

Intuitionistic Fuzzy Events and their Real-valued Probabilities

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ABSTRACT: A concept of a probability measure for intuitionistic fuzzy sets is introduced. The solution is given by an interval and is consistent with the probability measure proposed by Zadeh for fuzzy sets.

KEYWORDS: fuzzy set, intuitionistic fuzzy set, fuzzy event, intuitionistic fuzzy event, fuzzy probability, intuitionistic fuzzy probability.

1. INTRODUCTION

Probability theory has been a powerful tool for dealing with uncertainty. For instance, it has been able to handle uncertain events exemplified by the temperature of air tomorrow. Assuming the availability of some past data, one could have made it possible to calculate the probability of, say, the air temperature between 20° C and 25° C. However, events have had to be precisely defined.

In practice, however, people often use events which are imprecisely specified, and questions like "what is the probability of a nice weather tomorrow" are considered well valid in human discourse, though they are beyond the scope of classic probability theory.

Since the very beginning of fuzzy sets theory it has been clear that one of key issues would be to be able to devise some formal tools for expressing probabilities of such imprecisely specified events.

And, indeed, it was Zadeh (1968) who first introduced the concept of a fuzzy event, and introduced the concept of its probability by a natural extension of the probability of a nonfuzzy event. In case the probability was specified as a real number from the unit interval. Later on, some extensions have been proposed whose essence was the introduction of a fuzzy probability of a fuzzy event meant as a fuzzy number in the unit interval.

In the mid-1980s Atanassov introduced the concept of an intuitionistic fuzzy set [cf. Atanassov (1986)]. Basically, his idea was that unlike in conventional fuzzy sets in which imprecision is just modelled by the membership degree from $[0,1]$, and for which the nonmembership degree is just automatically the complementation to 1 of the membership degree, in an intuitionistic fuzzy set both the membership and nonmembership degrees are numbers from $[0,1]$, but their sum is not 1. Thus, one can express a well known psychological fact that a human being who expresses the degree of membership of an element in a fuzzy set, very often does not express, when asked, the degree of nonmembership as the complementation to 1.

This idea has led to an interesting theory whose point of departure is such a concept of an intuitionistic fuzzy set.

In this article we introduce the concept of an intuitionistic fuzzy event and a probability of an intuitionistic fuzzy event. The proposed probability lies in an interval and is such that when considered intuitionistic fuzzy event becomes a fuzzy event (by making the hesitancy margin equal to zero) the proposed interval is reduced to the probability of a fuzzy event defined by Zadeh.

Our further considerations (above the scope of this article) led to replacing an intuitionistic fuzzy event by the best corresponding fuzzy event [cf. Szmidt and Kacprzyk 1998b]. In effect the probability of such an event lies in the middle of the previously obtained interval and is consistent with the solution proposed by Gesternkorn and Manko (1990) and Zadeh. Some other properties of the interval in which a probability of an intuitionistic event lies were also considered.

2. MAIN DEFINITIONS AND RESULTS

As opposed to a fuzzy set (Zadeh, 1965) in $X = x$, given by:

$$A' = \{ \langle x, \mu_{A'}(x) \rangle \mid x \in X \} \quad (1)$$

where $\mu_{A'} : X \rightarrow [0, 1]$ is the membership function of the fuzzy set A' , an intuitionistic fuzzy set (Atanassov, 1986) $A \in X$ is given by

$$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X \} \quad (2)$$

where: $\mu_A : X \rightarrow [0, 1]$ and $\nu_A : X \rightarrow [0, 1]$ with the condition

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1 \quad (3)$$

and the numbers $\mu_A(x), \nu_A(x) \in [0, 1]$, denote the degree of membership and non-membership of x to A respectively.

Zadeh (1968) published the first work dealing with a probability of fuzzy events. By a fuzzy event A' he meant a fuzzy subset belonging to the elementary space E' , i.e. $A' \subset E'$ whose membership function $\mu_{A'}(x)$ was Borel measurable.

Definition 1. Let us assign to every element of a fuzzy event $A' \subset E' = \{x_1, \dots, x_n\}$ its probability of occurrence, i.e. $p(x_1), \dots, p(x_n)$. Probability of a fuzzy event is equal to

$$p(A') = \sum_{i=1}^n p(x_i) \mu_{A'}(x_i) \quad (4)$$

Definition 2. By an intuitionistic fuzzy event A we will mean an intuitionistic fuzzy subset belonging to the elementary event space E , i.e. $A \subset E$ whose membership function $\mu_A(x)$ and hesitancy margin $\pi_A(x)$ are Borel measurable.

Definition 3. Let us assign to every element of an intuitionistic fuzzy event $A \subset E = \{x_1, \dots, x_n\}$ its probability of occurrence, i.e. $p(x_1), \dots, p(x_n)$.

Minimal probability $p_{min}(A)$ of an intuitionistic fuzzy event A is equal to

$$p_{min}(A) = \sum_{i=1}^n p(x_i) \mu_A(x_i) \quad (5)$$

Maximal probability $p_{max}(A)$ of an intuitionistic fuzzy event A is equal to

$$p_{max}(A) = p_{min}(A) + \sum_{i=1}^n p(x_i) \pi_A(x_i) \quad (6)$$

so probability of an event A is a number from the interval $[p_{min}(A), p_{max}(A)]$, or

$$p(A) \in \left[\sum_{i=1}^n p(x_i) \mu_A(x_i), \sum_{i=1}^n p(x_i) \mu_A(x_i) + \sum_{i=1}^n p(x_i) \pi_A(x_i) \right] \quad (7)$$

Let us notice that the proposed probability (7) reduces to Zadeh's probability measure (4) in the case when the considered intuitionistic fuzzy event is replaced by ordinary fuzzy set (when assuming that hesitancy margin is equal to zero).

3. FINAL REMARKS

We proposed the probability measure for intuitionistic fuzzy events which is consistent with Zadeh's approach for fuzzy sets.

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