fuzzy controller: commutative law, neutral element, and symmetric elements. These properties and the forms of the characteristics of the fuzzy controller suggest using of circle's stability criterion, which is a general valid method for any kind of fuzzy controllers. The stability analysis serves to choose the value of the gain factor of the open loop control system.

KEYWORDS: fuzzy control, stability analysis, circle's criterion, d. c. drives.

1.INTRODUCTION

In the last year many papers presented speed control systems of electric drives based upon fuzzy logic [Bose 1994, Guillermin 1996, Volosencu 1998]. A stability analysis is necessary is this case. We present in this paper a stability analysis based upon circle's criterion, which serve to choose the gain factor of the open loop control system.
The structure of the speed fuzzy control system of the dc drive is presented in Fig. 1.

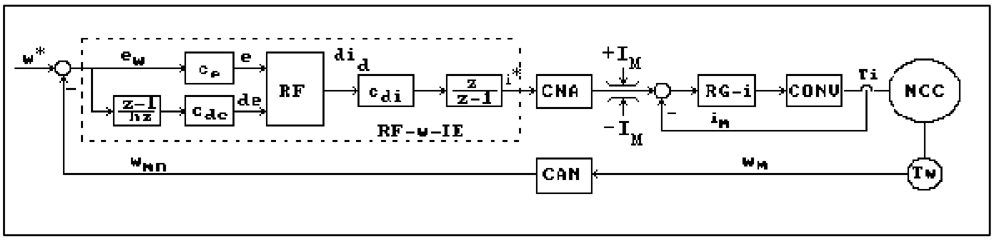

Figure 1. The speed fuzzy control system

The control system includes: the dc motor (*MCC*), fed with a current control system, made with a current PI linear controller (*RG-i*), a current sensor (*Ti*) and a power converter (*CONV*), the speed sensor (*T*ω), the analog to digital and digital to analog converters (*CAN*, *CNA*) and a fuzzy PI speed controller with output integration (*RF*-ω-*IE*).
The fuzzy speed controller (*RF*-ω-*IE*) has a fuzzy controller (*RF*), which includes the fuzzification, the inference and the defuzzification. The fuzzy controller (*RF*) has two inputs: *e* and *de* and one output di_d. At the controller's inputs there are two input factors c_e and c_{de}. The fuzzy controller has a derivation at the input and integration at the output. At the output of the fuzzy controller there is a gain compensation factor c_{di} of the open loop control system. The speed fuzzy controller (*RF*-ω-*IE*) gives the reference value of the current i^*.
We chose the following universes of discussion. The universe of discussion for the current increment is $di_d \in [-I_M, I_M]/I_N$, were I_M is the maximum motor current and I_N is the rated current. The universe of discussion for the speed error $e \in [-2\Omega_b, 2\Omega_b]/\Omega_b$, were Ω_b is the base speed. And the universe of discussion for the error derivative $de \in [-M_{Mt}/J, M_{Mt}/J]/(M_N/J)$, were M_N and M_{Mt} are the rated and the maximum torque and J is the inertial moment.

The input factors of the fuzzy controller may be chosen by a natural choice: $c_e=2/\Omega_b$, and $c_{de}=1/(M_{Mt}J)$, based on the static speed-torque characteristic and the second law of dynamics. Also, they must be chosen making equivalence between a fuzzy PI controller and a linear PI controller.

The current controller is a linear PI controller, designed based on Kessler's, module criterion, with the method took from [Leonhard 1985]. For the motor we took the linear equations.

The control system parameters are as following [3]. For the motor: P_N=1 kW, n_N=3000 rot/min, u_{aN}= 220 V, I_N =5 A, I_M=10,8 A, M_N=3,1 Nm, M_{Mt}=3,4 Nm, rotor resistance R_a=2,01 Ω, rotor inductance L_a=27mH. For the power converter: $H_{CONV}(s)=22/(1+0{,}001s)$. For the current sensor: $H_{Ti}(s)=1/(1+0{,}005s)$. For the speed sensor: $H_{Tw}(s)=0{,}1/\pi(1+0{,}01s)$. For the analog to digital and digital to analog converters: $K_{CAN}=2^{11}/10=1/K_{CNA}$. Also, we chose h=0,003 s. The current controller parameters are: K_{Ri}=0,5 and T_{Ri}=14 ms.

We made simulations using Matlab with Fuzzy Toolbox and Simulink [Mohan 1994].

CHARACTERISTICS OF THE FUZZY CONTROLLER

INTRODUCTION

A simple rule base with only 9 rules, the sum-prod inference and defuzzification with center of gravity are chosen [Buhler 1994, Volosencu 1997]. This controller is called a basic one. For the basic controller we illustrate some algebraic proprieties, which are valid also for more complex controllers with different rule bases, inference methods, membership functions and defuzzification methods.

For a fuzzy controller with 9 rules, with the rule base from Fig. 2, we chose the membership functions from Fig. 3.

di_d		e		
		NB	ZE	PB
de	NB	NB_2	NB_4	ZE_6
	ZE	NB_8	ZE_1	PB_9
	PB	ZE_7	PB_5	PB_3

Figure 2. The rule base

For the basic controller we illustrate the following properties: the commutative law of the rule base, the existence of the symmetric elements for the rule base, the commutative law of the fuzzy controller, the existence of the symmetric elements for the fuzzy controller. The stability analysis is made based on these properties.

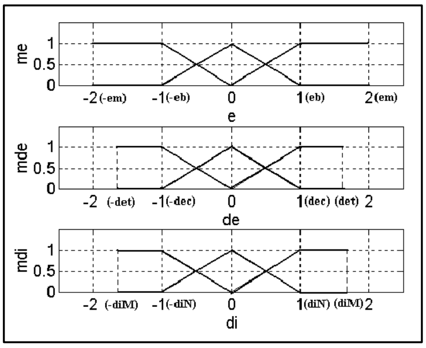

Figure 3. The membership functions

ALGEBRAIC PROPERTIES OF THE RULE BASE

The fuzzy controller has two input variable *e* and *de* and one output variable *di*. The fuzzy variable are defined on the following sets:

$$e \in E,\ de \in D_e,\ di \in D_i \tag{1}$$

We take in consideration the cases when the definition sets are equal:

$$E = D_e = D_i = U \tag{2}$$

The fuzzy set *U* has for the 9 rules base the following fuzzy values:

$$U_3 = \{NB,\ ZE,\ PB\} \tag{3}$$

With this sets we define the application:

$$f : UxU \to U,\ (e,de) \to f(e,de) \tag{4}$$

which we call a fuzzy composition law defined over the set *U*. The unique defined element $di=f(e,de)\in U$ through the application *f* we call the composed of *e* and *de* though the law *f*. A rule table describes the law *f*. We use the symbol * for the composition law *f*:

$$UxU \to U,(e,de) \to e * de \tag{5}$$

Commutative law:

$$e * de = de * e,\ \forall e, de \in U \tag{6}$$

The neutral element: An element $o\in U$ is called neutral for a composition law Ux$U\to U$, $(e, de)\to e*de$ if

$$o * x = x * o = x,\ \forall x \in U \tag{7}$$

The neutral element is unique. We notice from the rule table that the neutral element is the fuzzy value *ZE*.

Symmetric elements: An element $x\in U$ is called symmetric through the composition rule Ux$U\to U$, $(e, de)\to e*de$, (with a neutral element) if there is $x'\in U$ with the property:

$$x' * x = x * x' = o \tag{8}$$

Then $x'\in U$ is called the symmetric of x..
We notice the symmetric element is the opposed element: *NB*PB=ZE.*

ALGEBRAIC PROPERTIES OF THE FUZZY CONTROLLER

The fuzzy controller has two input real variable *e* and *de* and an output real variable di_d. It is implemented using the sum-prod inference and the defuzzification with the center of gravity. This controller has the attached application:

$$di_d = f(e, de) \tag{9}$$

The variable *e*, *de* and di_d take values on the real universes of discussion: *E*, D_e and D_i. We considered a general scaled universe of discussion: $E=D_e=D_i=U=[-1, 1]$. The application *f* is defined on the Cartesian product *U*x*U* with values in *U*:

$$f : UxU \to U,\ (e, de) \to f(e, de) \tag{10}$$

It is called the fuzzy composition law on *U*. The unique element $di_d = f(e,de) \in U$ defined through the application *f* is called the composed of *e* and *de* through the composition fuzzy law *f*. An extended universe of the scaled basic universe $U=[-1, 1]$ may be $\underline{U}=[-a, a]$, with $|a|>1$. If the fuzzy controllers are digital implemented the universes of discussion *E*, D_e, D_i and *U* are discrete. A discrete universe of discussion *U* has a finite number of discrete elements $u_i \in U$, $i=1,...,n$: $U=\{u_1, u_2,...,u_n\}$. A composition law, which describes a fuzzy controller, may be represented as a three-dimension surface (Fig. 4). We mark the composition law attached to the fuzzy controller with $\circ$:

$$UxU \to U, (e, de) \to e \circ de \tag{11}$$

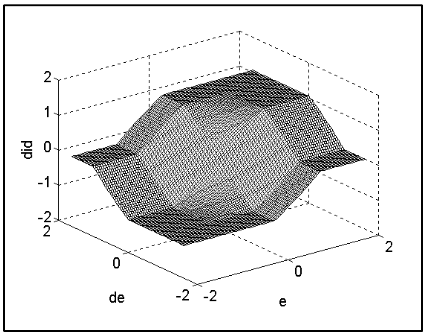

Figure 4. The controller surface

The fuzzy composition law has the following algebraic properties.
Commutative law.

$$e \circ de = de \circ e,\ \forall e, de \in U \tag{12}$$

An operation of inference and an operation of defuzzification compose the application *f*. We mark the inference with f_I and the defuzification with *d*. These functions are defined as follow. The inference function is:

$$di = f_I(e, de),\ e, de \in [-1, 1]$$

were *di* results as a fuzzy set of elements di_i, i=1,...,n: $di=\{di_1, di_2,...,di_n\}$. The value di_i are membership values resulted after the inference. The defuzzification function is uniquely defined:

$$di_d = d(di),\ di = \{di_i\},\ i = 1,...,n,\ di_d \in U \tag{14}$$

By the composition of two functions f_I and *d* the fuzzy controller results as the composed function:

$$di_d = d(f_I(e, de)),\ e, de, di_d \in U \tag{15}$$

The inference may be defined using the basic operation of summation and product. For example, for a 3-3-rule base we may write the following fuzzy relation:

$$\begin{aligned} di = {} & \{[m_{NB}(e) \wedge m_{NB}(de)] \wedge di_{NBi} | i = 1,...,n\} \vee \{[m_{ZE}(e) \wedge m_{NB}(de)] \wedge di_{NBi} | i = 1,...,n\} \vee ... \\ & \vee \{[m_{PB}(e) \wedge m_{PB}(de)] \wedge di_{PBi} | i = 1,...,n\} \end{aligned} \tag{16}$$

were

$$\begin{aligned} di_{NB} &= \{di_{NBi} = m_{NB}(di_{di}) | di_{di} \in U,\ i = 1,...,n\}, \\ di_{ZE} &= \{di_{ZEi} = m_{ZE}(di_{di}) | di_{di} \in U,\ i = 1,...,n\}, \\ di_{PB} &= \{di_{PBi} = m_{PB}(di_{di}) | di_{di} \in U,\ i = 1,...,n\} \end{aligned} \tag{17}$$

m_{NB}, m_{ZE}, m_{PB} are membership functions $m:U\to[-1, 1]$.
In the above relation we used the symbol $\wedge$ for product and the symbol $\vee$ for summation. The conclusion has associated the summation.
We may notice that for the existence of the commutative law of the membership functions must be equal:

$$m_{NB}(e) = m_{NB}(de),\ \ m_{ZE}(e) = m_{ZE}(de),\ \ m_{PB}(e) = m_{PB}(de),\ pt.\ \forall e = de \in U \tag{18}$$

If the defuzzification function *d* is univocal defined, for the same output fuzzy set of *di* results an unique defuzzified value di_d and $di_d=d(f_I(e,de))= d(f_I(de,e))$.
The neutral element. We call an element $o\in U$ a fuzzy neutral element for a composition law $UxU\to U$, $(e, de)\to e\circ de$, if

$$o \circ x = x \circ o = x,\ \forall x \in U \tag{19}$$

The neutral element is uniquely defined. The neutral element for the fuzzy composition rule of the fuzzy controller is the real number 0.
In Fig. 5 we present the characteristics of the fuzzy control $di_d=f(e)$, with *de* as parameter. We notice that the controller characteristic has the property $f(x, 0)=f(0, x)$=x.

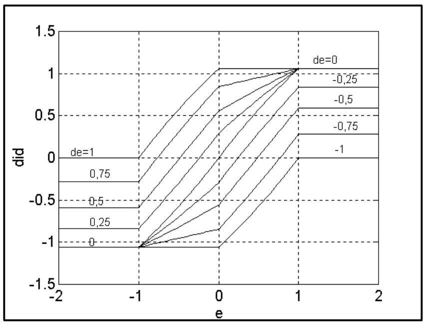

Figure 5. The controller characteristics

Symmetric elements. An element $x \in U$ is called symmetric related to the fuzzy composition rule (with neutral element) $U \mathrm{x} U \rightarrow U$, $(e, de) \rightarrow e \circ de$, if there is $x' \in U$ such as

$$x' * x = x * x' = o \tag{20}$$

Then the unique element $x' \in U$ with the property $x \circ x' = x' \circ x = o$ is called the symmetric of x, related to the operation $\circ$. We notice the symmetric of an input element is its opposed. The graphic of the function $f(x, -x)$ for all the value from the discussion universe is a 0 line.

The fuzzy composition function is a continuous function. The inference function f_I, resulted from the sum-prod inference is continuous over the universes of discussion U. The defuzzification function d is also continuous. So the composed function $d(f_I(e,de)$ is continuous. In our case there are not such points of discontinuity.

THE MAIN PROPERTY OF THE FUZZY CONTROLLER

The fuzzy controller has the nonlinear characteristic

$$di_d = F(x_e) \tag{21}$$

We may transform it, based on the above properties, in an interesting one:

$$di_d = K_R(e,de)(e+de), \ with \ \ 0 < K_R(e,de) < K_{\max} \tag{22}$$

This property helps us to use the circle criterion in the stability analysis of the fuzzy control system. Also, we may develop a method to choose the value of the open loop gain c_{di} for different fuzzy controllers with the above property.

The same properties are valid for others fuzzy controllers with a larger number of rules, with min-max inference and for the fuzzy controllers which use other defuzzification methods.

To use the main property demonstrated above we do a linear transformation of the state space. For this purpose we do a linear transformation of the input variables:

$$x_t = [x_{t1} \quad x_{t2}]^T = Tx_e = T[e \quad de]^T = [e+de \quad 0]^T, \quad x_e = T^{-1}x_t \tag{23}$$

The linear transformation gives:

$$di_d = K_R[1 \quad 1]x_e = K_R[1 \quad 1]T^{-1}x_t = K_R.(e+de) = K_R x_{t1} \tag{24}$$

were:

$$[1 \quad 1]T^{-1} = [1 \quad 0] \tag{25}$$

And for the transformation matrix results:

$$T = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \tag{26}$$

Now the nonlinear characteristic of the fuzzy controller may be expressed as follows:

$$di_d = K_R(x_t)x_{t1} \tag{27}$$

with

$$K_R = f(x_{t1}) = \frac{di_d}{x_{t1}} = \frac{di_d}{e+de} \tag{28}$$

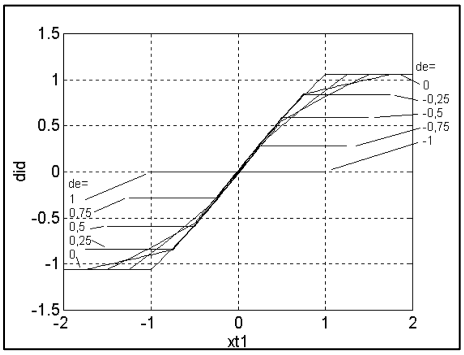

Figure 6. The translated characteristics

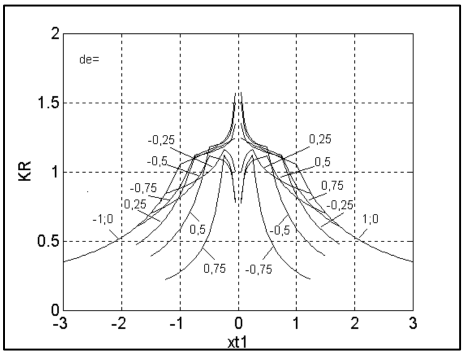

Figure 7. The gain characteristics

For the fuzzy controllers we present the following characteristics: $di_d=f(x_{t1})$ and $K_R=K_R(x_{t1})=di_d/x_{t1}$, with *de* as a parameter. The values of this parameter are -1, -0,75, -0,5, -0,25, 0, 0,25, 0,5, 0,75 and 1. The characteristics $di_d=f(x_{t1})$ presented in Fig. 6 are the translated characteristics from Fig. 5. The characteristics $K_R=K_R(x_{t1})=di_d/x_{t1}$ presented in Fig. 7 are obtained from the translated characteristics from Fig. 6. We notice that the translated characteristics are only in the first and the third quadrant of the coordinate plane, and the gain K_R has only positive values $K_R \in [0, K_{Rm}]$.

STABILITY ANALYSIS

After the analysis of the above characteristics we may say that the fuzzy controller has a nonlinear character for all kind of base rules, inference methods and defuzzification methods. The form of the above characteristics and the property of these controllers suggest the using of circle's criterion to analyze the stability [7]. In the stability analysis we use a block diagram of the fuzzy control, system in continuous time, as we shown in Fig. 8.

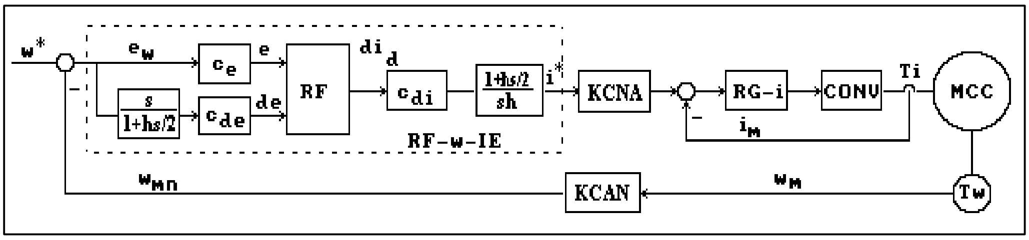

Figure 8. The control structure for stability analysis

To apply circle's criterion we transform the control system structure from Fig. 8 in an adequate one, which mach the standard structure [Voicu 1986] from Fig. 9. It has a linear part *L* and a nonlinear part $K_R(e,de)$. The linear part has the state variables *x*. *e* and *de* are two of these state variables. The structure from the controller's input is presented in Fig. 10.

In the block diagram from Fig. 9 we passed the gain and derivation blocks before the summation point. We obtained at the fuzzy controller *RF* input the structure from Fig. 11.The input variable is now $x_{t1}=e+de$. The nonlinear characteristic is a product between the input x_{t1} and the nonlinear function $K_R(e,de)$, that depend on the state variable *e* and *de*. The stability analysis is made for $w^*=0$ and $c_{di}=1$.

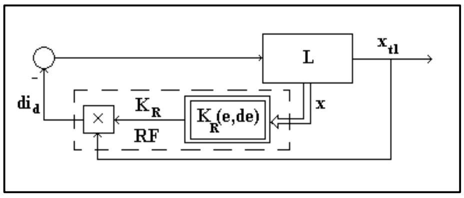

Figure 9.The standard diagram for the circle's criterion

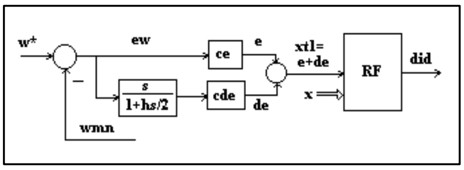

Figure 10. The summation at the input of the fuzzy controller

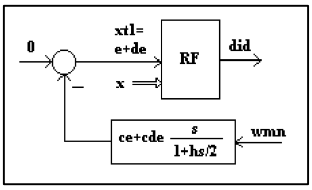

Figure 11. The structure at the input of the controller

Comparing the structures from Fig. 8 and Fig. 1 we easy can find which components are parts of the linear block L. The linear part has the transfer function:

$$H(s)=c_{di}\frac{1+hs/2}{hs}K_{CNA}H_i(s)k_m\frac{1}{Js+k_f}H_{Tw}(s)K_{CAN}\left(c_e+c_{de}\frac{s}{1+hs/2}\right)= \\ =\frac{1+hs/2}{hs}H_i(s)\frac{k_m}{Js+k_f}H_{Tw}(s)\left(c_e+c_{de}\frac{s}{1+hs/2}\right) \tag{29}$$

were $K_{CAN}=1/_{KCNA}$ and $c_{di}=1$. $H_i(s)$ is the equivalent transfer function of the current control loop.

The nonlinear function $K_R(e,de)$ has the property specific to circle's criterion degenerated case:

$$K_1 < K_R(e,de) < K_2,\ \ with\ \ K_1=0 \tag{30}$$

The gain characteristics $K_R=f(x_{t1})$ have a minimum gain 0 and a maximum gain $K_m=K2$. We may use the following theorem : The nonlinear system with the structure from Fig. 9, in which the linear part has the transfer function $H(s)$ and the nonlinear part has the above equations, has the equilibrate point $x_{t1}=0$ global asymptotic stable if the function

$$F(s)=\frac{K_2H(s)+1}{K_1H(s)+1} \tag{31}$$

is positive real.

In the degenerated case, for $K_1=0$, $F(s)$ becomes $F(s)=K_2H(s)+1$. The degenerated case is a particular one of the circle's criterion.

Definition of the positive real function: A rational function $F(s)$ is positive real if: 1.it has no pole in $Re\ s>0$; 2.the poles from the imaginary axis (when they exist) are simple and the corresponding residues are real and positive; 3.$Re\ F(j\omega) \geq 0$ for all $\omega \geq 0$.

From the third condition of the above definition we obtain:

$$\mathrm{Re}\{K_2 H(s)+1\} \geq 0,\ \ \omega > 0 \tag{32}$$

We substitute $H(s)=v(\omega)+w(\omega)$ and we obtain:

$$K_2 v + 1 \geq 0,\ or\ \ v \geq -1/K_2 \tag{33}$$

From a geometrical point of view, in the complex plain (v, w), this inequality represents a sector at the right of the vertical straight line which pass through the point of abscises $-1/K_m$.

The conditions 1 and 2 from the definition of the real positive function may be replaced with the condition in which the polynomial $P(s)+Q(s)$ is a Hurwitz polynomial, were $F(s)=P(s)/Q(s)$. The zeros of this polynomial are the poles of the following rational function:

$$F_1(s) = \frac{H(s)}{1/K_2 + H(s)} \tag{34}$$

We may apply Nyquist's criterion, using also the hodograph $H(j\omega)$. In our case the function $H(s)$ has no poles in the half-plane $Re\ s>0$: $n_+=0$ and has one pole on the axis $Re\ s=0$: $n_0=1$. So, we have the speed control system of a dc motor, with the structure from Fig. 1, with the speed controller based on fuzzy logic. It may use also other different fuzzy controllers. It has the linear part described by the transfer function $H(s)$, which has no poles in the half-plane $Re\ s>0$ and it has one pole on the axis $Re\ s=0$. Then this system has the equilibrium point $di_d=0$ global asymptotic stable when the hodograph $H(j\omega)$ is at the right of the vertical line.

We plot the hodograph $H(j\omega)$ in the complex plane in Fig. 12. We must take only the maximum gain K_m from the characteristics of the gain of the fuzzy controllers $K_R=f(x_{t1})$. Then we plot the vertical line through the point $(-1/K_m, 0j)$ in the complex plane of the hodograph $H(j\omega)$, as it is shown in Fig. 12. If the hodograph is not at the right of the vertical line we must decrease the open loop gain c_{di} until the hodograph passes at the right of the vertical line.

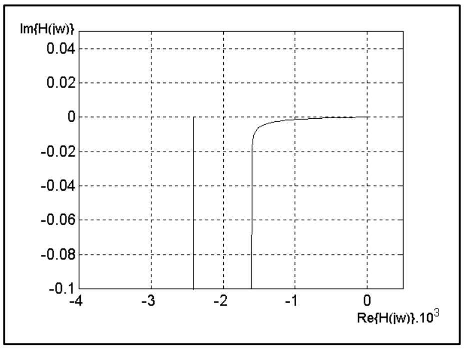

Figure 12. Nyquist's hodograph

CONCLUSION

In this paper we present a method to analyze the stability of a speed fuzzy control system for a dc drive which can be used to compute the gain factor of the open loop control system. We analyzed the gain characteristics of a basic fuzzy controller made using a 9 rules base, the sum-prod inference and the defuzzification with the center of gravity. We prssent for this fuzzy controller the algebraic properties of commutative law, the existence of the neutral element and the symmetric elements. These algebraic properties help us to say that the fuzzy controller have nonlinear characteristics that make possible the usage of circle's criterion in stability analysis. Many other fuzzy controllers have such properties.

We used the degenerated case of the circle's criterion, in a vertical line, based on the observation of the algebraic properties of the static characteristics of the fuzzy controllers. This method assures a sufficient condition for the stability.

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