Strong And Weak Stability In Possibilistic Linear Programming[*]

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ABSTRACT: In this article the questions of stability of possibilistic linear programming problems are investigated. Conditions of strong stability (stability by solution) for base models are received.

KEYWORDS: Possibility Measure, Possibilistic Linear Programming, Stability.

INTRODUCTION

This article is devoted to researching the problem of stability in possibilistic optimization raised in works by Kovacs, et. al. (1989) and Fuller (1989). In previous works by the authors — Rybkin and Yazenin (1998, 1999), the functional stability (or weak stability, stability by result) for base models of possibilistic optimization introduced by Yazenin (1987) was proved. It is shown that models based on system of linewise restrictions on possibility are unstable in general. In this connexion the method of stable regularization for such models based on lowering the possibility of satisfying the problem restrictions was proposed. In the present paper within the framework of the developed approach to investigation of stability the interrelation of stability of possibilistic optimization problem by result and by solution is established. The conditions of stability for considering models are determined.

1 BASIC CONCEPTS AND NOTATIONS

In our approach to formalization of fuzziness based on Zadeh (1978), Nahmias (1978), Yazenin (1987, 1996), Yazenin and Wagenknecht (1996) a triplet $(\Gamma, P(\Gamma), \pi)$ is a possibilistic space, Γ is a crisp set of elements $\gamma \in \Gamma$, $P(\Gamma)$ is the power set of Γ, π — a possibility measure.

Definition 1.1. *A possibility measure is a set function* $\pi : P(\Gamma) \to [0,1]$ *with the properties*

$$\pi\{\varnothing\} = 0, \quad \pi\{\Gamma\} = 1,$$
$$\text{ð}\{\bigcup_{i\in I} A_i\} = \sup_{i\in I}\{A_i\},$$

for any index set I and sets $A_i \in P(\Gamma)$.

Definition 1.2. *A necessity measure is a set function* $\nu : P(\Gamma) \to [0,1]$ *defined by*

$$\nu\{A\} = 1 - \pi\{\hat{A}\},$$

where $\hat{A}$ *denotes the complement of A in* $P(\Gamma)$.

Definition 1.3. *A possibilistic variable is a mapping* $X : \Gamma \to E^1$ *with values limited by function* $\text{ì}_X : E^1 \to [0,1]$

$$\text{ì}_X(x) = \pi\{\gamma \in \Gamma \mid X(\gamma) = x\}, \quad \forall x \in E^1.$$

$\text{ì}_X(x)$ is called *distribution* of X and gives the possibility of taking value x by variable X.

Definition 1.4. *A support of possibilistic variable X is a set*

$$\text{supp}(X) = \{x \in E^1 \mid \text{ì}_X(x) > 0\}.$$

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Definition 1.5. *An* α*-level set of possibilistic variable X is a set*

$$[X]^{\alpha} = \{x \in E^1 \mid \mu_X(x) \geq \alpha\}, \quad \alpha \in (0,1].$$

We shall hereinafter assume that $[X]^0 = \mathrm{cl}\,(\mathrm{supp}\,(X))$ is a compact.

Let $\boldsymbol{F}(E^1)$ be a set of all possibilistic variables that have compactly supported quasiconcave distributions with closed α-level sets $\forall\alpha \in (0,1]$. We define a metric in set $\boldsymbol{F}(E^1)$ following Puri and Ralescu (1986) by

$$d(X,Y) = \sup_{0 \leq \alpha \leq 1} d_H([X]^{\alpha}, [Y]^{\alpha}),$$

where d_H is a Hausdorf metric:

$$d_H(A,B) = \max\,[\Delta(A,B), \Delta(B,A)].$$

Here $\Delta(A,B) = \sup_{a \in A} \inf_{b \in B} \|a - b\|$, $\Delta(B,A) = \sup_{b \in B} \inf_{a \in A} \|a - b\|$ are the semimetrics.

Theorem 1.1. $(\boldsymbol{F}(E^1), d)$ *is a metric space*.

Further $\boldsymbol{F}_c(E^1) \subseteq \boldsymbol{F}(E^1)$ be a subset of possibilistic variables with continuous distributions.

2 FORMALIZATION AND STABLE CRITERIA OF POSSIBILISTIC OPTIMIZATION PROBLEMS

The problem of possibilistic optimization can be described by the following elements (see Yazenin and Wagenknecht (1996)):

1. set of decision variables of special structure $X \subseteq E^n$;
2. possibilistic space $(\Gamma, P(\Gamma), \pi)$;
3. set of possibilistic functions $f_i(\cdot,\cdot): E^n \times \Gamma \to E^1$, $i = 0,\ldots,m$, modelling the performance criteria and the set D of problem restrictions*;*
4. model of decision making $\mathrm{MD}(X, (\Gamma, P(\Gamma), \pi), f_0, \ldots, f_m)$.

We shall consider the problems of linear possibilistic programming. In this case the elements of a problem assuming the following form:

$$X = \{x \in E^n \mid x_1, \ldots, x_n \geq 0\}, \quad f_i(x,\gamma) = \sum_{j=1}^{n} a_{ij}(\gamma) x_j - b_i(\gamma),$$

$$D = \{x \in X \mid \sigma_i\{f_i(x,\gamma)\ R_i\ 0\} \geq \alpha_i,\ \alpha_i \in (0,1],\ i = 1,\ldots,m\},$$

where a_{ij}, b_i are min-related possibilistic variables defined on $(\Gamma, P(\Gamma), \pi)$, σ_i is measure π or measure ν, R_i is a binary relation on $E^1 \times E^1$, α_i are levels of possibility or necessity. Such model of restrictions is known as model of *linewise restrictions on possibility or necessity*.

Different models of criteria may be used for modeling possibilistic linear programming problem. We shall consider the following ones:

Model of modal value optimization:

$$\mathrm{M}\left\{\sum_{j=1}^{n} a_{oj}(\gamma) x_j\right\} \to \max_{x \in D},$$

where M is an operator of possibilistic function modal value determination.

Level optimization model:

$$k \to \max_{x \in D},$$

$$\pi\left\{\sum_{j=1}^{n} a_{oj}(\gamma) x_j = k\right\} \geq \alpha_o.$$

This model of criterion is also known as *maximax* model.

Model of maximization of the measure of satisfying criteria:

$$s\left\{\sum_{j=1}^{n} a_{oj}(\tilde{a})x_j \mathrm{R}_o b_o(\tilde{a})\right\} \to \max_{x\in D},$$

where σ is a measure π or measure ν **.**

For these models we give definitions of strong and weak stability in accordance with classic results by Tikhonov and Arsenin (1974), Fiodorov (1979).

Let $\Re$ be the optimal value of the criterion function (solving result) of the problem with possibilistic parameters $a_1, \ldots, a_p$, $\Re$ attained in $X_0 \subseteq X$, $\Re^{\varepsilon}$ is a result of solving this problem with parameters $a_1^{\varepsilon}, \ldots, a_p^{\varepsilon}$ such that

$$\max_{i=1,\ldots,p} d\left(a_i(\tilde{a}), a_i^{\mathring{a}}(\tilde{a})\right) \le \mathring{a},$$

where $\varepsilon \in E^1$, $\varepsilon > 0$, $\Re^{\varepsilon}$ attained in $X_0^{\varepsilon} \subseteq X$.

Definition 2.1. *Possibilistic optimization problem is stable by the result when* $|\Re^{\varepsilon} - \Re| \to 0$ *if* $\varepsilon \to 0+$.

Definition 2.2. *Possibilistic optimization problem is stable by the solution when* $\Delta(X_0^{\varepsilon}, X_0) \to 0$ *if* $\varepsilon \to 0+$.

3 MAIN RESULTS

In researching stability of possibilistic programming problems we need to clarify two important questions. First, we must fix models that have property of strong or weak stability under appropriate conditions, and, secondly, determine the conditions of strong stability for weak stable models.

Theorem 3.1. *If introduced above models are stable by result or by solution, they have a bounded set of decision.*

Under this condition the following theorems hold.

Theorem 3.2. *Let in model of modal optimization possibilistic parameters belong* $\boldsymbol{F}(E^1)$, *parameters of linewise restrictions belong* $\boldsymbol{F}_c(E^1)$. *Then model is stable by the result if and only if it is stable by the solution.*

Theorem 3.3. *Let in model of level optimization possibilistic parameters belong* $\boldsymbol{F}(E^1)$, *parameters of linewise restrictions belong* $\boldsymbol{F}_c(E^1)$. *Then model is stable by the result if and only if it is stable by the solution.*

Theorem 3.4. *Let possibilistic parameters of possibility or necessity maximization model and linewise restrictions belong* $\boldsymbol{F}_c(E^1)$. *Then model is stable by the result if and only if it is stable by the solution.*

The following theorem determines condition of stability for all considering problems. Let

$$D_{\eta} = \{x \in X \mid \pi\{ f_i(x,\gamma) = 0\} \ge \alpha_i + \eta,\ i=1,\ldots,m\}.$$

Theorem 3.5. *Let in a problem of possibilistic optimization model of criterion is stable by the result and set of restrictions D is bounded. If exists* η_0 *such that for all* $\eta < \eta_0$ *the set* D_{η} *is bounded then this problem is stable by the solution.*

SUMMARY

In this paper three base models of possibilistic otpimization are investigated — models of modal and level optimization and model of measure maximization. It is shown that in case of stability of the models (by result or by solution) they have a bounded set of decision. In this condition the equivalence of strong and weak stability is asserted. The condition of strong stability for models with system of linewise restrictions on possibility is received.

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