MECHANISMS OF THE FUZZY CHOICE

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ABSTRACT: Notions and formal models of the consequent choice fuzzy mechanism, parallel choice fuzzy mechanism, fuzzy mechanism for series-parallel choice of the decisions variants are determined here. Variants of the decisions adaptation have a fuzzy description.

INTRODUCTION

When the intelligent automated advising systems are working out, there is always the problem of the decision making models development in the manner of the choice from the certain variants set.
Fuzzy choice formalization is done in the work [Finaev V.I., Lankin A.V., Besshaposhnikov V.V. 1998]. The formalization main point was in the following.
A universal X variants set (plans, strategies or other choice objects) and space (X,M) are given. There *M* - is the variants algebra, which is known on the set X. Variants algebra can be set by some fuzzy multitude $\tilde{M}$.
When the fuzzy choice problem is solving, certain subset $\tilde{\mathbf{Y}}$ must be chosen from the presentation $\tilde{\mathbf{B}}$, coming from the $\tilde{\mathbf{g}}$ choice rule, which is set by the fuzzy relation $\tilde{\mathbf{g}} = \langle \tilde{M}, \tilde{\mathbf{A}} \rangle,$ where $\tilde{\mathbf{A}}$ is a fuzzy graph.
The choice rules structure contains fuzzy relations $\tilde{\mathbf{g}}_1, \tilde{\mathbf{g}}_2, \ldots, \tilde{\mathbf{g}}_k,$ used in the choice. The fuzzy relations $\tilde{\mathbf{g}}_1, \tilde{\mathbf{g}}_2, \ldots, \tilde{\mathbf{g}}_k$ are ranked according to their importance.
Fuzzy choice is operated according to certain algorithms, which will be named as mechanisms. Formal mathematical model must be worked out for each concrete choice mechanism.

MECHANISMS OF THE CONSEQUENT AND LEXICOGRAPHIC FUZZY CHOICE.

Let′s consider the choice variants, which are based on the approach, when separate fuzzy relations form any total choice function from the choice functions.
The choice is realized in several stages, and the variants decrease will take place on each of them. Approaching to the aim, the power of the sets $\mathbf{B} \subseteq \mathbf{X}$ will decrease after each stage.
Determination 1. If each fuzzy choice stage is realized on the base of some fuzzy relation, then the natural fuzzy choice process is called as the mechanism of the consequent fuzzy choice.
Consequent fuzzy choice mechanism will be known as the fuzzy relations set Q [Finaev V.I., Besshaposhnikov V.V. 1998] [2]

$$\mathbf{Q} = \{\tilde{\mathbf{q}}_1, \tilde{\mathbf{q}}_2, \ldots, \tilde{\mathbf{q}}_k\},$$

and its scheme will be presented by the following fig.1:
Abbreviation CFC will be used for marking the consequent fuzzy choice mechanism.
The following will be compared with the mechanism CFC

$$\tilde{\mathbf{C}}_{\mathbf{Q}}^{\text{Í ïñ}}(\mathbf{X}) = \tilde{\mathbf{C}}_{\tilde{\mathbf{g}}_k}(\tilde{\mathbf{C}}_{\tilde{\mathbf{g}}_{k-1}}(\ldots(\tilde{\mathbf{C}}_{\tilde{\mathbf{g}}_2}(\tilde{\mathbf{C}}_{\tilde{\mathbf{g}}_1}(\mathbf{X})))\ldots)). \quad (1)$$

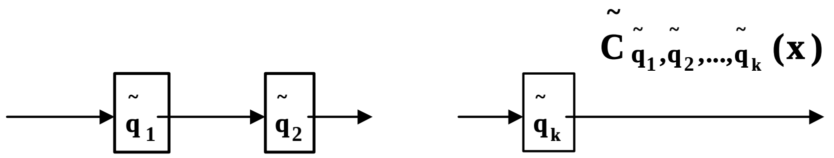

Fig.1

The analysis of the function (1) type shows, that first happens the choice by the fuzzy relation $\tilde{q}_1$. In consequence B_1 variants will be chosen and $B_1 = \tilde{C}_{\tilde{g}_1}(X) \subseteq X$. Choice by the fuzzy relation $\tilde{q}_2$ is realized on the chosen B1 variants, and then the $B_2 = \tilde{C}_{\tilde{g}_2}(B_1) \subseteq B_1$ variants choice will be done and etc. Number k will mean the depth of the consequent fuzzy choice. Function (1) is a fuzzy superposition.

If we consider that the set ***Q***={Q} of the fuzzy relations possible sets is done, then the class of the consequent fuzzy choice mechanisms will be marked by ***CFC(Q)***.

When there is a fuzzy lexicographic choice, then the fuzzy criteria are ranked by experts in such way, that the previous fuzzy criterion is more important then the other following fuzzy criteria. The fuzzy lexicographic choice rule is determined in the work [Finaev V.I., Besshaposhnikov V.V. 1998]. This rule can be considered as the particular case of the consequent fuzzy choice. The fuzzy lexicographic choice can be determined by:

$$\tilde{C}_{\tilde{\pi}}(X) = \tilde{C}_{\tilde{r}_k}(\tilde{C}_{\tilde{r}_{k-1}}(\ldots(\tilde{C}_{\tilde{r}_2}(\tilde{C}_{\tilde{r}_1}(X)))\ldots)).$$

where $\tilde{r}_i$ is a relation of the fuzzy faint order, corresponding to the fuzzy criterion $\tilde{\varphi}_i$.

MECHANISM OF THE FUZZY PARALLEL CHOICE

Determination 2. If the fuzzy choice is realized independently by fuzzy relations $\tilde{q}_1, \tilde{q}_2, \ldots, \tilde{q}_k$ and the total choice comes from the variants sets $\tilde{C}_{\tilde{g}_1}(X), \tilde{C}_{\tilde{g}_2}(X), \ldots, \tilde{C}_{\tilde{g}_k}(X)$ with the help of some fuzzy function $\tilde{F}$, then such mechanism will be known as fuzzy parallel choice by the set of the relations $Q = \{\tilde{q}_1, \tilde{q}_2, \ldots, \tilde{q}_k\}$. Abbreviation and indication FPC(Q, $\tilde{F}$) will be introduced for the fuzzy parallel choice mechanism. There is a scheme of the mechanism FPC(Q, $\tilde{F}$) on the fig. 2.

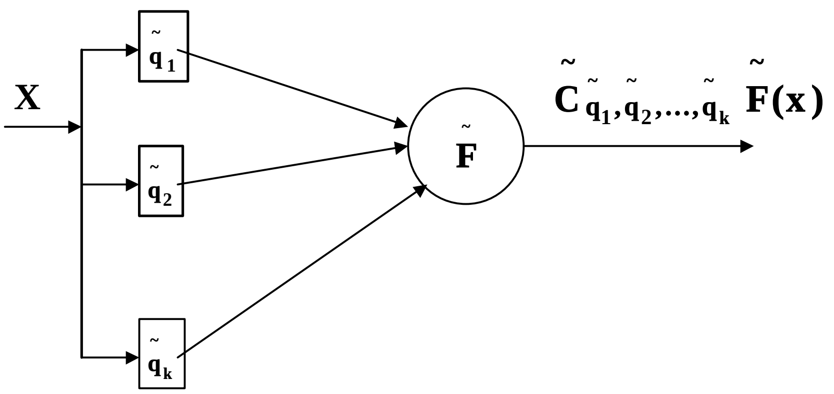

Fig.2

The following function corresponds to the mechanism FPC(Q, $\tilde{F}$)

$$\tilde{C}^{\text{íïð}}_{Q,\tilde{F}}(X) = \tilde{F}(\tilde{C}_{\tilde{g}_1}(X), \tilde{C}_{\tilde{g}_2}(X), \ldots \tilde{C}_{\tilde{g}_1}(X)). \qquad (2)$$

Fuzzy function (2) is got by the composition according to the fuzzy rule $\tilde{F}$ from $\tilde{C}_{\tilde{g}_1}, \tilde{C}_{\tilde{g}_2}, \ldots, \tilde{C}_{\tilde{g}_k}$.

Theorist - plural operations are implied under the fuzzy rule $\tilde{F}$.

Necessary and sufficient condition of the fuzzy composition application in the manner of the choice function (2) is the condition that $\tilde{F}(\tilde{\varnothing}, \tilde{\varnothing}, \ldots \tilde{\varnothing})$ is fuzzy equal to $\tilde{\varnothing}$. Otherwise, fuzzy choice will not satisfy the fuzzy inclusion.

$$\tilde{C}(X) = \tilde{C}_{Q,\tilde{F}}^{\text{í ï ð}}(X) \subseteq X. \tag{3}$$

Necessity of the mentioned condition comes from the fact, that the choice from the fuzzy - empty sets must be fuzzy empty. Here is the sufficiency of the condition. If we take any pair $< \mu_C(x)\ /(x) > \in \tilde{C}(X)$ and $\tilde{C}(X)$ is fuzzy unequal to the fuzzy empty set, then it is followed from (2), (3) and fuzzy operation $\tilde{F}$ determination, that it is always possible to find such fuzzy relation $\tilde{q}_i$, when $< \mu_C(x)\ /(x) > \in \tilde{C}(X)$, and that is why x∈X. x was taken arbitrarily, so $\tilde{C}(X) \subseteq X$.

Class of all parallel fuzzy choice mechanisms will be known as ***FPC(Q).*** Fuzzy rules $\tilde{F}_+$ can be used in the capacity of the fuzzy composition methods**.**

Fuzzy superposition $\tilde{C}_{\tilde{r}2}(\tilde{C}_{\tilde{r}1}(X))$ operation and fuzzy composition $\tilde{F}(\tilde{C}_{\tilde{g}_1}(X), \tilde{C}_{\tilde{g}_2}(X), \ldots \tilde{C}_{\tilde{g}_1}(X))$ operation can be used to arbitrary around determined choice functions. Using these fuzzy operations it is possible to create more difficult fuzzy choice mechanisms on the base of $Q = \{\tilde{q}_1, \tilde{q}_2, \ldots, \tilde{q}_k\}$.

It is necessary to distinguish the consequent - parallel fuzzy choice mechanism, the main idea of which is presented on the fig. 3.

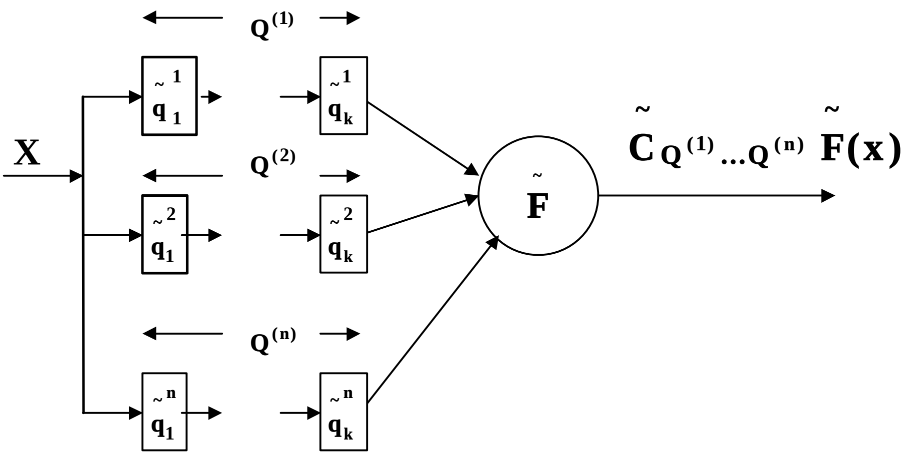

Fig.3

That choice is realized in the form of the procedure with experts participation. Each expert i make a consequent choice on the fuzzy relations set $Q^{(i)}$

$$Q^{(i)} = \{\tilde{q}_1^{(i)}, \tilde{q}_2^{(i)}, \ldots, \tilde{q}_k^{(i)}\}.$$

Final choice is always formed from the variants set with the help of the fuzzy composition $\tilde{F}(B_1, B_2, \ldots, B_k)$, where

$$B^{(i)} = \tilde{C}_{Q_i}^{\text{í ñ}}(X) = \tilde{C}_{\tilde{g}_k^{(i)}}(\tilde{C}_{\tilde{g}_{k-1}^{(i)}}(\ldots(\tilde{C}_{\tilde{g}_2^{(i)}}(\tilde{C}_{\tilde{g}_1^{(i)}}(X)))\ldots)).$$

Class of all the consequently - parallel fuzzy choice mechanisms with fuzzy relations Q sets will be marked in ***CPFC(Q)***.

GENERAL PROBLEMS AT FUZZY CHOICE MECHANISMS STUDY

When fuzzy choice models are working out, such constructive type traditional problems as analysis, synthesis and optimization appear.
According to the work [Lyapunov A.A., Yablonsky S.V. 1963] choice mechanism M must be considered in the unity of the mechanism structure and fuzzy choice function $\tilde{C}_M(X)$.

Basing on the mechanism $M = \{B, \tilde{\pi}\}$, fuzzy choice function $\tilde{C}_M(X)$ search and description form the analysis task.

This problem can be presented easier as the chosen variants fuzzy set $\tilde{C}_{<B,\tilde{\pi}>}(X) = \tilde{C}_M(X)$ search on the structure $B \subseteq X$ accordingly to the rule $\tilde{\pi}$. After that it is sufficiently to form a table, containing the list of all the set X elements with the indication of all possible fuzzy choice rules $\tilde{C}_M(X)$ [Finaev V.I., Besshaposhnikov V.V. 19.98].
Taking into consideration the fact that this table will have too large dimensionality, then, using this method, the fuzzy functions $\tilde{C}_M(X)$ entry can not be realized from the standpoint of the computing efficiency.
It is well - known [Ahoe A., Hopkroft D., Ulman D. 1979, Gary M., Jdonson D. 1982] that the efficient algorithms are the algorithms, for which machine working time is limited by some multinomial from the problem initial data dimensions. Algorithms, requiring more then polynomial number of steps, are practically unrealized.

Line quantity of the table, assigning the fuzzy choice function realization, will be equal to $m^n \left|\tilde{C}_M(X)\right|$ at best. There $n = |X|$, and m - is the power of the alphabet, used for set X elements description. So, lines quantity will increase quicker then any multinomial from the dimensionality. Consequently, all the fuzzy choice mechanisms analysis methods, based on the tabular task of the fuzzy choice functions will be inefficient when min n values are large.
So, it is worth while to chose an analysis language in the form of the fuzzy conclusion rules, which cover the most typical situations and they are realized with the fuzzy logic technique usage [Melihov A.N., Bershteyn L.S., Korovin S.Y. 1990., Averkin A.N., Batirshin I.Z., Blinshun A.F., Silaev B.V., Tarasov B.N. 1986].
The synthesis problem is in the fuzzy choice mechanism search on the fuzzy choice function, which is realized by it.

Experts describe the fuzzy choice function $\tilde{C}(X)$ as the set of the decision inference rules, which satisfy the fuzzy criteria $\tilde{\varphi}_1, \tilde{\varphi}_2, \dots, \tilde{\varphi}_d$ totality.

When experts work out fuzzy choice functions $\tilde{C}(X)$, they are based on the decision-making processes [Larichev O.I. 1981], deciding what is more preferable in these or those hypothetical situations.
As a rule synthesis problems do not lead to the only decision in the traditional analytical problems. This defect is formally overcomed by the fuzzy choice.
Fuzzy choice function contains the set of rules, formulated by experts, and different choice mechanisms from the examined class can correspond to them.
Numerical values I(M) which characterize difficulty are connected [Larichev O.I. 1981] with the choice mechanism.
The optimum synthesis problem is in the creation of the minimum difficulty mechanism, which realizes the fuzzy choice function.
Different parameters can be understood under the word ”difficulty”. It can be the number of choice stages (choice depth for the consequent fuzzy choice), number of fuzzy criteria and fuzzy relations and their interaction in a choice, that can be also any other engineering - exploratory evaluations, which numerically characterize the fuzzy choice mechanism.
In general case the optimum synthesis problem is known as an insoluble one. The proof of the similar problems NP-difficulty is known [Gary M. 1982].

Well-known [Shannon S.E. 1949, .Lupanov Î.B. 1963] synthesis methods elaboration guarantees the difficulty realization evaluation of the choice functions with given parameters, where requirements to the synthesis methods are not so strong.
A great number of works, survey of which is in the work [Sholomov L.À. 1983], can be devoted to difficulty evaluations and difficulty qualitative conduct research, interaction of different difficulty characteristics. These results can also be used for fuzzy choice mechanisms without essential changes.
A very important conclusion is the fact that the fuzzy choice mechanism optimization problem closely sides with the optimum synthesis problem. The main idea of it is in the following.

Two fuzzy choice functions $\tilde{C}_{M_1}(X)$ è $\tilde{C}_{M_2}(X)$ will be fuzzy equal functions if the variants sets, got as the result of their operations, are equal.
Fuzzy choice mechanism will be a minimum one, if it has the least difficulty amongst all the M class mechanisms equal to it.
According to the practice of solving choice problems on the binary relations [Sholomov L.P. 1989], these problems practically has not any analytical decision.
When choice variants and criteria are fuzzy set, then fuzzy choice mechanisms are realized. Experts analyze certain set of choice variants (choice scenarios), then fuzzy choice criteria are given, fuzzy relation $\mathbf{Q}$ is set between choice variants, and then a choice algorithm can be worked out according to the considered models.

CONCLUSIONS

When choice variants $x \in X$ (plans, technique or other choice objects) and criteria are fuzzy determined on the set X, then models of successive, lexicographic, parallel and successively - parallel fuzzy choices are formally determined. These models will find their application at the decision making intellectual systems working out.

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