Globally Stable Fuzzy Adaptive Control

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Abstract

In this paper a novel model reference fuzzy adaptive control system is introduced. The adaptation of control parameters is based on Lyapunov stability criterion. The adaptive parameters of the system are fuzzified. The main advantage of the proposed approach is the extension of globally stable adaptive control to nonlinear processes. The development of the novel algorithm has been done for ideal case on the first order system.

Keywords: Adaptive control, Fuzzy control

1 Introduction

The model reference adaptive control systems are proven to be globally stable under the certain assumption on the unknown process: phase minimal; no disturbance or unmodelled dynamics; linearity; time invariance; and knowledge of the process relative degree and the sign of b_0. Unfortunately, the assumptions given above are often violated in practice and 'adaptive algorithms as published in the literature is likely to produce unstable control systems if they are implemented on physical systems directly as they appear in the literature' [1]. It has been also shown that in the case of bounded disturbance and no persistent excitation the adaptive schemes may become unstable. Another difficulty is unmodelled dynamics. Robustness of the adaptive systems with unmodelled dynamics is treated in [2] ,[3].

The basic idea of model reference adaptive control is to introduce a global stability criterion into the design procedure and to choose the adaptive control law in such a way that the requirements of the stability criterion are fulfilled [4]. In our paper an extension of model reference adaptive control to nonlinear systems is made using fuzzy sets theory. We are introducing a novel globally stable model reference fuzzy adaptive control. The algorithm is based on globally stable model reference adaptive control obtained by Lyapunov criterion. The main idea of our approach is fuzzification of adaptive parameters. The parameters are fuzzified corresponding to the process input, output or state variables of the process. The paper is focused only on problem of nonlinearity. All other typical problems which are common for adaptive system in general can be treated and solved as proposed in literature [3], [5],[6].

The paper is organised as follows: in the Section 2 the description of globally stable model reference fuzzy adaptive control is given, in Section 3 a simulation example is described. In Conclusion some main observations are discussed.

2 Globally Stable Model Reference Fuzzy Adaptive Control

The globally stable continuous model-reference adaptive control dynamics is given first. The goal of the model-reference adaptive system is to design a controller which forces the process to follow the model output, which is in the case of first order given by the following equation

$$G_m(s) = \frac{y_m(s)}{w(s)} = \frac{b_{0m}}{s + a_{0m}}. \tag{1}$$

To obtain perfect model following a pre-filter with gain f and the gain q in the feedback loop should be designed. Assuming the process transfer function

$$G_p(s) = \frac{y_p(s)}{u(s)} = \frac{b_0}{s + a_0} \tag{2}$$

the basic-loop transfer function is given by

$$G_w(s) = \frac{f b_0}{s + a_0 + b_0 q} \tag{3}$$

The discrepancies between the model parameters and basic-loop parameters manifest in the following equation

$$\tilde{b}_0 = f b_0 - b_{0m} \tag{4}$$

$$\tilde{a}_0 = a_0 + b_0 q - a_{0m} \tag{5}$$

Applying this and subtracting the differential equations of the basic loop and reference model, the error equation is obtained

$$\dot{e} + a_{0m} e = \tilde{b}_0 w - \tilde{a}_0 y_p \tag{6}$$

where e defines the error between the basic loop and model reference response

$$e = y_p - y_m \tag{7}$$

Introducing a Lyapunov function

$$V = e^2 + \frac{1}{\gamma_0}\tilde{b}_0^2 + \frac{1}{\gamma_1}\tilde{a}_0^2 \tag{8}$$

the adaptive control laws are obtained

$$f = -\frac{\gamma_0}{b_0}\int_0^t ew\,dt + f(0) \tag{9}$$

$$q = \frac{\gamma_1}{b_0}\int_0^t e y_p\,dt + q(0) \tag{10}$$

The adaptive control law of simple globally stable model reference adaptive system is the following

$$u = fw - q y_p \tag{11}$$

Global stability is obtained in small operation region where the process can be sufficiently described by linear model. Problems arise in the case of unmodelled dynamics and nonlinear process plants. Our main motivation was to find a simple solution for adaptive control of nonlinear processes. The algorithm of Globally Stable Fuzzy Adaptive Control (GSFAC) will be presented next.

The proposed globally stable fuzzy adaptive control system assume the fuzzification of forward gain $\boldsymbol{f}$ and feedback gain $\boldsymbol{q}$. The choice of fuzzification variables depends to the process behaviour and is similar problem to those of structural identification in case of Takagi-Sugeno (TS) model [7]. The fuzzified gains are described in the way of fuzzy numbers $\boldsymbol{f}$ and $\boldsymbol{q}$

$$\begin{aligned} \boldsymbol{f}^T &= [f_1, f_2, \dots, f_K] \\ \boldsymbol{q}^T &= [q_1, q_2, \dots, q_K] \end{aligned} \tag{12}$$

where K stands for number of rules.

We assume that the process under investigation can be modelled by the TS fuzzy model of the form [8]

$$\mathbf{R}^i : \textbf{ if } u \textbf{ is } \mathbf{A}_{i_a} \textbf{ and } y_p \textbf{ is } \mathbf{B}_{i_b} \textbf{ then } \dot{y}_p = a_i y_p + b_i u + r_i \qquad i = 1, \dots, K \tag{13}$$

where u and y_p are input variables of the fuzzy system, $\dot{y}_p$ is an output variable, A_{i_a}, B_{i_b} are fuzzy membership functions where $i_a = 1, \dots, n_a$ and $i_b = 1, \dots, n_b$. The number of membership functions for the first and the second input variables defines the number of rules $K = n_a \times n_b$. The membership functions have to cover the whole operating area of the closed-loop system. The output of TS model is than given by the following equation

$$\dot{y}_p = \frac{\sum_{i=1}^{K} \left(\beta_c^i(\boldsymbol{\varphi}(k))(a_i y_p + b_i u + r_i) \right)}{\sum_{i=1}^{K} \beta_c^i(\boldsymbol{\varphi}(k))} \tag{14}$$

where $\boldsymbol{\varphi}(k)$ represents the regressor which consists of input and output signals. The degree of fulfilment $\beta_i(\boldsymbol{\varphi}(k))$ is obtained using T-norm which is in this case a simple algebraic product

$$\beta_c^i(\boldsymbol{\varphi}(k)) = T\left(\mu_{A_{i_a}}(y_p), \mu_{B_{i_b}}(u) \right) = \mu_{A_{i_a}}(y_p).\mu_{B_{i_b}}(u) \tag{15}$$

The degrees of fulfillment for the whole set of rules can be written as

$$\boldsymbol{\beta}_c = [\beta_c^1, \beta_c^2, \dots, \beta_c^K] \tag{16}$$

and given in normalized form as

$$\boldsymbol{\beta} = \frac{\boldsymbol{\beta}_c}{\sum_i \beta_c^i} \tag{17}$$

Due to the Eq.(14) and Eq.(17) the process can be modelled in fuzzy form as

$$\dot{y}_p = -\boldsymbol{\beta}^T \boldsymbol{a} y_p + \boldsymbol{\beta}^T \boldsymbol{b} u \tag{18}$$

where $\boldsymbol{a}$ and $\boldsymbol{b}$ stands for fuzzified parameters of the process which have constant elements

$$\begin{aligned} \boldsymbol{a}^T &= [a_1, a_2, \dots, a_K] \\ \boldsymbol{b}^T &= [b_1, b_2, \dots, b_K] \end{aligned} \tag{19}$$

and the following condition should be satisfied

$$\boldsymbol{\beta}^T \boldsymbol{a} > 0 \tag{20}$$

Model in Eq.(18) represents a linear model with changeable parameters which are called the global linear parameters and are given in the following equations

$$\begin{aligned} \hat{a}(\boldsymbol{\varphi}(k)) &= \boldsymbol{\beta}^T \boldsymbol{a} \\ \hat{b}(\boldsymbol{\varphi}(k)) &= \boldsymbol{\beta}^T \boldsymbol{b} \end{aligned} \tag{21}$$

The adaptive control signal in the case of proposed globally stable fuzzy adaptive system is than the following

$$u = \boldsymbol{\beta}^T \boldsymbol{f} w - \boldsymbol{\beta}^T \boldsymbol{q} y_p \tag{22}$$

Implementing the control law in the basic closed loop of the control system the following differential equation is obtained

$$\dot{y}_p = -\boldsymbol{\beta}^T \boldsymbol{a} y_p + \boldsymbol{\beta}^T \boldsymbol{b} \boldsymbol{\beta}^T \boldsymbol{f} w - \boldsymbol{\beta}^T \boldsymbol{b} \boldsymbol{\beta}^T \boldsymbol{q} y_p \tag{23}$$

Writting the reference model in the form of fuzzy model in Eq.(24)

$$\dot{y}_m = -\boldsymbol{\beta}^T \boldsymbol{a}_m y_m + \boldsymbol{\beta}^T \boldsymbol{b}_m w \tag{24}$$

and substracting this differential equations with fuzzy parameters from Eq.(23) where the control law from Eq.(22) is implemented, the following error model is obtained

$$\dot{e} + \boldsymbol{\beta}^T \boldsymbol{a}_m e = \boldsymbol{\beta}^T (\boldsymbol{\beta}^T \boldsymbol{f} - \boldsymbol{b}_m) w - \boldsymbol{\beta}^T (\boldsymbol{\beta}^T \boldsymbol{q} + \boldsymbol{a} - \boldsymbol{a}_m) y_p \tag{25}$$

The following equations are written to simplify further description

$$\begin{aligned} \tilde{\boldsymbol{b}} &= \boldsymbol{b} \boldsymbol{\beta}^T \boldsymbol{f} - \boldsymbol{b}_m \\ \tilde{\boldsymbol{a}} &= \boldsymbol{b} \boldsymbol{\beta}^T \boldsymbol{q} + \boldsymbol{a} - \boldsymbol{a}_m \end{aligned} \tag{26}$$

Implementing expressions from Eq.(26) to Eq.25 the simplified equation is given by

$$\dot{e} + \boldsymbol{\beta}^T \boldsymbol{a}_m e = \boldsymbol{\beta}^T \tilde{\boldsymbol{b}} w - \boldsymbol{\beta}^T \tilde{\boldsymbol{a}} y_p \tag{27}$$

Introducing the Lyapunov criterion

$$V = e^2 + \frac{1}{\gamma_b} \tilde{\boldsymbol{b}}^T \tilde{\boldsymbol{b}} + \frac{1}{\gamma_a} \tilde{\boldsymbol{a}}^T \tilde{\boldsymbol{a}} \tag{28}$$

the following adaptive laws of model error parameters are obtained

$$\begin{aligned} \dot{\tilde{\boldsymbol{b}}} &= -\gamma_b e w \boldsymbol{\beta} \\ \dot{\tilde{\boldsymbol{a}}} &= \gamma_a e y_p \boldsymbol{\beta} \end{aligned} \tag{29}$$

Derivation of Eq.(26) results in the following equations

$$\begin{aligned} \dot{\tilde{\boldsymbol{b}}} &= \boldsymbol{b} \dot{\boldsymbol{\beta}}^T \boldsymbol{f} + \boldsymbol{b} \boldsymbol{\beta}^T \dot{\boldsymbol{f}} \\ \dot{\tilde{\boldsymbol{a}}} &= \boldsymbol{b} \dot{\boldsymbol{\beta}}^T \boldsymbol{q} + \boldsymbol{b} \boldsymbol{\beta}^T \dot{\boldsymbol{q}} \end{aligned} \tag{30}$$

which can be simplefied as follows assuming the relatively smooth changes of process parameters

$$\begin{aligned} \dot{\tilde{\boldsymbol{b}}} &= \boldsymbol{b} \boldsymbol{\beta}^T \dot{\boldsymbol{f}} \\ \dot{\tilde{\boldsymbol{a}}} &= \boldsymbol{b} \boldsymbol{\beta}^T \dot{\boldsymbol{q}} \end{aligned} \tag{31}$$

Assuming the relations in Eq.(29) and Eq.(31) the following equations are obtained

$$\begin{aligned} \boldsymbol{b} \boldsymbol{\beta} \dot{\boldsymbol{f}} &= -\gamma_b e w \boldsymbol{\beta} \\ \boldsymbol{b} \boldsymbol{\beta} \dot{\boldsymbol{q}} &= -\gamma_a e y_p \boldsymbol{\beta} \end{aligned} \tag{32}$$

Further development of Eq.(32) results in the following equations

$$\begin{aligned} \boldsymbol{\beta}^T \dot{\boldsymbol{f}} &= -\gamma_b e w \frac{\boldsymbol{b}^T \boldsymbol{\beta}}{\boldsymbol{b}^T \boldsymbol{b}} \\ \boldsymbol{\beta}^T \dot{\boldsymbol{q}} &= -\gamma_a e y_p \frac{\boldsymbol{b}^T \boldsymbol{\beta}}{\boldsymbol{b}^T \boldsymbol{b}} \end{aligned} \tag{33}$$

Assuming the condition

$$M_1 < \boldsymbol{\beta}^T \boldsymbol{b} < M_2, \;\; \forall \boldsymbol{\beta}, \;\; M_1, M_2 > 0, \;\; M_1, M_2 \in \mathcal{R} \tag{34}$$

where M_1 and M_2 stands for constrained positive real numbers, Eq.(33) is written in the following form

$$\begin{aligned} \boldsymbol{\beta}^T \dot{\boldsymbol{f}} &= -\gamma_f e w \\ \boldsymbol{\beta}^T \dot{\boldsymbol{q}} &= -\gamma_q e y_p \end{aligned} \tag{35}$$

Solving Eq.(35) on adaptive parameters $\boldsymbol{f}$ and $\boldsymbol{q}$ the adaptive laws of globally stable fuzzy adaptive control system become

$$\begin{aligned} \dot{\boldsymbol{f}} &= -\gamma_f e w \frac{\boldsymbol{\beta}}{\boldsymbol{\beta}^T \boldsymbol{\beta}} \\ \dot{\boldsymbol{q}} &= \gamma_q e y_p \frac{\boldsymbol{\beta}}{\boldsymbol{\beta}^T \boldsymbol{\beta}} \end{aligned} \tag{36}$$

Adaptive law of the GSFAC written in the matrix form is presented in the next equation

$$\begin{bmatrix} \dot{\boldsymbol{f}} \\ \dot{\boldsymbol{q}} \end{bmatrix} = \begin{bmatrix} \gamma_f & 0 \\ 0 & \gamma_q \end{bmatrix} \begin{bmatrix} -w \frac{\boldsymbol{\beta}}{\boldsymbol{\beta}^T \boldsymbol{\beta}} \\ y_p \frac{\boldsymbol{\beta}}{\boldsymbol{\beta}^T \boldsymbol{\beta}} \end{bmatrix} e \tag{37}$$

or

$$\dot{\boldsymbol{\Theta}} = \boldsymbol{,} \, \boldsymbol{\Psi}_f e \tag{38}$$

The adaptive controller parameters $\boldsymbol{\Theta}$ can be adjusted by different algorithms as it is reported in Narendra and Lin [16]. The gradient I algorithm is presented in Eq.(38). The gradient II algorithm is written in following equation

$$\dot{\boldsymbol{\Theta}} = \frac{\boldsymbol{,} \, \boldsymbol{\Psi}_f e}{1 + c \, \boldsymbol{\Psi}^T \boldsymbol{,} \, \boldsymbol{\Psi}} \tag{39}$$

and gradient III

$$\dot{\boldsymbol{\Theta}} = \frac{\boldsymbol{,} \, \boldsymbol{\Psi}_f e}{1 + c \mid \boldsymbol{\Psi} \mid^2} \tag{40}$$

where **,** is a positive definited real symmetric matrix, c is a positive constant and $\boldsymbol{\Psi}$ stands for

$$\boldsymbol{\Psi} = \begin{bmatrix} -w \\ y_p \end{bmatrix} \tag{41}$$

The least squares algorithm is described in Eq.(42, 43)

$$\dot{\boldsymbol{\Theta}} = \frac{\boldsymbol{P} \boldsymbol{\Psi}_f e}{1 + c \, \boldsymbol{\Psi}^T \boldsymbol{P} \boldsymbol{\Psi}} \tag{42}$$

where $\boldsymbol{P}$ is the covariance matrix updated by

$$\dot{\boldsymbol{P}} = -\frac{\boldsymbol{P} \boldsymbol{\Psi}_f \boldsymbol{\Psi}_f^T \boldsymbol{P}}{1 + c \, \boldsymbol{\Psi}^T \boldsymbol{P} \boldsymbol{\Psi}} \tag{43}$$

Covariance resetting and projection algorithms are reviewed in Anderson *et al.* [9]. If $c > 0$ the estimation algorithm is said to be normalised. The main advantage of normalised algorithms is that they yield bounded $\boldsymbol{\Theta}$ evan for unbounded signals $\boldsymbol{\Psi}$, which is a crucial point for the stability analysis of adaptive systems. The stability of model reference adaptive control systems is treated in Narendra and Annaswamy [2], Narendra and Valavani [10].

The continuous-time model reference adaptive systems is stable if the relative degree of $G_p(s)$ is known, the relative degree of $G_m(s)$ is equal to or greater than the relative degree of $G_p(s)$, an upper bound on the number of poles is known, the zeros of the process transfer function lie in the left half of the complex plane, that is, the process must be of minimum phase, the sign of $\hat{b}_0$ is known.

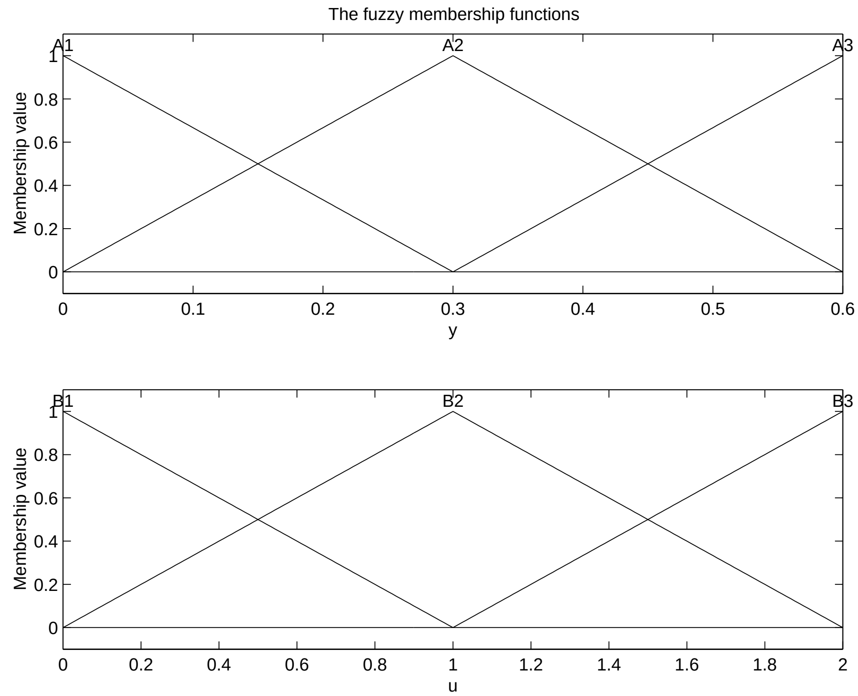

Figure 1: The fuzzy membership functions of a) the process output signal and b) the process input signal

3 Simulation example of model-reference fuzzy adaptive control for nonlinear process model

The globally stable fuzzy model-reference adaptive controller has been tested on the nonlinear process which is presented with the following differential equation

$$\dot{y} = -0.05y + (0.05y^2 + 0.015)u \tag{44}$$

The model reference transfer function is given as

$$G_m(s) = \frac{0.1}{s + 0.1} \tag{45}$$

In this simulated example the gradient II adaptive algorithm was used with $\gamma_f = 20$, $\gamma_q = 20$ and $c = 10$. The operating domain of the process has been divided into nine subspaces, both variables were divided into three membership functions which are shown in Fig.(1). The results of fuzzy adaptive control using the proposed globally stable fuzzy adaptive control are shown in Fig.(2). In Fig.(3) the fuzzified feedforward $\boldsymbol{f}$ and feedback gain $\boldsymbol{q}$ are shown. The system response approximately converge to the reference model response. The simulation example has been made under assumption of ideal conditions, because only the nonlinearity has been treated in this paper.

The fuzzy adaptive control has been compared to the classical adaptive approach. The results of the classical approach are presented in Fig.(4) The feedforward f and feedback gain q of classical adaptive system are shown in Fig.(5)

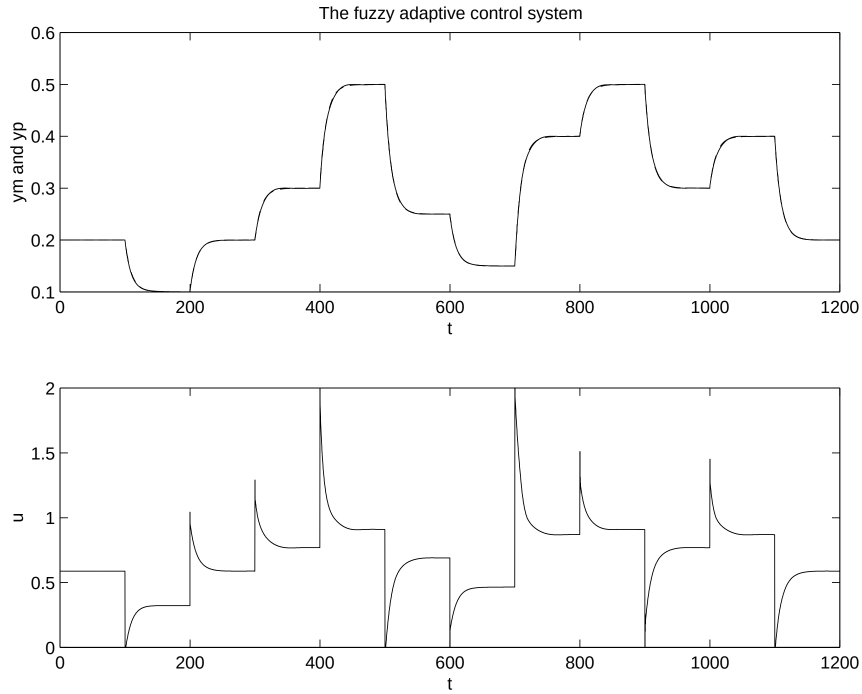

Figure 2: a) The model reference (–) and the output signal of the fuzzy adaptive system −− b) The control signal

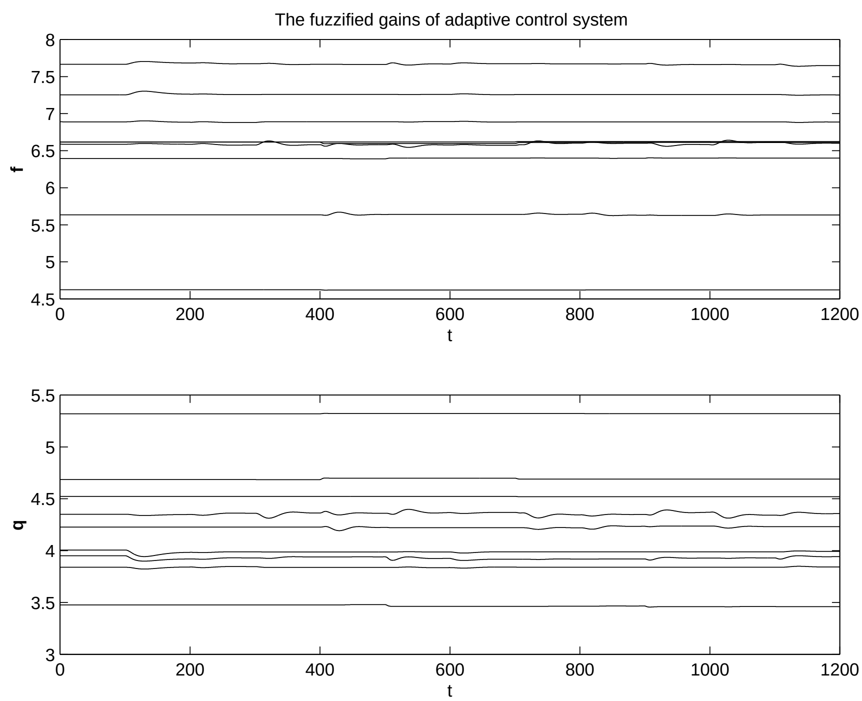

Figure 3: a) The fuzzified feedforward gain $\boldsymbol{f}$ b) The fuzzified feedback gain $\boldsymbol{q}$

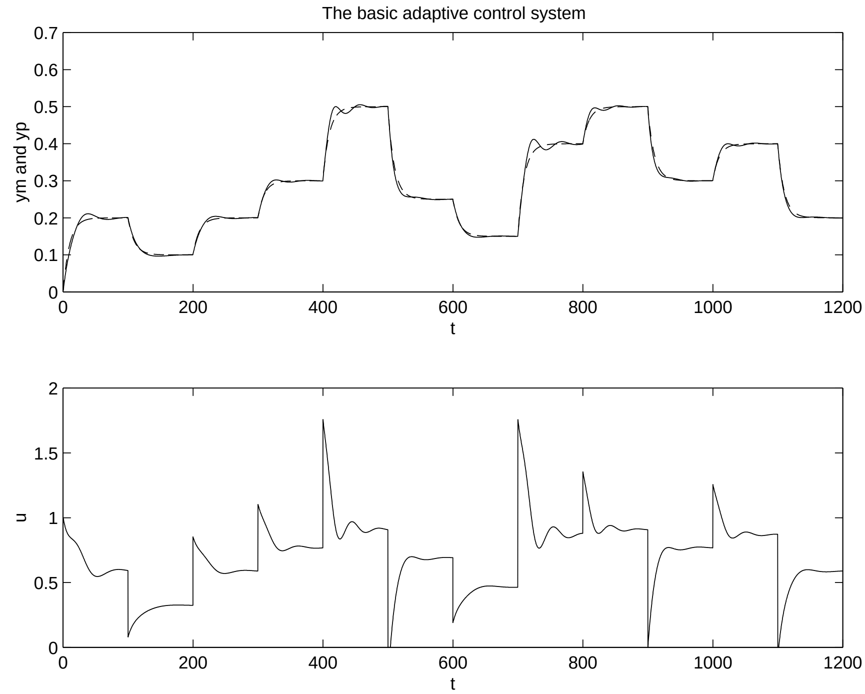

Figure 4: a) The model reference (–) and the output signal of the classical adaptive system – – b) The control signal

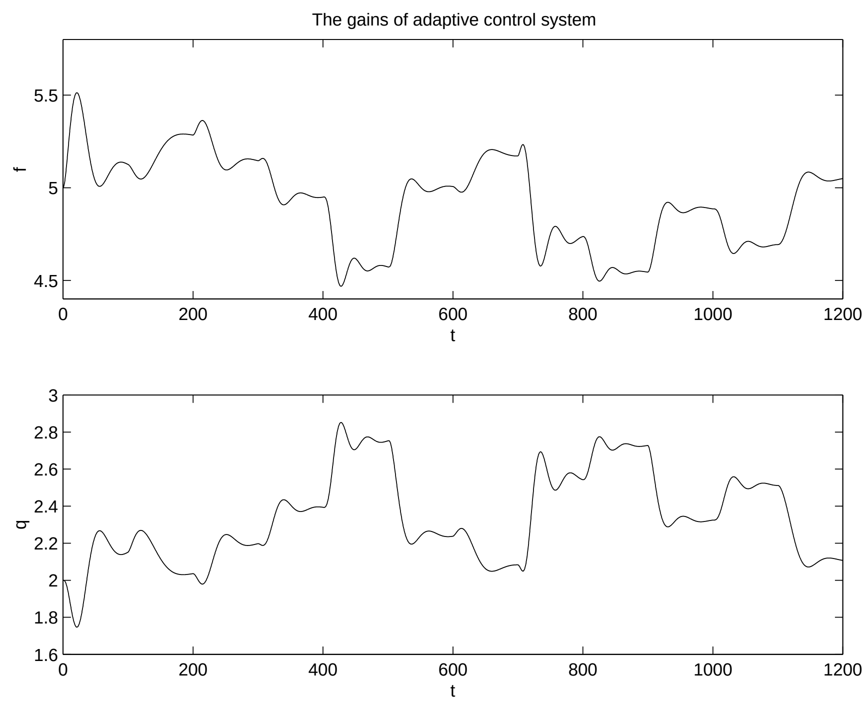

Figure 5: a) The feedforward gain f b) The feedback gain q

4 Conclusion

In this paper a novel model reference fuzzy adaptive control system is introduced. It is based on Lyapunov stability criterion. The adaptive parameters of the system are fuzzified. The main advantage of the proposed approach is the extension of globally stable adaptive control to nonlinear processes. The parameters are fuzzified corresponding to the process input, output or state variables of the process. The paper is focused only on problem of nonlinearity. The main advantage of the proposed approach is capability of globally stable adaptive control to the nonlinear processes. The development of the novel algorithm has been done for ideal case on the first order system. The extension to higher order systems can be easily done. The ideal case has been studied because the goal was to study only the behaviour of fuzzy adaptive control in the case of nonlinear processes. All problems which are common for adaptive control systems and have been mentioned in the introduction can be solved on the similar way, as it is reported in literature.

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