Regression on Heterogeneous Fuzzy Data

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ABSTRACT: A novel formalism is presented, which enables the processing of heterogeneous data including numeric-, interval-valued-, or fuzzy- data. The data in question are represented herein as interval-supported fuzzy sets with suitable membership functions. The term *Fuzzy Interval Number* (*FIN*) denotes one of the aforementioned types of data. A *FIN* can have either a positive or a negative membership function. We show a number of mathematical tools/properties in the set of *FIN*'s including, a partial ordering relation, a distance function, an addition and a multiplication operation; hence we can talk of *FIN-arithmetic*. The presented formalism is used to carry out regression on *FIN*'s. Potential applications of regression on *FIN*'s include reduction in the number of rules in a fuzzy system, generalization of fuzzy rules, etc. Two examples demonstrate the merits of the presented formalism.

KEYWORDS: Fuzzy Systems, Heterogeneous Data, Algebra, Regression, Linguistic Variables, Fuzzy Lattices.

1. INTRODUCTION

The volume, versatility, and ambiguity of the data one has to cope with characterize "contemporary information processing"; this is particularly true in view of the upcoming proliferation of information networks. Within the context in question the need arises to deal with *heterogeneous data*, these are disparate types of data including numeric, interval-valued, and fuzzy data. Note that *interval-valued*, and *fuzzy* data are referred to herein as *linguistic data*.

On the one hand, previous works, which have dealt with the aforementioned *heterogeneous data*, rely on conventional fuzzy set theory e.g. Bortolan *et al* (1997), Pedrycz *et al* (1998), Zeng *et al* (1997). On the other hand, the authors in Kaburlasos *et al* (1997), and in Petridis *et al* (1998) have introduced the framework of *fuzzy lattices* (*FL*-framework) as a convenient platform for carrying out two types of data processing, namely clustering and classification. In particular, clustering and classification have been considered on disparate types of data including jointly vectors of numbers, intervals, complex numbers, fuzzy sets, multi-dimensional functions, events in a probability space, symbols, propositions in prepositional calculus, etc. Herein we extend our formalism in such a way that another type of data processing, namely regression, can be performed on *heterogeneous data*.

Rather than based upon "fuzzy logic", the approach to regression presented herein is based upon a novel "algebra of fuzzy sets". Section 2 delineates a theoretical foundation for our algebraic approach to fuzzy sets. Section 3 makes a connection with fuzzy sets. Section 4 demonstrates two regression examples. In the last section 5 we discuss how our novel algebraic approach to fuzzy sets implies several significant improvements.

2. *M*-TUPLE ARITHMETIC

In this section the space R^M of *M*-tuples of real numbers is dealt with. An *M*-tuple will typically be denoted by an italicized capital letter, e.g. $A\in\mathsf{R}^M$. The i^{th} component of $A\in\mathsf{R}^M$ will be denoted by an italicized small letter, e.g. $a_i\in\mathsf{R}$, where $i=1\ldots M$. We define two operations, namely *addition* and *multiplication*, as follows.

Definition 1 An *addition* operation $\boxplus$: $\mathsf{R}^M\times\mathsf{R}^M\to\mathsf{R}^M$ is defined by the map $(A,B)\to A\boxplus B$ given by $A\boxplus B=(a_1,\ldots,a_M)\boxplus(b_1,\ldots,b_M)=(a_1+b_1,\ldots,a_M+b_M)$.

Definition 2 A *multiplication* operation $\odot$: $R^M \times R^M \to R^M$ is defined by the map $(A,B) \to A \odot B$ given by $A \odot B = (a_1,\ldots,a_M) \odot (b_1,\ldots,b_M) = (a_1b_1,\ldots,a_Mb_M)$.

In view of definitions *1* and *2*, the symbol $P(M)$ will be used in the sequel to denote the collection of *M*-tuples of real numbers endowed with the aforementioned operations for addition and multiplication in order to avoid confusion with the conventional set R^M.

Recall that the set R of real numbers with its two conventional operations, namely *addition* and *multiplication* is a (*mathematical*) *field*; that is (1) the set R with the conventional addition operation constitutes an *Abelian group* with *zero element* equal to *0*, the *inverse* of $x \in R$ is $-x$ (for definition of an Abelian group the reader may refer to the Encyclopedic Dictionary of Mathematics (1987)), (2) the set $R \backslash \{0\}$ with the conventional multiplication operation constitutes an *Abelian group* with *unit element* equal to *1*, the *inverse* of an element $x \in R$ is x^{-1},and (3) multiplication is distributive over addition. Nevertheless, we remark that space $P(M)$ with $M>1$ and operations $\boxplus$ and $\odot$ defined above is not a field because division cannot be defined in the set $P^* = \{X: X=(x_1,\ldots,x_M) \text{ and } \exists i \in \{1,\ldots,M\} \text{ such that } x_i=0\}$.

3. FROM GENERALIZED INTERVALS TO FUZZY INTERVAL NUMBERS (*FIN*'s)

We extend the definition for a fuzzy set so as to include negative degrees of membership.

Definition 3 A **fuzzy set** is a pair (U,μ), where U is the universe of discourse and $\mu: U \to [-1,1]$ is a real function.

The notion "negative degree of membership", implied by the above definition, may sound counter-intuitive; nevertheless we show in this section that it allows for powerful analytical tools involving fuzzy sets.

Note, in the first place, that most of "fuzzy logic based" applications employ fuzzy sets defined on the *totally ordered set* R of real numbers; moreover the fuzzy sets in question are typically characterized by an interval support. For instance, fuzzy sets employed in applications may correspond to such linguistic terms as "fast", "heavy", "long", etc. Motivated by the aforementioned practical considerations we deal in this work with interval-supported- fuzzy sets. Moreover, *convex* fuzzy sets will be considered. In view of definition *3*, we deal with fuzzy sets whose membership function is either non-negative or non-positive. We call those fuzzy sets *fuzzy interval numbers* or *FIN's*. Note that a FIN could be regarded as a generalized LR-type fuzzy number, e.g. Dubois *et al* (1980), Zimmermann (1991). We study in the sequel a particular subset of FIN's, that is the collection P of *generalized intervals* whose elements are defined underneath.

Definition 4 A *generalized interval* is a fuzzy interval number (FIN) with membership function $p(x;a,b)$, $x,a,b \in R$, such that

If *a£b* (*positive generalized interval*) then $p(x;a,b) = \begin{cases} 1, & a \le x \le b \\ 0, & otherwise \end{cases}$

If *b£a* (*negative generalized interval*) then $p(x;a,b) = \begin{cases} -1, & b \le x \le a \\ 0, & otherwise \end{cases}$

By a *trivial generalized interval*, symbolically $p(x;a,a)$, is meant both the positive and the negative generalized intervals $p(x;a,a)$. Even though, a *trivial generalized interval* corresponds to a pair of generalized intervals, it has been shown by the authors that $p(x;a,a)$ can be treated as a single element in computations; details will be given elsewhere for lack of space. The set of all positive (negative) generalized intervals is denoted by P^+ (P^-). The set of trivial generalized intervals is denoted by P^0. The following proposition points out an ordering in P.

Proposition 5 The set P of generalized intervals is a non-complete mathematical lattice.

For lack of space, a proof of the above proposition will be given elsewhere. For the definition of a *mathematical lattice* the reader may refer to Encyclopedic Dictionary of Mathematics (1987). Fig.1 illustrates the *lattice join* ($p_1 \vee_P p_2$) and *lattice meet* ($p_1 \wedge_P p_2$) of two generalized intervals p_1 and p_2.

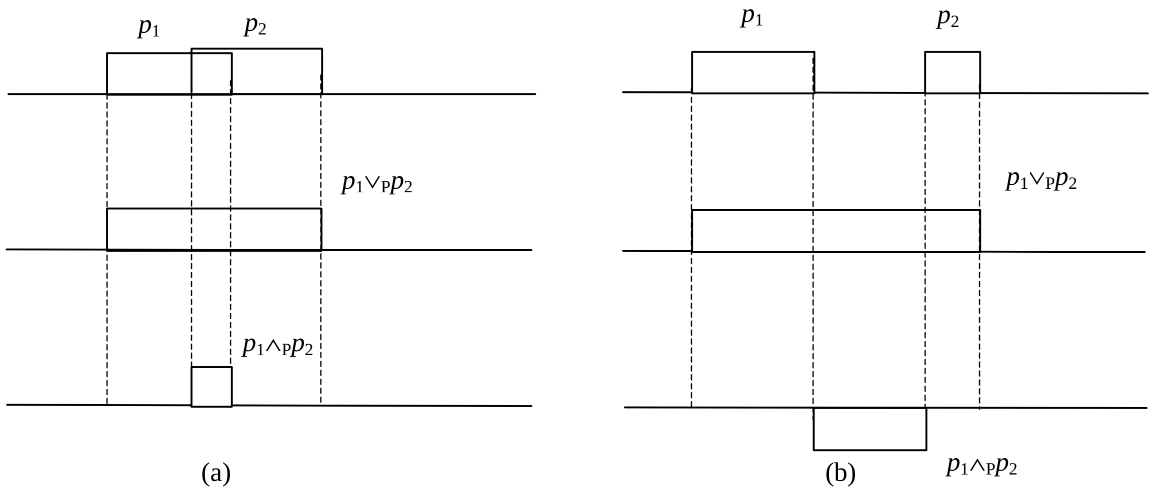

Figure 1: The *join* ($p_1 \vee_P p_2$) and *meet* ($p_1 \wedge_P p_2$) for selected pairs p_1 and p_2 of generalized intervals $p_1, p_2 \in \mathsf{P}$.
(a) "Intersecting positive generalized intervals" p_1 and p_2,
(b) "Non-intersecting positive generalized intervals" p_1 and p_2. Note that the *meet* $p_1 \wedge_P p_2$ is *a negative generalized interval*, that is a generalized interval with a negative membership function.

The notion *fuzzy lattice* has been convenient in interpreting specific and efficient schemes for clustering and classification in Kaburlasos *et al* (1997) and in Petridis *et al* (1998). It will be shown soon in the sequel that a fuzzy lattice can be defined out of P. The definition for fuzzy lattice is cited next for reader's convenience.

Definition 6 A **fuzzy lattice** is a pair (L,μ), where L is a conventional lattice and $(\mathsf{L}\times\mathsf{L},\mu)$ is a fuzzy set such that $\mu(x,y)=1$ if and only if $x \leq_L y$ in L.

The collection of fuzzy lattices is referred to as *framework of fuzzy lattices* or *FL-framework* in Kaburlasos *et al* (1997), and in Petridis *et al* (1998). It is known that if σ is an *inclusion measure*[1] in lattice L then the pair (L,σ) can define a *fuzzy lattice*. Moreover, it is known from Kaburlasos *et al* (1997) that an inclusion measure σ can be defined in a lattice L by a *positive valuation function* in L. The following proposition indicates a positive valuation function in lattice P.

Proposition 7 Function v: $\mathsf{P}\to\mathsf{R}$, which maps a positive (negative) generalized interval to its corresponding area signed by a plus (minus) sign, is a positive valuation function in P.

For lack of space, a proof of the above proposition will be given elsewhere. Note that function v maps a trivial generalized interval to number "zero". A positive valuation function v in P implies two useful functions. First, a *metric* (a *distance*) d_P between elements of P is given by $d_P(p_1,p_2)=\frac{1}{2}[v(p_1 \vee_P p_2)-v(p_1 \wedge_P p_2)]$. Second, an inclusion measure σ in P is given by $\sigma(p \leq_P q)=k(p \leq_P q)=\frac{v(q)}{v(p \vee_P q)}$, $p,q \in \mathsf{P}$.

We point out the obvious bijection (one-one mapping) between the set $\mathsf{P}(2)$ of 2-tuples and the set P of generalized intervals. Hence the two operations, namely addition ($\boxplus$) and multiplication ($\odot$), can be extended from $\mathsf{P}(2)$ to P. In conclusion the set P is further equipped with an addition ($\boxplus$) and a multiplication ($\odot$) operation.

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[1] In Petridis *et al* (1998) an inclusion measure σ is defined on a *complete lattice* L as a real function σ: $\mathsf{L}\times\mathsf{L}\to[0,1]$ which satisfies the three conditions: (*C0*) $\sigma(x,O_L)=0$, $x \neq O_L$, O_L is the *least element* of the complete lattice L, (*C1*) $\sigma(x,x)=1$, $\forall x \in \mathsf{L}$, and (*C2*) $u \leq_L w \Rightarrow \sigma(x,u) \leq \sigma(x,w)$, $\forall x \in \mathsf{L}$ (*Consistency Property*). Herein the definition of σ is enhanced so as to include non-complete lattices L. An enhanced definition for σ considers only two out of the previous three conditions; more specifically only conditions (*C1*) and (*C2*) will be considered.

We study now the relation between the sets R and P^0. In particular, the sets R and P^0 are *isomorphic (to each other) with respect to their two operations*, namely addition and multiplication, in the following sense: consider the bijection (one-one mapping) $x \leftrightarrow [x,x]$, $x \in R$, $[x,x] \in P^0$; then for $x,y \in R$ both $(x+y) \leftrightarrow [x,x] \boxplus [y,y] = [x+y,x+y]$ and $xy \leftrightarrow [x,x] \odot [y,y] = [xy,xy]$. Apart from the aforementioned isomorphism, it can be shown that the distance $d_R(x,y)=|x-y|$ between two real numbers $x,y \in R$ equals the distance $d_P([x,x],[y,y])$ between their corresponding images $[x,x]$, $[y,y]$ in P^0, and vice-versa. Nevertheless, the two sets R and P^0 differ in their own ordering relations. In particular, on the one hand the set R is a *chain* (or, *totally ordered*), that is for $x,y \in R$ either $x \leq y$ or $y \leq x$; on the other hand, the set P^0 is an *antichain*, that is $[x,x] \leq_P [y,y] \Leftrightarrow [x,x]=[y,y]$.

We recapitulate our results regarding the set P of generalized intervals: (1) P is lattice-ordered, (2) there exist a metric (a distance) in P, and (3) there exist two operations in P, namely addition ($\boxplus$) and multiplication ($\odot$). The previous exposition shows that the space P of generalized intervals is equipped with powerful tools for analysis involving jointly such heterogeneous data as intervals and real numbers.

We suggest tentatively the following interpretation for a signed membership function of a generalized interval. In particular, the sign of a membership function could be interpreted as the direction of scanning its interval-support. That is, a positive membership function may be interpreted as scanning the interval-support of a generalized interval from left to right, whereas a negative membership function may be interpreted as scanning it from right to left.

The next step is to consider FIN's other than generalized intervals so as to enhance flexibility in applications. In this introductory work we consider only the set Λ of isosceles triangular FIN's both positive and negative. Apparently there exists a *bijection* (one-one mapping) between the set P of generalized intervals and the set Λ of isosceles fuzzy sets. We define (1) an ordering, (2) a metric, and (3) an addition ($\boxplus$) and a multiplication ($\odot$) operation in Λ, all in the same manner as they have been defined previously between their corresponding generalized intervals. The computation that involves FIN's and employs the aforementioned analytical tools is called *FIN-arithmetic*.

In conclusion, we can handle either "numbers and intervals" or "numbers and isosceles FIN's" by operating on generalized intervals. In other words, in the context of the present work, we can deal with either "numbers and intervals" or "numbers and isosceles FIN's" but we can not deal simultaneously with all the three of: numbers, intervals, and isosceles FIN's.

4. EXAMPLES

In this section we demonstrate results of FIN-regression on a couple data sets originating from the literature, which involve triangular fuzzy sets. We have enhanced those data sets by noisy numeric data in order to demonstrate the capacity for regression on heterogeneous data. We do not provide examples involving intervals since accommodation of intervals is straightforward as it has been explained in the previous section 3. We define the *average distance error*, to be used in the following examples, between two series X_i, Y_i, $i \in \{1,\ldots,n\}$ of FIN's by $\frac{1}{n}\sum_{i=1}^{n} d_P(X_i, Y_i)$, where d_P is the distance (metric) function in P introduced in the previous section 3.

EXAMPLE-4.1

The data set in this example originates from Kóczy *et al* (1997) and it includes *11* pairs (*X,Y*) of triangular FIN's. For all triangular fuzzy sets in Kóczy *et al* (1997) we have retained their interval support, nevertheless we have redefined the triangle tops so as all triangles are isosceles ones. We have employed a quadratic FIN-regressor $Y=A \odot X^2 \boxplus B \odot X \boxplus C$, where $A,B,C,X,Y \in \Lambda$. A LSE quadratic regression was applied individually for the left- and for the right- ends of all interval-supports shown in Kóczy *et al* (1997). The values obtained for the coefficients *A*, *B*, and *C* have been $A=(-0.17,-0.18)$, $B=(1.57,2.32)$, and $C=(0.55,-1.14)$. Using the distance d_P between generalized intervals, the average distance error on the training data was calculated to be *0.167*. The results of the FIN-regression are shown comparatively in Table I, where the column labeled "measured FIN's" denotes isosceles triangular fuzzy sets with supports given in Kóczy *et al* (1997), and the column labeled "estimated FIN's" denotes isosceles triangular fuzzy sets which have resulted by FIN-quadratic regression.

Note that the calculated quadratic FIN-regression has effected a replacement of *11*2=22* FIN's, given in Kóczy *et al* (1997), by *3* FIN-coefficients *A*, *B*, and *C*. In this sense we have moved from the representation of fuzzy rules described by *22*2=44* parameters (real numbers) to the representation of a FIN-model described by *3*2=6* parameters (real numbers). That is a significant reduction in data representation complexity.

"measured FIN's"	"estimated FIN's"
(*0,1*)	(*0.55, 1.00*)
(*1,3*)	(*0.55, 2.77*)
(*2,4*)	(*1.95, 4.17*)
(*3,5*)	(*3.01, 5.20*)
(*4,6*)	(*3.72, 5.86*)
(*4,6*)	(*4.10, 6.16*)
(*4,6*)	(*4.13, 6.08*)
(*4,6*)	(*3.81, 5.63*)
(*3,5*)	(*3.16, 4.82*)
(*2,4*)	(*2.16, 3.63*)
(*1,3*)	(*0.82, 3.63*)

TABLE I

In the sequel, we have introduced *5* numeric data Y_i= $-0.18X^2+2X-0.3$, $i=1,\ldots,5$, where X has been a random variable uniformly distributed over interval [*0,10*]. Regression on heterogeneous (numeric & fuzzy) data was straightforward with an average distance error on the training data equal to *0.598*. An increase of the numeric data to *10* and then to *20* resulted in average distance errors *0.379* and *0.251*, respectively.

FIN's with an interval support outside the domain [*0,10*] have also been employed. Calculations of the corresponding FIN-output was straightforward by employing regressor Y=(*-0.17*,*-0.18*)⊙X^2⊞(*1.57*,*2.32*)⊙X⊞(*0.55*,*-1.14*). For instance, FIN (*-1,0*) has inferred (*-1.19*,*-1.14*), whereas FIN (*11,15*) has inferred (*-2.75*,*-6.74*).

Defuzzification has been effected in this paper by a very simple defuzzification scheme; in particular, an output FIN has been defuzzified by calculating the middle point of its corresponding interval support. Hence (*-1.19*,*-1.14*), above, has been defuzzified to -1.165; whereas (*-2.75*,*-6.74*), above, has been defuzzified to -4.745.

Another capacity of FIN regression should be pointed out; that is the capacity to produce new rules. For instance a new rule which maps the linguistic variable with isosceles membership function over interval (*-1,0*), to the linguistic variable with isosceles membership function over interval (*-1.19*,*-1.14*), has been produced by the aforementioned FIN-regressor Y=(*-0.17*,*-0.18*)⊙X^2⊞(*1.57*,*2.32*)⊙X⊞(*0.55*,*-1.14*).

EXAMPLE-4.2

The data set in this example originates from Yang *et al* (1997); it includes *40* pairs (X,Y) of triangular FIN's. For all triangular fuzzy sets in Yang *et al* (1997) we have retained their interval support, nevertheless we have redefined the triangle tops so as all triangles are isosceles ones. We have built on the preprocessing results, by Yang *et al* (1997), which correspond to two clusters by one-stage fuzzy regression analysis (Fig.4 from Yang *et al* (1997)).

Linear FIN-regression was applied individually for the left- and for the right- ends of all interval supports shown in Yang *et al* (1997). The regressor for category-1 of Fig.4 in Yang *et al* (1997) was calculated herein to be Y_1=(*0.79*, *0.76*)⊙X⊞(*-1.22*, *-0.40*). Using the distance d_P between generalized intervals, the average distance error of Y_1 on the training data was calculated to be *0.773*. The regressor for category-2 of Fig.4 in Yang *et al* (1997) was calculated to be Y_2=(*-1.11*, *-1.11*)⊙X⊞(*21.29*, *24.65*) giving an average distance error on the training data equal to *1.088*.

Note that with the aforementioned linear FIN-regressors we have replaced the *40*2=80* FIN's in Yang *et al* (1997) by *4* FIN-coefficients. In other words, we have moved from a representation complexity of *80*2=160* parameters (real numbers) in Yang *et al* (1997) to a representation complexity of *4*2=8* parameters (real numbers) herein; that is an order of magnitude reduction in data representation complexity.

In the sequel we have introduced *10* numeric data for each one of the categories- 1 and 2 using, respectively, formulas $Y_1=0.77X-0.8$, and $Y_1=-1.11X+23$, $i=1,\ldots,10$, where X is a random variable uniformly distributed over interval domain [*0*,*22*]. Regression on heterogeneous (numeric & fuzzy) data was straightforward giving an average distance error on the training data for category-1 and category-2 equal to *1.534* and *13.658*, respectively. An increase of the numeric data to *20* for each category resulted in average distance errors *0.870* and *11.264* for categories 1 and 2, respectively. A further increase of the numeric data to *40* for each category resulted in average distance errors *0.525* and *8.052* for categories 1 and 2, respectively.

Finally, as it has been illustrated in the last two paragraphs of the previous example-4.1, FIN regressors for categories- 1 and 2 can be used for calculating rapidly defuzzified outputs even when input data fall outside the domain [*0*,*22*]. Moreover new fuzzy rules can be inferred likewise as before.

5. DISCUSSION & CONCLUSION

In this work we have introduced a new formalism for *heterogeneous data* including numeric, interval-valued, and fuzzy data. The formalism in question has been effected within the framework of *fuzzy lattices* (*FL*-framework); the latter framework has been introduced in Kaburlasos *et al* (1997), and in Petridis *et al* (1998). Concrete examples have demonstrated the capacity for regression based on the aforementioned *heterogeneous data*.

Our work has enhanced the notion “fuzzy set” by introducing fuzzy sets with negative membership functions. The notion *fuzzy interval number* (*FIN*) has been introduced as a convex fuzzy set with interval support. We have introduced a novel arithmetic involving FIN’s, namely *FIN-arithmetic*, with several and substantial advantages including: (1) the capacity to deal rigorously with heterogeneous data, (2) the capacity to reduce the representation complexity of a fuzzy system by orders of magnitude; note that by looking in perspective at the aforementioned reduction in data complexity, it can be argued that FIN-regression could be employed for reducing substantially the number of rules in a fuzzy system within a specified average distance error, (3) the capacity for generalization outside the domain of given fuzzy rules, (4) the capacity for a “new type” of function approximation and regression involving jointly numeric and linguistic data, both in the function domain and range, (5) the capacity to elicit new rules by a FIN-regressor function, (6) the potential for “rapid fuzzy data processing” by a FIN-regressor function, that is an important advantage especially during defuzzification in the face of a large number of rules.

A characteristic of *FIN-arithmetic* constitutes the treatment of one fuzzy set (one FIN) as a single-datum /element of a mathematical lattice, whereas other techniques which employ the conventional fuzzy set theory may treat a fuzzy set as a constraint to be satisfied in an optimization problem, e.g. in system structure identification in Zeng *et al* (1997). Furthermore note, on the one hand, that the majority of fuzzy models employed in the literature deal with multiple-input single-output (MISO) fuzzy systems as in Bolognani *et al* (1998). On the other hand, FIN-arithmetic provides the mathematical tools for a rigorous study of multiple-input multiple-output (MIMO) fuzzy models. In addition note that FIN-arithmetic could convey additional information regarding linguistic data; for instance, it has been suggested herein that a FIN may determine the direction of traversing its interval support. Finally we remark that FIN-arithmetic is fully compatible with existing fuzzy systems.

Plans for future research include considering an employment of FIN’s with membership functions other than isosceles triangles so as to increase their utility. More sophisticated defuzzification techniques should also be considered.

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