VALIDITY OF FUZZY CHOICE

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ABSTRACT: It is offered to modificate the probability standard of fuzzy choice mechanisms validity value coming from probability aspects of functions presentation of fuzzy variables accessories.

INTRODUCTION

A validation, technique or numerical evaluation of validity or accuracy of the research must accompany engineering and research works with the help of the methods of fuzzy inference.
Well-known traditional criteria for accuracy value are the following [Temnikov F.E., Aphonin V.A., Dmitriev V.I. 1971] - criterion of even approximation or the most deflection, criterion for root-mean-square approximation, criterion of integral approach.
If simulation modeling operates the research, the validity value of the modeling results is obtained by using the probability criterion [Buspenko N.P. 1978, Sovetov B.Y. 1985], which is set by:

$$\mathbf{P} = \{|\varepsilon(t) - \varepsilon^*(t)| \le \delta\} = \alpha, \tag{1}$$

where $\varepsilon^*(t)$ - is the meaning of random quantity, got as the result of simulation modeling, $\varepsilon(t)$ - is the theoretical (real) value of random quantity, δ - is the evaluation accuracy, α - is the probability of the fact that an empirical evaluation of the random quantity $\varepsilon(t)$ theoretical value will not be higher then the δ value.
Technique for the fussy choice validity evaluation is necessary for organization and investigation of fussy choice problems and mechanisms of its realization [Finaev V.I., Lankin A.V., Besshaposhnikov V.V. 1998].
Obviously that using of the known criteria is impossible in this case, but the probability criterion (1) is the most close one to it.
Let′s set the problem of the probability criterion modification (1) and of the working out the approach to the fuzzy choice validity evaluation using some of the choice Ì mechanisms.

NOTION OF THE FUZZY CHOICE

A universal X variant set is given (plans, strategies or other choice objects). Some fuzzy sets $\tilde{\mathbf{B}}$ with the qualitative variant description from the set X are determined by experts on the X variants set. The choice occurs when the variants qualitative evaluation of presentations happens and the set of choice functions in the manner of fuzzy relations takes place.
The method of the fuzzy choice is mainly defined by the fuzzy choice rule, which structure contains fuzzy relations $\tilde{g}_1, \tilde{g}_2, \ldots, \tilde{g}_k$ participating in the choice. These fuzzy relations appear when there is an analysis of choosing variants aspects, as well as at the participation in the choice of experts with mismatched standpoints. The fuzzy relations $\tilde{g}_1, \tilde{g}_2, \ldots, \tilde{g}_k$ are ranked on importance.
Parts forming all the structures must be found for each concrete choice mechanism.

Fuzzy criterions $\tilde{\varphi}_1, \tilde{\varphi}_2, \dots, \tilde{\varphi}_d$ are set by experts on the X variant multitude. Fuzzy criterions $\tilde{\varphi}_1, \tilde{\varphi}_2, \dots, \tilde{\varphi}_d$ can be ranked according to importance and they influence on the structure of the multicriterious choice.

The choice is realized in several stages, on each of which the reduction of variants occurs, i.e. the power of the sets $B \subseteq X$ will decrease after each stage of choice while the moving to the purpose.

As each stage of fuzzy choice is realized on the basis of fuzzy relations, so the process of fuzzy choice is identified as a mechanism of consequent fuzzy choice [Finaev V.I., Lankin A.V., Besshaposhnikov V.V. 1998].

The consequent fuzzy choice mechanism is defined by the fuzzy relations set Q

$$Q = \{\tilde{q}_1, \tilde{q}_2, \dots, \tilde{q}_k\}.$$

Fuzzy choice mechanisms which were considered in [Finaev V.I., Lankin A.V., Besshaposhnikov V.V. 1998] must satisfactory forecast the result of the choice even in the situations, when the result is not observed, i.e. a priori entropy of the result is as large as you want.

So, in any case fuzzy choice occurs on the original, arbitrarily given presentation sets - an initial set X and the variants multitudes of the following choice stages $B_i \subseteq X, \quad i = \overline{1,k}$, where k is the total number of stages.

A standpoint on the efficient using of simple mechanisms is given and explained in the work [Vapnic V.N., Chervonenkis A.Y. 1974]. Its results have a deep statistical basis.

Approaches of the work [Vapnic V.N., Chervonenkis A.Y. 1974] are developed in work [Shopomov L.A., Yudin D.B. 1986] and carried on problems of choice mechanisms building while using the binary relations.

PROBABILITY FUZZY CHOICE VALIDITY EVALUATIONS

Let′s consider a variant of the forecast problem organization of the fuzzy choice within certain given validity [Finaev V.I., Besshaposhnikov V.V. 1998].

Let′s enter a notion of the fuzzy choice $\tilde{\tilde{N}}(X)$ fuzzy mistake $\tilde{\varepsilon}_{\tilde{C}}(M, X)$ on the X variants presentation set while using a mechanism M.

Let′s consider the following for notion and way illustration of fuzzy mistake $\tilde{\varepsilon}_{\tilde{C}}(M, X)$ formalization.

Set X elements are ranked according to the uncertainty growth in connection of choice validity by empirical way.

Let′s enter a notion of linguistically variable α_i - "mistake of the fuzzy choice mechanism ¹ i".

$\grave{O}(\alpha_i)$ - linguistically variable α_i therms-set, is defined by experts, as well as functions of fuzzy variables accessories from the therms-sets $\grave{O}(\alpha_i)$.

Let $\grave{O}(\alpha_i) = \{\beta_1^i, \beta_2^i\}$ be defined by experts, where β_1^i - is "an insignificant mistake of the fuzzy choice mechanism ¹ i ", β_2^i - is an "essential mistake of the fuzzy choice mechanism ¹ i".

Fuzzy variables β_j^i are assigned in the manner of three-tupled sets $< \beta_j^i, X, \tilde{C}(\beta_j^i) >$, where $\tilde{C}(\beta_j^i)$ - fuzzy sets, assigned on the set X, so that $\tilde{C}(\beta_j^i) = \{< \mu_{\tilde{C}(\beta_j^i)}(x) / (x) >\}$.

Fuzzy mistake $\tilde{\varepsilon}_{\tilde{C}}(M, X)$ is assigned as a fuzzy variable on the basic set X and has notion "fuzzy realization $\tilde{\tilde{N}}_M(X)$ is wrong".

Let fuzzy choice function $\tilde{\tilde{N}}(X)$ be used on the set $B = \{B_1, B_2, ..., B_k\}$.

Frequency of mistakes, which appear under realization of the fuzzy choice $\tilde{\tilde{N}}$ on the set B by means of the mechanism $\grave{I}_i$ will be defined as $\Delta(M_i, B)$.

If to consider a theorist-plural sense of membership functions definition, then the frequency of mistakes, which appear during the realization of the $\grave{I}_i$ mechanism on the set $B_l \subseteq X$, $l = \overline{1, k}$, will be defined in the following way.

Let′s define memberships degrees $\mu_{\tilde{C}(\beta^i)}(x_p)$, which characterize the mistake, for each variable β^i_j .

If the linguistically variable α_i has single therms β^i with semantic value "mistake of the fuzzy choice mechanism ¹ i", then the frequency of mistakes is determined by

$$\Delta_l(M_i, B_l) = \frac{1}{|B_l|}\sum_{p=1}^{|B_l|}\mu_{\tilde{C}(\beta^i)}(x_p), \quad x_p \in B_l.$$

If the linguistically variable α_i has several therms β^i_j, $j = \overline{1, z}$, then for each j therms the value of the mistakes frequency of the fuzzy choice will be defined by

$$\Delta_j(M_i, B_l) = \frac{1}{|B_l|}\sum_{p=1}^{|B_l|}\mu_{\tilde{C}(\beta^i_j)}(x_p), \quad x_p \in B_l,$$

then the value of the mistake frequency of fuzzy choice $\tilde{\tilde{N}}$ will be determined as an average value by

$$\Delta_l(M_i, B_l) = \frac{1}{z}\sum_{j=1}^{z}\Delta_j(M_i, B_l).$$

Total mistake of the mechanism $\grave{I}_i$ at the realization of fuzzy choice $\tilde{\tilde{N}}$ on the set B is defined as an average value

$$\Delta(M_i, B) = \frac{1}{k}\sum_{l=1}^{k}\Delta_l(M_i, B_l).$$

Let an average value of the fuzzy mistake $\tilde{\varepsilon}_{\tilde{C}}(M_i, X)$ be considered as a probability of the mistake $P_{\tilde{C}}(M_i)$ at fuzzy choice function $\tilde{\tilde{N}}_{M_i}(X)$ realization of the choice mechanism i.

Let M be the class, embracing all the mechanisms of the fuzzy choice $\grave{I}_i$. The elements of the class are considered to be priori determined.

The number of the fuzzy choice functions $\tilde{\tilde{N}}_{M_i}(X)$ will be defined by $G = \left|\tilde{\tilde{N}}_{M_i}(X)\right|$.

Then H(*M*)=log2G will be an entropy of the class *M*.

It is possible to get an evaluation under the law of greater numbers from the Gephding inequality [Hoeffding W. 1963]

$$P\{\left|P_{\tilde{C}}(M_i) - \Delta(M_i, B)\right| \geq \delta\} \leq l^{-2\delta^2 k}$$

for the fixed mechanism $\grave{I}_i$.

Value $\eta = G l^{-2\delta^2 k}$ is limited by the probability of the fact that at least for one fuzzy choice mechanism $\grave{I}_i$ $\left| P_{\tilde{C}}(M_i) - \Delta(M_i, B) \right| \geq \delta$ exists, where δ - is a fuzzy choice accuracy, realized by the fuzzy function $\tilde{\tilde{N}}(X)$.

Coming from the condition that $\eta < 1$ it is possible to find the presentations set B value according to the formula

$$k \geq \frac{1}{2\delta^2}(H(M) \ln 2 - \ln \eta) \quad (2)$$

It is necessary to make a conclusion that admitting fuzzy choice on each of the sets $X, B_1, B_2, \dots, B_l, \dots, B_k$ it is possible to confirm with $1 - \eta$ probability that if the set B power k satisfies inequality (2), then

$$P_{\tilde{C}}(M_i) \leq \Delta(M_i, B) + \delta \quad (3)$$

is true for any fuzzy choice mechanism $\grave{I}_i \in M$.

In other words this conclusion means that if this mechanism $\grave{I}_i$, realizing fuzzy choice $\tilde{\tilde{N}}(X)$ on the set B is found and condition (3) is executing, the fuzzy choice will be realized on the whole variants set X with the accuracy δ.

The qualitative analysis of the condition (2) allows making one important conclusion more.

With the fuzzy choice mechanisms class M entropy reducing, the set B power can be reduced if the condition (3) is carried out, i.e. if the required accuracy of the whole choice variants set X is provided. Really, coming from the reasoning logic, more information for fuzzy choice realization is demanded with the class M "Universality" growth.

The connection between entropy and such k evaluation as (2) means that it is better to use simple mechanisms for good fuzzy choice prediction.

CONCLUSIONS

The fuzzy choice validity determination technique is worked out. On the basis of the choice entropy notion and the criterion got from the Gephding′s inequality, the probability values of the fuzzy choice validity have been received.

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