Approximate Spatial Reasoning via Rough Mereology

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Abstract. Rough mereology is a paradigm allowing to blend main ideas of two potent paradigms for approximate reasoning : fuzzy set theory and rough set theory. Essential ideas of rough mereology and schemes for approximate reasoning in distributed systems based on rough mereological logic were presented in Polkowski(1994), Polkowski(1996), Skowron(1998). In this contribution, we introduce the general context for rough mereology i.e. Ontology and Mereology of Stanisław Leśniewski and we define within this framework the rough mereological predicate " to be a part in a degree". In this context, we introduce a basic scheme for spatial reasoning.In particular, we demonstrate that in our context mereotopologies arise naturally and we may define graded notions of connection allowing us to work as well in the mereotopological paradigm stemming therefrom.

keywords: *rough sets, fuzzy sets, rough mereology, mereology, mereotopology, spatial reasoning*

1 Introduction

Rough mereology has been proposed in Polkowski(1994) and developed into a paradigm for approximate reasoning in Polkowski(1996). Its applications to problems of approximate synthesis, control , design and analysis of complex objects have been discussed in Skowron(1998) and in Polkowski(1998) a granular semantics for computing with words was proposed based on rough mereology. The fundamental predicate on which this theory is centered is that of being a part in a degree. This predicate invokes the predicate of being a part around which the theory of Mereology i.e. the theory of collective classes proposed by Stanisław Leśniewski in Leśniewski(1927) was developed. Mereology in turn was constructed by Leśniewski in the framework of Ontology cf. Lejewski(1958), Słupecki(1955) i.e. a Theory of Names or distributive classes. In this contribution, we outline the development of Rough Mereology in the logical framework of Ontology of Leśniewski . Then, we sketch a scheme for approximate spatial reasoning constructed within Rough Mereology. In the following sections, we discuss briefly Ontology, Mereology and Rough Mereology concluded by a final section on topologies induced by rough mereological predicate and its applications to spatial reasoning.

2 Ontology

Ontological theory of Leśniewski is concerned with objects of either of two categories: propositional and nominal. Although it has been based by its inventor on a logical calculus called *protothetics* yet it may be for a very large part exposed in language of ordinary predicate logic; this is a way we prefer here.

This very concise reminder of Ontology we begin by selecting symbols X, Y, Z etc to denote names of objects; the primitive symbol of ontology is ε (read "is").

The sole Axiom of Ontology is a formula coding the meaning of ε as follows

2.1 Ontology Axiom

$X\varepsilon Y \Longleftrightarrow \exists Z.Z\varepsilon X \wedge \forall U,W.(U\varepsilon X \wedge W\varepsilon X \Longrightarrow U\varepsilon W) \wedge \forall T.(T\varepsilon X \Longrightarrow T\varepsilon Y)$

This axiom determines the meaning of the formula $X\varepsilon Y$ ("X is Y") as the conjunction of three conditions:

$\exists Z.Z\varepsilon X$ ("something is X");

$\forall U,W.(U\varepsilon X \wedge W\varepsilon X \Longrightarrow U\varepsilon W)$ ("any two objects which are X are identical" i.e. X is an individual name);

$\forall T.(T\varepsilon X \Longrightarrow T\varepsilon Y)$ ("everything which is X is Y").

Therefore the meaning of the formula $X\varepsilon Y$ is as follows: X is a non-empty name of an individual (X is an individual) and any object which is X is also Y.

We may introduce the following abbreviations:

$X = Y$ for $X\varepsilon Y \wedge Y\varepsilon X$;

$X \subseteq Y$ for $\forall T.(T\varepsilon X \Longrightarrow T\varepsilon Y)$;

Hence: $X\varepsilon Y \Longleftrightarrow \exists Z.Z\varepsilon X \wedge \forall U,W.(U\varepsilon X \wedge W\varepsilon X \Longrightarrow U = W) \wedge X \subseteq Y$.

The extensional identity $=_E$is introduced as follows:

$X =_E Y \Longleftrightarrow \forall T.(T\varepsilon X \Longleftrightarrow T\varepsilon Y)$; i.e. $X =_E Y \Longleftrightarrow X \subseteq Y \wedge Y \subseteq X$.

One can clearly see that the relation $\subseteq$ is reflexive, transitive and weakly - anti- symmetric i.e. it is a partial order. Similarly, one can easily prove that

$X\varepsilon Y \wedge Z\varepsilon X \Longrightarrow Z\varepsilon Y$; $X\varepsilon Y \Longrightarrow X\varepsilon X$; $X\varepsilon Y \wedge Z\varepsilon X \Longrightarrow X\varepsilon Z$.

Finally, we introduce a name V defined via : $X\varepsilon V \Longleftrightarrow \exists Y.X\varepsilon Y$.

The copula ε formalized as above permits to accomodate not only distributive classes as shown above but also collective classes. The latter are discussed in the framework of mereology.

3 Mereology

Mereology of Leśniewski cf. Leśniewski(1927), Sobociński(1954) can be based on each of a few primitive notions; we prefer the orthodox way here starting with the notion of a part conceived as a name - forming functor *pt* on individual names.

3.1 Mereology Axioms

We start with basic axioms for *pt*.

(Me1) $X\varepsilon pt(Y) \Longrightarrow \exists Z.Z\varepsilon X \wedge Y\varepsilon V$;

(Me2) $X\varepsilon pt(Y) \wedge Y\varepsilon pt(Z) \Longrightarrow X\varepsilon pt(Z)$;

(Me3) $non(X\varepsilon pt(X))$.

It will be manifest that $X\varepsilon pt(Y)$ means that the individual denoted X is a proper part of the individual denoted Y.

The notion of an improper part is reflected as the notion of an element *el*; this is a name - forming functor defined as follows:

$X\varepsilon el(Y) \Longleftrightarrow X\varepsilon pt(Y) \vee X = Y$.

We will require that the following inference rule is valid.

(Me4) $\forall T.(T\varepsilon el(X) \Longrightarrow \exists W.W\varepsilon el(T) \wedge W\varepsilon el(Y)) \Longrightarrow X\varepsilon el(Y)$.

3.2 Classes

The notion of a collective class may be introduced at this point; this is effected by means of a name - forming functor Kl defined as follows.

$$X\varepsilon Kl(Y) \Longleftrightarrow$$
$$\exists Z.Z\varepsilon Y \wedge \forall Z.(Z\varepsilon Y \Longrightarrow Z\varepsilon el(X)) \wedge$$
$$\forall Z.(Z\varepsilon el(X) \Longrightarrow \exists U,W.U\varepsilon Y \wedge W\varepsilon el(U) \wedge W\varepsilon el(Z)).$$

The notion of a class is subjected to the following restrictions

(Me5) $X\varepsilon Kl(Y) \wedge Z\varepsilon Kl(Y) \Longrightarrow Z\varepsilon X$ ($Kl(Y)$ is an individual);

(Me6) $\exists Z.Z\varepsilon Y \Longleftrightarrow \exists Z.Z\varepsilon Kl(Y)$ (the class exists for each non-empty name).

One can also introduce a less restrictive name viz. of a set:

$$X\varepsilon set(Y) \Longleftrightarrow$$
$$\exists Z.Z\varepsilon Y \wedge \forall Z.(Z\varepsilon el(X) \Longrightarrow \exists U,W.U\varepsilon Y \wedge W\varepsilon el(U) \wedge W\varepsilon el(Z)).$$

One can prove (op.cit., op.cit.) the following theses

(i) $X\varepsilon X \Longrightarrow X\varepsilon Kl(\varepsilon X)$ where $Z\varepsilon(\varepsilon X)$ iff $Z\varepsilon X$;

(ii) $X\varepsilon X \Longrightarrow X\varepsilon Kl(ptX)$ where $Z\varepsilon(ptX)$ iff $Z\varepsilon pt(X)$;

(iii) $X\varepsilon X \Longrightarrow X\varepsilon Kl(elX)$ where $Z\varepsilon(elX)$ iff $Z\varepsilon el(X)$;

(iv) $X\varepsilon Y \Longrightarrow X\varepsilon set(Y)$;

(v) $X\varepsilon Kl(Y) \Longrightarrow X\varepsilon set(Y)$;

(vi) $X\varepsilon Kl(set(Y)) \Longleftrightarrow X\varepsilon Kl(Y)$.

One may also introduce the notion of a subset viz. $X\varepsilon sub(Y) \Longleftrightarrow X\varepsilon V \wedge Y\varepsilon V \wedge X \subseteq Y$.

Then clearly

$X\varepsilon sub(Y) \Longleftrightarrow X\varepsilon el(Y)$.

3.3 Mereotopology

Within mereology one may define some functors expressing relative position of objects. The functor *ext* expresses disjointness in terms of parts:

$X \varepsilon ext(Y) \Longleftrightarrow non(\exists Z.Z \varepsilon el(X) \wedge Z \varepsilon el(Y))$.

The notion of a complement is expressed by the functor *comp* :

$X \varepsilon comp(Y, rel Z) \Longleftrightarrow Y \varepsilon sub(Z) \wedge X \varepsilon Kl(el Z | ext Y)$

where $U \varepsilon el Z | ext Y$ iff $U \varepsilon el(Z) \wedge U \varepsilon ext(Y)$.

3.4 Models

Ontology and Mereology have natural models related to boolean algebras. Formally, letting $<$ to denote a proposition - forming functor of two nominal variables defined as follows cf. Lejewski(1958):

$X < Y \Longleftrightarrow \exists Z.Z \varepsilon X \wedge X \subseteq Y$

one proposes an alternative axiomatization of Ontology. The functor $<$ has the following basic properties:

(i) $\exists Z.Z \varepsilon X \Longleftrightarrow \exists Z.Z < X$;

(ii) $\forall U, W.(U \varepsilon X \wedge W \varepsilon X) \Longleftrightarrow \forall U, W.(U < X \wedge W < X)$;

(iii) $X < Y \Longleftrightarrow$

$$\exists Z.Z \varepsilon X \wedge$$
$$\forall Z.(Z < X \Longrightarrow \exists W.W < Z \wedge W < Y \wedge \forall P, Q.(P < W \wedge Q < W \Longrightarrow P, Q)).$$

The property (iii) may be taken as the Axiom for Ontology (op.cit.) and it is easily seen that this is as well the axiom for an atomic boolean algebra without 0, atoms defined via the nominal constant *at* :

$X \varepsilon at \Longleftrightarrow X \varepsilon X \wedge \forall P, Q.(P < X \wedge Q < X \Longrightarrow P < Q)$.

Thus, it follows that models for Ontology are models for atomic boolean algebras without 0.

For Mereology, one may check that the following statement :

(E) $X \varepsilon el(Y) \Longleftrightarrow X \varepsilon X \wedge Y \varepsilon Y \wedge Y \varepsilon el(Y) \Longrightarrow$

$$[\forall U, W.Y \varepsilon U \wedge \forall C.(C \varepsilon W \Longleftrightarrow \forall D.D \varepsilon U \Longrightarrow D \varepsilon el(C) \wedge \forall D.D \varepsilon el(C) \Longrightarrow$$
$$\exists E, F.E \varepsilon U \wedge F \varepsilon el(D) \wedge F \varepsilon el(E)) \Longrightarrow X \varepsilon el(W)]$$

may be taken as Axiom for Mereology cf. Sobociński(1954).

Replacing $X \varepsilon el(Y)$ by $X \leq Y$ and $X \varepsilon X$ by $X \varepsilon U$, one may check that (E) is an axiom for a complete boolean algebra without 0 cf. Clay(1974).

Thus, models for mereology are models for complete boolean algebras without 0 as observed very early by Tarski(1935).

In particular, for any topological space (U, τ),the families $RO^*(U), RC^*(U)$ of , respectively, regular open non-empty, regular closed non-empty sets are models for mereology (a set U is regular open if $U = IntClU$ and regular closed if its complement is regular open (cf. also Asher(1995)).

4 Rough Mereology

Approximate Reasoning carried out under Uncertainty needs a weaker form of part predicate: of being a part in a degree. The degree of being a part may then be specified either on the basis of a priori considerations and findings or directly from data. In our construction of rough mereoogical predicate, we are guided by the tendency to preserve Mereology as an exact skeleton of reasoning.

Rough Mereology has been proposed and studied in e.g. Polkowski(1994), Polkowski(1996), Skowron(1998) as a first-order theory. Here, we propose a new formalization in the framework of Ontology; hence, rough mereology becomes now a genuine extension of mereology in a unified framework.

We assume that a mereological predicate *el* of an element is given and ε is a symbol for ontological copula as defined above.

4.1 Rough Mereology Axioms

The following is a list of axiomatic postulates for Rough Mereology. We introduce a graded family $\mu_r, r \in [0,1]$ any real number from the unit interval, which would satisfy

(RM1) $X \varepsilon \mu_1(Y) \Longleftrightarrow X \varepsilon el(Y)$;

(RM2) $X \varepsilon \mu_1(Y) \Longrightarrow \forall Z.(Z \varepsilon \mu_r(X) \Longrightarrow Z \varepsilon \mu_r(Y))$;

(RM3) $X = Y \wedge X \varepsilon \mu_r(Z) \Longrightarrow Y \varepsilon \mu_r(Z)$;

(RM4) $X \varepsilon \mu_r(Y) \wedge s \leq r \Longrightarrow X \varepsilon \mu_s(Y)$;

we introduce a following notational convention:

$X\varepsilon\mu_r^+(Y) \iff X\varepsilon\mu_r(Y) \wedge non(\exists s > r.X\varepsilon\mu_s(Y))$.

In some versions of our approach, we adopt one more axiom

(RM5) $X\varepsilon ext(Y) \Longrightarrow X\varepsilon\mu_0^+(Y)$.

4.2 Mereological Topologies

We define an object Kl_rX, each $X, r < 1$, as follows:

$Z\varepsilon Kl_rX \iff Z\varepsilon Kl(\mu_rX)$

where $Z\varepsilon\mu_rX \iff Z\varepsilon\mu_r(X)$.

Thus Kl_rX is the class of all objects Z such that $Z\varepsilon\mu_r(X)$.

A simplified description of Kl_rX may be provided as follows.

Let B_rX be defined via: $Z\varepsilon B_rX \iff \exists T.Z\varepsilon el(T) \wedge T\varepsilon\mu_rX$.

Then we have

$Kl_rX = B_rX$.

Proof. Let $Z\varepsilon el(B_rX)$; there is T such that $Z\varepsilon el(T)$ and $T\varepsilon\mu_rX$. Hence the following is true: $\forall Z.Z\varepsilon el(B_rX) \Longrightarrow \exists U.U\varepsilon el(Z) \wedge U\varepsilon el(Kl_rX)$ and $B_rX\varepsilon el(Kl_rX)$ follows by (Me4). Similarly, for $Z\varepsilon el(Kl_rX)$, we have P, Q with $P\varepsilon el(Z)$, $P\varepsilon el(Q)$, $Q\varepsilon\mu_r(X)$. Hence $P\varepsilon el(B_rX)$ and (Me4) implies that $Kl_rX\varepsilon el(B_rX)$ so finally, $Kl_rX = B_rX$.

There is another property

For $s \leq r$, $Kl_rX\varepsilon el(Kl_sX)$.

Indeed, by the previous fact, $Z\varepsilon el(Kl_rX)$ implies that $Z\varepsilon el(T)$ and $T\varepsilon\mu_rX$ for some T hence $T\varepsilon\mu_sX$ and a fortiori $Z\varepsilon el(Kl_sX)$.

Introducing a constant name Λ (the empty name) via the definition:

$X\varepsilon\Lambda \iff X\varepsilon X \wedge non(X\varepsilon X)$

and defining the interior $IntX$ of an object X as follows:

$IntX\varepsilon Kl(int\_X)$

where $Z\varepsilon int\_X \iff \exists T.\exists r < 1.Z\varepsilon el(Kl_rT) \wedge Kl_rT\varepsilon el(X)$

i.e. $IntX$ is the class of objects of the form Kl_rT which are elements of X.

Then we have:

(i) $Int\Lambda\varepsilon\Lambda$;

(ii) $X\varepsilon el(Y) \Longrightarrow IntX\varepsilon el(IntY)$;

(iii)$IntV = V$.

Properties (i)-(iii) witness that the family of all classes $Kl_rT, r < 1$, is a base for a Čech topology (cf. Čech(1966); we call this topology the *rough mereological topology* (*rm - topology*).

5 From Čech mereotopologies to mereotopologies

We introduce here a class of predicates μ_r which induce a topology on universe of our objects.

5.1 A *t-norm* model

To this end, we recall that a *t-norm* is a 2-argument functor $\top(x, y) : [0, 1]^2 \longrightarrow [0, 1]$ satisfying the conditions:

(i) $\top(x, y) = \top(y, x)$; (ii) $\top(x, 1) = x$; (iii) $x' \geq x, y' \geq y \longrightarrow \top(x', y') \geq \top(x, y)$; (iv) $\top(x, \top(y, z)) = \top(\top(x, y), z)$

and that the residual implication induced by $\top$,in symbols $\overset{\rightrightarrows}{\top}$, is defined via $\overset{\rightrightarrows}{\top}(r, s) \geq t \iff \top(t, r) \leq s$.

We apply here the ideas developed in [9] and we define, given a part in degree predicate μ,a new measure of partiality in degree, $\mu_\top$, defined as follows

(*) $X\varepsilon\mu_\top(r)(Y) \iff \forall Z.(Z\varepsilon\mu(u)(X) \wedge Z\varepsilon\mu(v)(Y) \Longrightarrow \overset{\rightrightarrows}{\top}(u, v) \geq r)$.

It turns out that our measure $\mu_\top$ satisfies axioms (RM1)-(RM5).

Indeed, we may check (RM1): $X\varepsilon\mu_\top(1)(Y)$ implies that from $Z\varepsilon\mu(u)(X) \wedge Z\varepsilon\mu(v)(Y)$ it follows that $u \leq v$ for each Z hence: $Z\varepsilon el(X) \Longrightarrow Z\varepsilon el(Y)$ follows for any Z i.e. $X\varepsilon el(Y)$. Similarly, $X\varepsilon el(Y)$ implies via (Me2) for μ that $Z\varepsilon\mu(u)(X) \wedge Z\varepsilon\mu(v)(Y)$ yields $u \leq v$ i.e. $\overset{\rightrightarrows}{\top}(u, v) \geq 1$ for any Z thus $X\varepsilon\mu_\top(1)(Y)$.

(RM2), (RM3), (RM4) are checked similarly, for (RM5), we begin with the premise $X\varepsilon ext(Y)$ hence $X\varepsilon\mu_0^+(Y)$; assuming $X\varepsilon\mu_\top(r)(Y)$ we get by (*) for $Z = X$ that $\overset{\rightrightarrows}{\top}(1, 0) \geq r$ i.e. $\top(r, 1) = r \leq 0$.

Thus $\mu_\top$ is a partiality in degree predicate.

Modifying a proof given in (Polkowski(1996), Prop.14), we find that the following deduction rule is valid for $\mu_\top$:

(RMP) $\frac{X\varepsilon\mu_\top(r)(Y), Y\varepsilon\mu_\top(s)(Z)}{X\varepsilon\mu_\top(\top(r,s))(Z)}$.

We denote with the symbol $Kl_{r,\top}X$ the class $Kl_r X$ with respect to $\mu_\top$.

We may give a new characterization of $Kl_{r,\top}X$.

$Y\varepsilon el(Kl_{r,\top}X) \Longleftrightarrow Y\varepsilon\mu_\top(r)(X)$.

Indeed, $Y\varepsilon el(Kl_{r,\top}X)$ means by ... that $Y\varepsilon el(Z)$ and $Z\varepsilon\mu_\top(r)(X)$ for some Z. From $Y\varepsilon\mu_\top(1)(Z)$ and $Z\varepsilon\mu_\top(r)(X)$ it follows by (RMP) that $Y\varepsilon\mu_\top(\top(1,r))(X)$.

We may regard therefore $Kl_{r,\top}X$ as a "ball of radius r centered at X" with respect to the "metric" $\mu_\top$.

Furthermore, we have

$Y\varepsilon el(Kl_{r,\top}X)$ and $s_o = \min\_\arg(\top(r,s) \geq r)$ imply $Kl_{s_0,\top}Y\varepsilon el(Kl_{r,\top}X)$.

It follows that the family $\{Kl_{r,\top}X : r < 1, X\}$ induces a topology on our universe of objects (under the assumption that $\top(r,s) < 1$ whenever $rs < 1$).

6 Connections

We refer to alternative scheme for mereological reasoning based on Clarke's formalism in Clarke(1981) in Calculus of Individuals of Leonard(1940); see in this respect Aurnague(1995).

The basic primitive in this approach is the predicate $C(X,Y)$ (read "X and Y are *connected*") which should satisfy : (i) $C(X,X)$; (ii) $C(X,Y) \Longrightarrow C(Y,X)$; (iii) $\forall Z.(C(X,Z) \Longleftrightarrow C(Y,Z)) \Longrightarrow X = Y$.

From C other predicates are generated and under additional assumptions (cf. Aurnague(1995), Clarke(1981)) a topology may be generated from C.

We will define a notion of connection in our model; clearly, as in our model topological structures arise in a natural way via "metrics" μ, we may afford a more stratified approach to connection and separation properties. So we propose a notion of a graded connection $C(r,s)$.

6.1 Graded connections

We let

$Bd_r X\varepsilon Kl(\mu_r^+ X)$ where $Z\varepsilon\mu_r^+(X) \Longleftrightarrow Z\varepsilon\mu_r(X) \wedge non(Z\varepsilon\mu_s(X), s > r)$

and then

$X\varepsilon C(r,s)(Y) \Longleftrightarrow \exists W.W\varepsilon el(Bd_r X) \wedge W\varepsilon el(Bd_s Y)$.

We have then clearly: (i) $X\varepsilon C(1,1)(X)$; (ii) $X\varepsilon C(r,s)(Y) \Longrightarrow Y\varepsilon C(s,r)(X)$;

the status of property (iii) depends on the context. Our discussion splits here into the cases:

I. We assume that μ satisfies (RM1)-(RM5). Then if $X \neq Y$, there exists a Z such that either $Z\varepsilon el(X) \wedge Z\varepsilon ext(Y)$ or $Z\varepsilon el(Y) \wedge Z\varepsilon ext(X)$; consider e.g. the first case. Then $Z\varepsilon\mu(1)(X)$ and $Z\varepsilon\mu^+(0)(Y)$ hence $Z\varepsilon C(1,1)(X)$ and $Z\varepsilon C(1,0)(Y)$; thus:

$X \neq Y \Longrightarrow \exists Z.(Z\varepsilon C(1,1)(X) \wedge Z\varepsilon C(1,0)(Y) \vee Z\varepsilon C(1,0)(X) \wedge Z\varepsilon C(1,1)(Y))$

is a version of (iii).

This case may be termed "exact reasoning with uncertainty" due to (RM5): we are able to perceive clearly that objects are exterior to each other.

Let us observe that in this case the " normalized metric " $\mu_\top$ is discrete: $X\varepsilon\mu_\top(1)(Y)$ when $X\varepsilon el(Y)$ and $X\varepsilon\mu_\top^+(0)(Y)$ otherwise.

II. A more realistic is the case when we replace (RM5) with

(RM5)* $X\varepsilon ext(Y) \Longrightarrow \exists r < 1. X\varepsilon\mu^+(r)(Y)$

meaning that we may not be sure that $X\varepsilon ext(Y)$ but we do not confuse this case with that of being an element.

Under (RM5)*,

$X \neq Y \Longrightarrow \exists Z.(Z\varepsilon C(1,1)(X) \wedge Z\varepsilon C(r,s)(Y) \vee Z\varepsilon C(r,s)(X) \wedge Z\varepsilon C(1,1)(Y))$

with some $r, s < 1$.

In both cases we may granulate graded connection by setting a threshold $\alpha \in (0,1)$ and letting

$X\varepsilon C(Y) \Longleftrightarrow \exists r, s \geq \alpha. X\varepsilon C(r,s)(Y)$.

Then

$X = Y \Longleftrightarrow \forall Z.(Z\varepsilon C(X) \Longleftrightarrow Z\varepsilon C(Y))$

follows for an appropriate level α.

Therefore in our setting we may have as well the mereotopological setting of Clarke(1981), Asher(1995) and Aurnague(1995).

Let us observe that in case I., under $\mu_\top$, $X \varepsilon C(Y) \iff X \varepsilon OV(Y)$ where $X \varepsilon OV(Y) \iff \exists Z. Z \varepsilon el(X) \wedge Z \varepsilon el(Y)$; however, in general setting of case II, $C \neq OV$.

7 Conclusion

We have presented a scheme for developing conceptual spatial reasoning under uncertainty. In this framework, as it will be presented elsewhere, we may develop various approaches to spatial reasoning, including metric geometry based on predicates μ. These concepts will be applied in our work on synthesis of control algorithms in mobile robotics.

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