Fuzzy Reactive Motion Control among Dynamic Obstacles for Robot Manipulators

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Abstract: This paper focuses on autonomous motion planning of manipulators in known environments and with unknown dynamic obstacles. The navigation technique of robot control using artificial potential functions is based on fuzzy logic and stability is guaranteed by Lyapunov theory. A fuzzy logic system or fuzzy system is a universal approximator which provides a rule-based mapping between the input and the output space, while classical approaches make use of analytic harmonic functions to solve the navigation problem. In this particular application, the fuzzy system proposed is used to approximate the gradient of the harmonic functions.

Keywords: Artificial potential fields, Autonomous motion planning, Dynamic obstacle avoidance, Fuzzy controller, Lyapunov stability, and Robot manipulators.

I. INTRODUCTION

Robots are widely used for tasks such as material handling, spot and welding, spray painting, mechanical and electronic assembly, material removal and water jet cutting etc. Most of such tasks include a primary problem of getting a robot to move from one position to another without bumping into any obstacles. This problem, denoted as the Robot Motion Planning problem, has been the great subject among researchers.

The obstacle avoidance problem is important for both mobile robots and manipulators. A robust obstacle scheme should be capable of dealing with moving obstacles. For a manipulator, the problem is more complex. Not only must the end effector move to the desired destination without collisions with obstacles, but also the links of the arm must avoid collisions. Because this precedent requirement is more restrictive, a strategy that works for manipulators can be applied to mobile robots.

Every method concerning robot motion planning has its own advantages and application domains as well as its disadvantages and constraints. Therefore it would be rather difficult either to compare methods or to motivate the choice of a method upon others.

In contrast to many methods, robot motion planning through artificial potential fields (APF) considers simultaneously the problem of obstacle avoidance and that of trajectory planning. In addition the dynamics of the manipulator are directly taking into account, which leads in our opinion to a more natural motion of the robot. However, the major problem in the potential field motion planning approach is the occurrence of local minima in the potential field, which cause the untimely termination of the motion of the robot. A robust APF should have no local minima. These kind of robust artificial potential fields include the employment of the so-called harmonic functions.

Using a potential function to accomplish a certain motion implies that the trajectory of the robot is not known or calculated in advance, which means that the robot chooses autonomously its way to reach its goal.

The problem addressed in this paper is the autonomous motion planning of manipulators in known environments and with unknown, possible moving, obstacles. The navigation technique of robot control using artificial potential functions is based on fuzzy logic and stability is guaranteed by Lyapunov theory. A fuzzy logic system or fuzzy system is a universal approximator which provides a rule-based mapping between the input and the output space, while classical approaches make use of analytic harmonic functions to solve the navigation problem. In this particular application, the fuzzy system proposed is used to approximate the gradient of the harmonic functions.

This paper is organised as follows. In Section II preliminaries, we present the dynamic model of the manipulator and its properties. Section III examines the motion planning through the artificial potential functions. After some definitions and requirements, harmonic functions are introduced. Section IV proposes a fuzzy control strategy involving artificial potential functions. The stability is proved by Lyapunov theory. Section V presents some simulation results. Finally, Section VI gives some concluding remarks.

II. ROBOT MANIPULATOR MODEL AND ITS PROPERTIES

The model for an n-link robot manipulator is given by [8]

$$T = M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + F_d\,\dot{q} + F_s(\dot{q}) + T_d \tag{1}$$

where $q, \dot{q}, \ddot{q} \in R^n$ denoting the link position, velocity, and acceleration vectors, respectively, $M(q) \in R^{n\times n}$ the inertia matrix, $C(q,\dot{q}) \in R^n$ the coriolis and centrifugal torque vectors, $G(q) \in R^n$ the gravitational torque vectors, $F_d \in R^{n\times n}$ the diagonal matrix of viscous and/or dynamic friction coefficients, $F_s(\dot{q}) \in R^n$ the vectors of unstructured friction effects such as static friction terms, and $T_d \in R^n$ the vectors of any generalised input due to disturbances or unmodeled dynamics.

For the structure of the classical PD controller, asymptotic stability can only be achieved if the static friction $F_s(\dot{q})$ and disturbance vectors T_d are negligible. This means that (1) becomes

$$T = M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + F_d\,\dot{q} \tag{2}$$

On the other hand

$$T = T' + F_d\,\dot{q} \tag{3}$$

where $T' = M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q)$ is an ideal robot model equation.

III. MOTION PLANNING THROUGH THE ARTIFICIAL POTENTIAL FIELDS

The problem of moving in space while avoiding collision with the environment is known as obstacle avoidance or path planning. The path-planning problem has generally been approached in two ways: using graph methods and using APF approach. This paper proposes the use of an APF approach, which can best be described as a local methodology.

The essence of the APF approach is that points along the robot's path may be considered to be attractive forces and obstacles in the environment are repulsive forces. The interaction between the APF causes a net force to act on the robot, which will guide it along an obstacle free path.

A. GENERAL CONCEPTS

The APF approach may be described as follows. If q_d designates the goal position, the guidance of the robot with respect to n obstacles O_i, $(i = 1,....,n)$, can be achieved by subjecting it to the APF:

$$\varphi_{art}(q) = \varphi_{q_d}(q) + \sum_{i=1}^{n} \varphi_{O_i}(q) \tag{4}$$

where $\varphi_{art}(q$ is the total strength of the APF at the point q, $\varphi_{q_d}(q)$ the APF strength contribution from the attractive goal and $\varphi_{O_i}(q)$ is the contribution from the i^{th} repulsive obstacle. This field causes the following artificial force to act on the robot:

$$F_{art}(q) = F_{q_d}(q) + \sum_{i=1}^{n} F_{O_i}(q) \tag{5}$$

where $F_{q_d}(q) = -\nabla\varphi_{q_d}(q)$ and $F_{O_i}(q) = -\nabla\varphi_{O_i}(q)$.

For illustration, consider the two-dimensional obstacle avoidance situation illustrated in figure 1, in which the planar is constrained to move. Consider the planar to be a negatively charged particle trying to reach the positively charged goal position. If there were no obstacles in the environment, it would be a simple matter of moving in the straight line between the start and goal. However, when there are obstacles present their negative charge repels the links of robot.

The first use of the APF concept for obstacle avoidance was presented by Khatib [2]. He proposed a Force Involving an Artificial Repulsion from the Surface (in French FIRAS) which should be a non-negative, continuous and differentiable. However, the potential field introduced exhibits local minima other than the goal position of the robot. To solve the precedent problem, Volpe and Khosla [13] developed a new elliptical potential functions called

Superquadric Artificial Potential Functions, which do not generate local minima in the physical space. They have shown that superquadric potential fields can be constructed only for simple shapes like square or triangular figures. The problem of local minima remains, because the superquadric potential functions do not generate local minima in the workspace but the local minima can occur in the C-space of the robot. The contributions of Koditschek in [4], [5], [9], and [10], are worth to be mentioned because they introduced an analytic potential field in the C-space without local minima. However the topology of the application range is limited to obstacles, which have to be ball-, or star-shaped otherwise no solution can be found. The contributions of Connolly [1] and of Kim and Khosla [3] are in our opinion the most successful methods concerning robot motion planning with potential field. They are in the same time, used the harmonic functions to build a potential field in the C-space without local minima. The harmonic functions attain their extreme values at the boundary of the domain. The harmonic functions used in this paper are related to the work of Kim and Khosla [3]. In next subsection, we will give a brief description of this methodology.

B. ANALYTIC HARMONIC FUNCTIONS

Kim and Khosla [3] introduced an artificial potential approach based on harmonic functions to guide a robot in the C-space. The harmonic functions are functions, which satisfy the following equation

$$\nabla_q^2 \varphi(q) = 0 \tag{6}$$

also called the Laplace equation.

The most important properties of harmonic functions are that they are free of local minima and that any linear combination of two harmonic functions is also harmonic function. The Laplace equation (6) can be written in general polar co-ordinates [3] as

$$\nabla^2 \varphi = \frac{\partial^2 \varphi}{\partial r} + \frac{n-1}{r}\frac{\partial \varphi}{\partial r} + angular\ terms \tag{7}$$

We assume that the harmonic function φ is a function of r only, the angular terms are zero. After rearranging and integrating with respect to r (7) becomes

$$\frac{\partial \varphi}{\partial r} = \frac{c}{r^{n-1}} \tag{8}$$

For $n = 2$, the solution of (8) is

$$\varphi = c \log r + c_1 \tag{9}$$

If n > 2, the solution is

$$\varphi = \frac{c/(2-n)}{r^{n-2}} + c_1 \tag{10}$$

where $r = \sqrt{(q_1 - q_{o1})^2 + (q_2 - q_{o2})^2 + ... + (q_n - q_{on})^2}$ with q_o being an arbitrary point. Depending on the sign of the constant scalar c , (9) and (10) can be used to represent a source or a sink. A source can be used to represent a point obstacle (see figure 2) at q_o and a sink can be used to represent the goal position at $q_o = q_d$.

The obstacles in C-space with an arbitrary shape can be represented by a number of panels (for more details see [3]).

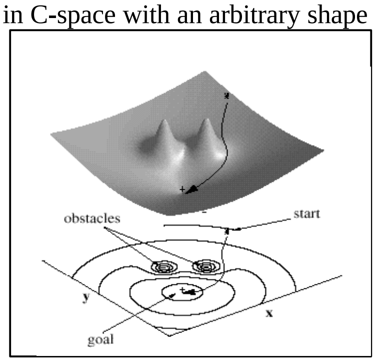

Figure 1: Two-dimensional APF.

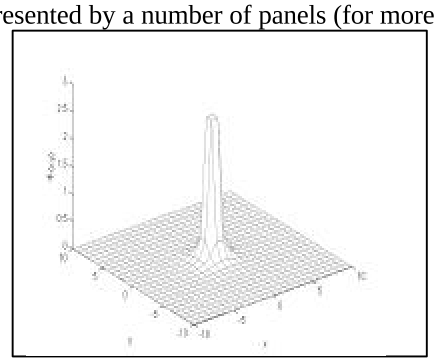

Figure 2: Example of harmonic function.

C. OBSTACLE AVOIDANCE STRATEGY

In order to avoid obstacles effectively, a system must be informed about the obstacles possible positions, and likely future positions. For this work we assume that the environments are known. The distance d_o is the distance between nearest obstacle position and current position of the link of the robot. In real-time, the sensors measure the distance d_o . It was decided to always calculate the direction of the repulsive force from the obstacle as being along the vector from the centre of the obstacle to the link of the robot. The distance d used in the repulsion calculation may be determined by decreasing the previously distance d_o, by the minimum approach distance d_a, i.e.:

$$d = d_o - d_a \qquad (11)$$

It is one of the two inputs of our fuzzy controller. This is illustrated in two dimensions in figure 3.

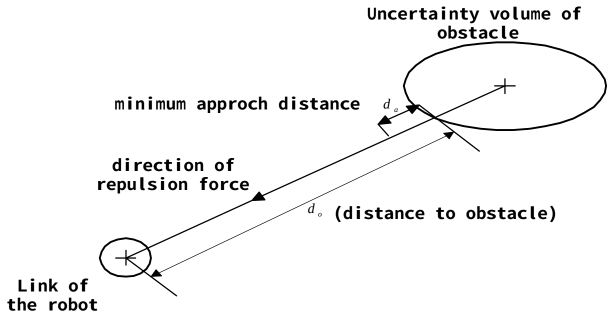

Figure 3: Obstacle repulsion based on APF

IV. ROBOT CONTROL

The development of the fuzzy controller in motion planning of manipulator using APF to inform the system about the obstacles is the main contribution of this paper.

A. CONTROL OBJECTIVE

The control objective is to develop a controller for robot dynamics given by (2) to accomplish a certain motion to reach the prescribed goal q_d; the trajectory of the robot is not known or calculated in advance. To accomplish this purpose we first define the position tracking error

$$e(t) \in R^n \text{ as } e = q_d - q \qquad (12)$$

and its first derivative

$$\dot{e}(t) \in R^n \text{ as } \dot{e} = -\dot{q} \qquad (13)$$

In addition, we also define a modification PD control called PD-Plus-Gravity control as

$$T = G(q) + K_P e + K_D \dot{e} \qquad (14)$$

The global asymptotic stability of the PD-Plus-Gravity controller is proof (refer appendix B).

Using an APF and substituting $K_P e$ by F_{art} in (14), we have:

$$T = G(q) + F_{art} + K_D \dot{e} \qquad (15)$$

We know that $F_{art} = -\nabla \varphi_{art}(q)$, and (15) becomes

$$T = G(q) - \nabla \varphi_{art}(q) - K_D \dot{q} \qquad (16)$$

We can prove that the system (2) is globally asymptotically stable for the new control strategy (16) (Proof refer appendix B).

B. CONTROLLER STRUCTURE

The control strategy introduced here is an extended version of the one already described in control objective. Using a fuzzy logic control we define the following control strategy for the system described by (2), given by the torque:

$$T = G(q) + \tau - K_D \dot{q} \qquad (17)$$

where τ is the output of the fuzzy controller. The figure 4 shows the structure of this control strategy.

For $\tau = -\nabla \varphi_{art}(q)$, we prove in the precedent subsection that the system is globally asymptotically stable.

C. FUZZY CONTROLLER

The fuzzy control of robot models implemented in this work is a version of Sugeno's models (position reasoning method proposed by Takagi and Sugeno [12]). Due to its computational efficiency, these models are commonly preferred to the Mamdani's model [6] in real time applications. The Sugeno's model is a simple fuzzy system. Its structure is shown in figure 5. The Sugeno's controller can be easily implemented by digital logic circuits. The basic fuzzy logic inference rules are very simple in mathematics. As a result, they can be realised by simple hardware (microprocessors, operational amplifiers, etc.). Tanaka and Sugeno [11] proved also that this controller is globally asymptotically stable.

In this paper, we develop an assembled fuzzy logic controller (the structure is shown in figure 4) for tracking control of multi-link revolute-joint robot manipulator system. A two-link revolute-joint robot arm is used as a prototype for this investigation, and it is illustrated by figure 8. There is one controller at each joint. We first develop a general design procedure consisting of selection of membership functions and establishment of a rule base for low level two-inputs/one output fuzzy logic controller, where the rule base has thirty rules.

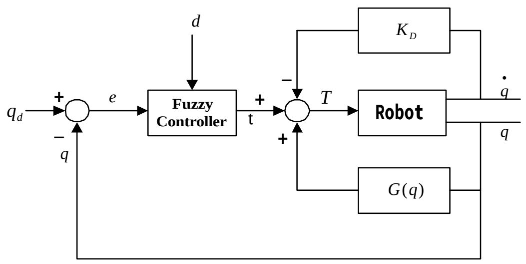

Figure 4: Fuzzy Controller Structure.

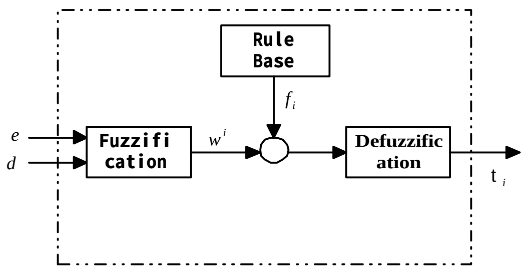

Figure 5: Structure of Sugeno Model.

a) Fuzzification

Each controller has two inputs. The distance, d , between the link and the nearest obstacle, and the tracking error, e . The output of each fuzzy logic controller is the torque τ , which is required to be bounded: $|\tau| < \infty$. All these variables can be positive as well as negative, thus, they do not only inform about the magnitude, but also about the sign of displacement.

The tracking error, e , input is partitioned into five fuzzy sets: big negative (BN), small negative (SN), zero (Z), small positive (SP), and big positive (BP). Its fuzzy membership functions are symmetric and shown in figure 6.

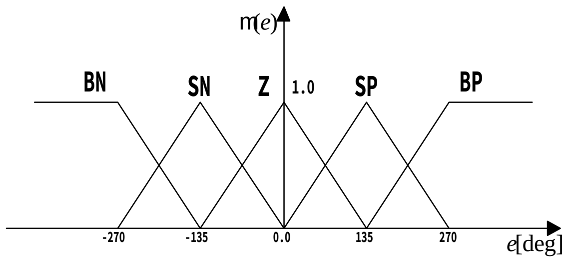

Figure 6: Fuzzy membership functions for e .

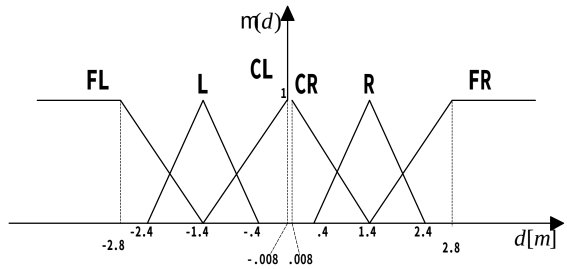

Figure 7: Fuzzy membership functions for d .

The distance, d , input is partitioned into six fuzzy sets: far left (FL), left (L), close left (CL), close right (CR), right (R), and far right (FR). When the obstacle is in the left of the link, the distance, d , is negative, and when it is in right of the arm, the distance, d , is positive. Its fuzzy membership functions are asymmetric and shown in figure 7.

b) Rule Base

The rule base is generalized as follows:

R^i :If $e(k)$ is $\mu_1^i(e(k))$ and …and $e(k-n+1)$ is $\mu_n^i(e(k-n+1))$ and $d(k)$ is $\mu_1^i(d(k))$ and …and $d(k-m+1)$ is $\mu_m^i(e(k-m+1))$ Then $\tau_i(k+1)$ is r (18)

And our thirty rule bases are arranged into a look-up table, as shown in table 1.

$\mu(d)$ \ $\mu(e)$	FL	L	CL	CR	R	FR
BN	LS	RB	RVB	LVB	LB	LB
SN	LS	RVS	RB	LB	LB	LS
Z	Z	Z	RS	LS	Z	Z
SP	RS	RB	RB	LB	LVS	RS
BP	RB	RB	RVB	LVB	LB	RS

Table 1: Rule Base

The two inputs, $\mu(d)$ and $\mu(e)$, represent the fuzzy sets, which describe the distance between robot and obstacle, and the tracking error, respectively. The outputs of the base are τ_j which describe the torque output. And it is partitioned into 9 fuzzy sets: left very big (LVB), left big (LB), left small (LS), left very small (LVS), zero (Z), right very small (RVS), right small (RS), right big (RB), and right very big (RVB). For example, the rule 1 is:

$$R^1 : \text{If } d \text{ is FL and } e \text{ is BN Then } \tau_i \text{ is LS} \tag{19}$$

c) Defuzzification

For each of controllers, the following defuzzification formula is used.

$$\tau = \frac{\sum_{i=1}^{n} \tau_i w^i}{\sum_{i=1}^{n} w^i} \tag{20}$$

where the weight is

$$w^i = \prod_{p=1}^{n} \mu_p^i (e(k-p+1)) \times \prod_{h=1}^{n} \mu_h^i (d(k-h+1))$$

When the torque, τ, is positive, the link moves to the left, and when it is negative, the link moves to the right.

V. SIMULATIONS

A comprehensive simulation study has been carried out using two DOF experimental robot manipulator systems (figure 8), whose parameters can approximately be described by the system matrices

$$M(q) = \begin{bmatrix} 9.77 + 2.02\cos(q_2) & 1.26 + 1.01\cos(q_2) \\ 1.26 + 1.01\cos(q_2) & 1.12 \end{bmatrix}$$

$$G(q) = g\begin{bmatrix} 8.1\sin(q_1) + 1.13\sin(q_1 + q_2) \\ 1.13\sin(q_1 + q_2) \end{bmatrix}$$

$$C(q,\dot{q}) = 1.01\sin(q_2)\begin{bmatrix} -\dot{q}_2 & -(\dot{q}_1 + \dot{q}_2) \\ \dot{q}_1 & 0 \end{bmatrix}$$

The controller gain is selected as

$$K_D = \begin{bmatrix} 8 & 0 \\ 0 & 15 \end{bmatrix} [Nms]$$

And the rule base for link is shown in table 2.

m(d) \ m(e)	FL	L	CL	CR	R	FR
BN	0.1	-0.3	-0.5	0.5	0.3	0.3
SN	0.1	-.001	-0.3	0.3	0.3	0.1
Z	0.0	0.0	-0.1	0.1	0.0	0.0
SP	-0.1	-0.3	-0.3	0.3	.001	-0.1
BP	-0.3	-0.3	-0.5	0.5	0.3	-0.1

Table 2: Rule base.

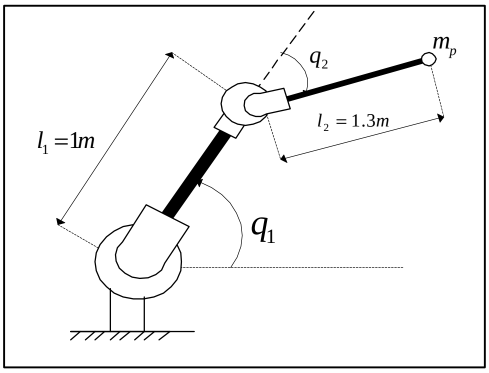

Figure 8: Two DOF experimental robot systems.

Two cases were used to demonstrate and evaluate the effectiveness of the proposed fuzzy control strategy for robot manipulator navigation using APF in known environments and with unknown, possible moving, obstacles.

Case 1: Static Obstacles

Due to the fact that the face A is longer than the face B, robot, referring figure 9, prefers to take longer route to attain a required goal. Figure 11 shows the position of the links, the tracking error, and the distance between end effector and the nearest obstacle for robot navigation (figure 9).

On the other hand, figure 10, the robot chooses the short trajectory to reach the desired destination, because the obstacle 1 is moved. Figure 12 shows the position of the links, the tracking error, and the distance between end effector and the nearest obstacle for robot navigation (figure 10).

This phenomenon proves that, from the information received by robot from APF about the obstacle, the robot autonomously chooses its way to reach the aimed target.

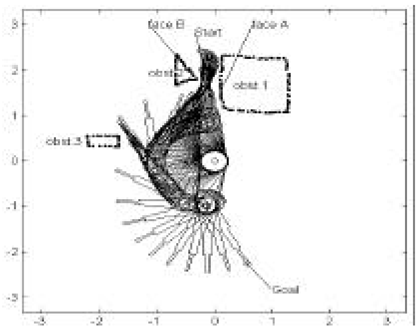

Figure 8: First Robot Navigation.

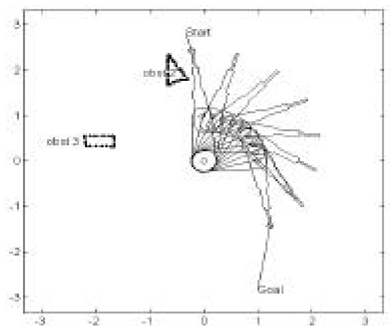

Figure 9: Second Robot Navigation

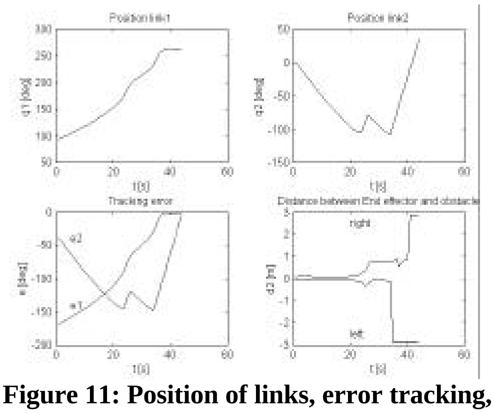

Figure 11: Position of links, error tracking, and distance between End effector and obstacle for robot navigation of figure 9

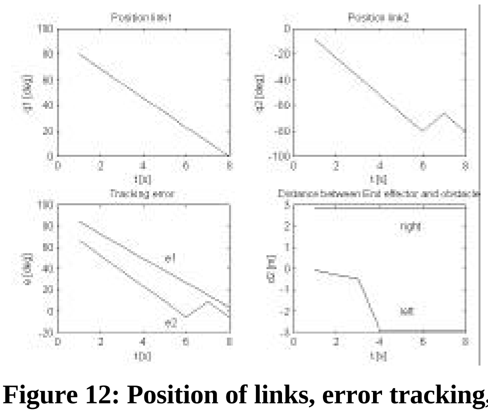

Figure 12: Position of links, error tracking, and distance between End effector and obstacle for robot navigation of figure 10

For the second robot navigation, it can be observed that, in the right of the links, there are no obstacles and the robot moves in the straight line from the start point to the goal.

Case 2: Dynamic Obstacles.

Figure 13 shows the robot navigation with dynamic obstacle. In this particular case, we prove that our system avoids the unknown obstacles successfully. Figure 14 shows the position of the links, the tracking error, and the distance between end effector and the nearest obstacle for robot navigation with moving obstacle.

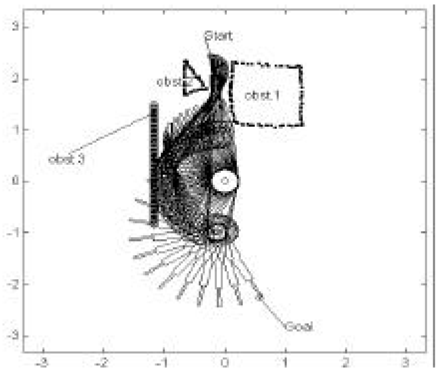

Figure 13: Robot navigation with dynamic obstacle.

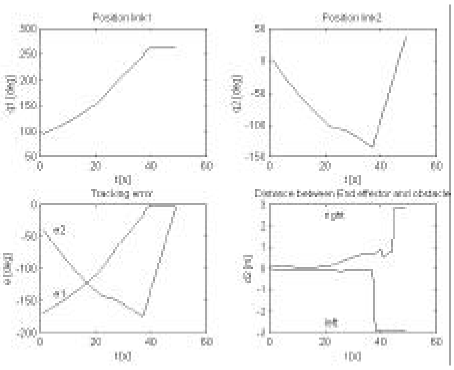

Figure 14: Position of links, error tracking, And distance between End effector and obstacle for robot navigation With unknown obstacle

For the both cases, it can be observed that the robot automatically reduces its velocity to avoid collision.

VI. CONCLUSION

A new control strategy for robot manipulator navigation using reactive method with a simple fuzzy logic controller has been proposed. The effectiveness of the proposed strategy has been demonstrated using this new fuzzy system architecture for behavior based control of robot navigation, autonomous motion planning in known environments with dynamic obstacles is greatly improved. The computational efficiency of the proposed control scheme makes it particularly suitable for real-time implementation. This system can be also applied in unknown environments.

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APPENDIX A: PROPERTIES OF MANIPULATORS

It is well known that the rigid dynamic (2) has the following properties [7], which can be exploited to facilitate control system design.

Property[1] 1- Boundedness of the Inertia Matrix

The inertia matrix $M(q)$ is symmetric and positive definite, and satisfies the following inequalities:

$$m_1\left\|\dot{q}\right\|^2 \le \dot{q}^T M(q)\dot{q} \le m_2\left\|\dot{q}\right\|^2 \ , \ \dot{q}\in R^n$$

where m_1 and m_2 are known positive constants and $\|\bullet\|$ denotes the standard Euclidean norm.

Property 2- Skew Symmetry:

The inertia and centripetal-coriolis matrices have the following property:

$$\dot{q}^T(\dot{M}(q)-2C(q,\dot{q}))\dot{q}=0\ ,\ \dot{q}\in R^n$$

where $\dot{M}(q)-2C(q,\dot{q})$ is a skew symmetric matrix.

APPENDIX B: PROOFS

Proof 1: stability of the PD-Plus-Gravity controller

The global asymptotic stability of the PD-Plus-Gravity controller is proof for any positive definite symmetric matrices K_P and K_D by considering the following Lyapunov function.

$$V=\frac{1}{2}[\dot{q}^T M(q)\dot{q}+e^T K_P e]>0$$

Taking time derivative yields

$$\dot{V}=\frac{1}{2}\dot{q}^T \dot{M}(q)\dot{q}+\dot{q}^T M(q)\ddot{q}+e^T K_P\dot{e}=\dot{q}^T[\frac{1}{2}\dot{M}(q)-C(q,\dot{q})]\dot{q}-\dot{q}^T F_d\dot{q}-\dot{q}^T K_D\dot{q}$$

$$\dot{V}=-\dot{q}^T F_d\dot{q}-\dot{q}^T K_D\dot{q}$$

K_D is positive definite, and the friction coefficients F_d are positive implies that $\dot{V}\le 0$. The system is globally asymptotically stable.ÿ

Proof 2: stability of the APF controller

Consider the positive definite Lyapunov function V :

$$V=\varphi_{art}(q)+\frac{1}{2}\dot{q}^T M(q)\dot{q}$$

where $\varphi_{art}(q)>0$ for all $q\neq q_d$.

The time derivative of V is given by:

$$\dot{V}=\dot{q}^T\nabla\varphi_{art}(q)+\frac{1}{2}\dot{q}^T\dot{M}(q)\dot{q}+\dot{q}^T M(q)\ddot{q}=\dot{q}^T[\frac{1}{2}\dot{M}(q)-C(q,\dot{q})]\dot{q}-\dot{q}^T F_d\dot{q}-\dot{q}^T K_D\dot{q}$$

$$\dot{V}=-\dot{q}^T F_d\dot{q}-\dot{q}^T K_D\dot{q}$$

K_D is positive definite, and the friction coefficients F_d are positive implies that $\dot{V}\le 0$. The system is globally asymptotically stable.ÿ

[1] Strictly speaking boundedness of the inertia matrix requires in general that all joints be revolute.