# Designing a Fuzzy Controller for a Multi-Story Frame Tested on a Shaking Table

Lucia Faravelli[*], Roberto Rossi[**] and Guido Torelli[**]
[*]Dept. of Structural Mechanics, University of Pavia
Via Ferrata, 1 – 27100 Pavia, Italy
Phone: +39 0382 505459, Fax: +39 0382 528422
e-mail: lucia@dipmec.unipv.it
[**]Dept. of Electronics, University of Pavia
Via Ferrata, 1 – 27100 Pavia, Italy
Phone: +39 0382 505215
e-mail: roberto@ele.unipv.it, g.torelli@ele.unipv.it

ABSTRACT: In this paper, the design of a fuzzy controller for earthquake engineering applications is presented. The controller is conceived to control the response of a frame specimen to a shaking table excitation. The controller uses a subset of the measured kinematic response variables: in particular no more than two velocities are used as feedback variables to control a three-story frame. In order to speed up the development process, an electronic circuit board is designed to allow a real-time emulation of the real test structure, thus making lengthy numerical simulations unnecessary. Experimental results obtained on the system including the fuzzy controller and the electronic board are provided.

KEYWORDS: active control, fuzzy control, earthquake engineering, electronic circuit, real-time emulation, shaking table

## INTRODUCTION

The implementation of an active controller for earthquake engineering applications can be troublesome if the classic approach is pursued (Battaini et al., 1997). Indeed, uncertainties in the mathematical model, nonlinearities and hysteretic behavior must be managed. Fuzzy logic theory is intrinsically able to deal with all these problems because it is based upon a linguistic description of the controller behavior, thus making the control system more flexible and suitable for operation with an incomplete model of the structural system. Also, a nonlinear control law can be quite easily described in terms of fuzzy sets and rules. Nevertheless, a fuzzy logic controller requires a lot of experimental work aimed at providing adequate countermeasures in all real-world conditions as well as appropriate stability of the closed-loop system (Faravelli et al., 1996; Casciati et al., 1996).

Tuning the controller can be a very lengthy and error-prone process. Therefore, it cannot be carried out on the real structural systems, as failure in a controller component may result in a definitive damage of the mechanical specimen. To avoid such a situation, two techniques apply:

1. thorough computer simulations, including the system under control and its controller;
2. emulation of the system under control by means of an electronic circuit board (Casciati et al., 1998; Battaini et al., 1998).

Despite its greater flexibility, a major drawback of the first method is its inability to run in real time, thus requiring a careful interpretation of time scale dilatation in addition to lengthening the whole design process. For this reason, the second approach was adopted and will be outlined in this paper.

## THE SYSTEM UNDER CONTROL

The specimen system is a three-story frame which is conveniently modeled by three degrees of freedom in series. Adding braces at each story, the actual number of degrees of freedom of the structure can be changed into two or one as in the case of Fig. 1. This paper deals with the system in its three-degree-of-freedom configuration.

An active mass located on the top story is driven to control the structure. The whole system is mounted on a shaking table that allows the simulation of seismic ground excitations.
Accelerometers are positioned on every floor, which enables us to reconstruct velocities and displacements to monitor the structure response and to feed the controller with adequately chosen input variables (Casciati et al., 1998; Battaini et al., 1998). The fuzzy controller is implemented using STMicroelectronics WARP Application Development Board 2.0, connected to a PC through the serial RS232 port. By means of the Fuzzystudio™ development system, the fuzzy project can be compiled and downloaded to the fuzzy chip (SGS-THOMPSON Microelectronics, 1996).
The fuzzy board is also connected to an Advantech PCL-818 data acquisition PC card, which allows both data plotting and easy bidirectional data transfer between the accelerometers and the fuzzy board (Cobelli, 1998; Casciati and Giorgi, 1996).
In the work presented in this paper, the test frame is replaced with a real-time emulation circuit to prevent the frame from being damaged by possible errors. Numerical simulations were not considered due to their inability to proceed in real time.

## REAL-TIME EMULATION OF THE STRUCTURAL SYSTEM

### MECHANICAL MODEL

The starting point for the design of the emulation circuit is the mathematical model of the system to be emulated. It was formerly formulated in (Casciati et al., 1998), but the actual implementation of the scheme is repeated for convenience in this section. This task can be easily accomplished after choosing a suitable coordinate system. Also, it is worth pointing out that, since the purpose of the controller is the minimization of the structural response, a linear model seems an appropriate approximation.
Let $x_i$ be the absolute position of the $i$-th story of the frame and $x_0$ the absolute position of the shaking table, which simulates ground motion during an earthquake. The following set of equations can be written:

$$\begin{cases} m_3\ddot{x}_3 + c_{32}(\dot{x}_3 - \dot{x}_2) + k_{32}(x_3 - x_2) = F_3 \\ m_2\ddot{x}_2 + c_{21}(\dot{x}_2 - \dot{x}_1) + c_{23}(\dot{x}_2 - \dot{x}_3) + k_{21}(x_2 - x_1) + k_{23}(x_2 - x_3) = F_2 \\ m_1\ddot{x}_1 + c_{10}(\dot{x}_1 - \dot{x}_0) + c_{12}(\dot{x}_1 - \dot{x}_2) + k_{10}(x_1 - x_0) + k_{12}(x_1 - x_2) = F_1 \end{cases} \quad (1)$$

where the $k_{ij}$ coefficients ($k_{ij} = k_{ji}$) represent the stiffness of the columns, while the $c_{ij}$ ( $= c_{ji}$) account for their damping. $F_i$ denotes the control force at each story. Only $F_3$ will be different from zero in the following elaboration.
As an alternative, let $X_i$ be the position of the $i$-th story relative to ground (i.e. the shaking table). We have:

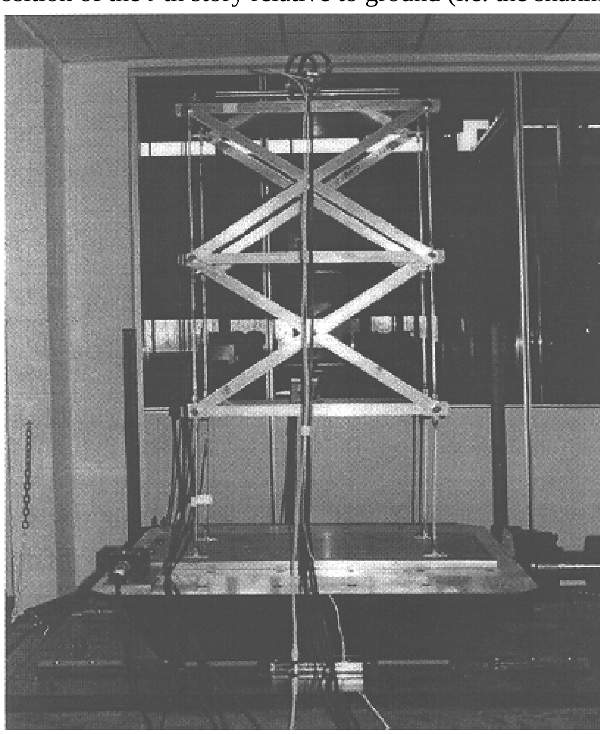

Figure 1: Single-degree-of-freedom frame. By removing all the braces, three degrees of freedom are activated.

$$X_i = x_i - x_0 \qquad (2)$$

Denoting the ground acceleration with $a_G$ ( $a_G = \ddot{x}_0$ ), equations (1) become:

$$\begin{cases} m_3 \ddot{X}_3 + c_{32}(\dot{X}_3 - \dot{X}_2) + k_{32}(\dot{X}_3 - \dot{X}_2) = F_3 - m_3 a_G \\ m_2 \ddot{X}_2 + c_{21}(\dot{X}_2 - \dot{X}_1) + c_{23}(\dot{X}_2 - \dot{X}_3) + k_{21}(X_2 - X_1) + k_{23}(X_2 - X_3) = F_2 - m_2 a_G \\ m_1 \ddot{X}_1 + c_{10}\dot{X}_1 + c_{12}(\dot{X}_1 - \dot{X}_2) + k_{10}X_1 + k_{12}(X_1 - X_2) = F_1 - m_1 a_G \end{cases} \qquad (3)$$

The two sets of equations (1) and (3) are quite similar, allowing one to implement both of them in the same circuit, thus enhancing flexibility during operation. If the following equation is adopted for each story:

$$m_i \ddot{x}_i + c_{i,i-1}(\dot{x}_i - \dot{x}_{i-1}) + c_{i,i+1}(\dot{x}_i - \dot{x}_{i+1}) + k_{i,i-1}(x_i - x_{i-1}) + k_{i,i+1}(x_i - x_{i+1}) = F_i - m_i a_G \qquad (4)$$

then relationships (1) and (3) can be viewed as particular cases of (4). Setting $a_G = 0$, equation (4) reduces to (1); on the other hand, setting $x_0$ to zero, (4) reduces to (3). It should also be remarked that, by simply setting $c_{34} = 0$ and $k_{34} = 0$, this equation can be used for the top story as well.

## ELECTRONIC MODEL

The design of an electronic circuit governed by the set of equations described before can be carried out in many ways. However, a few issues should be taken into account:

1. modularity is welcome because the circuit is designed to work in a laboratory environment: it is practical and cost-effective to have a circuit block suitable for any number of degrees of freedom (i.e. for any number of equations);
2. positions, velocities and accelerations of each story should be delivered as circuit outputs;
3. the circuit should be completely adjustable;
4. the number of operational amplifiers should be kept to a minimum, as in practice they are the only sources of non-ideal system behavior in the frequency band of interest, which spans from 0.5 Hz to 25 Hz for earthquake engineering applications.

To meet the first requirement, one should conceive a single circuit block valid for each equation of the model. As

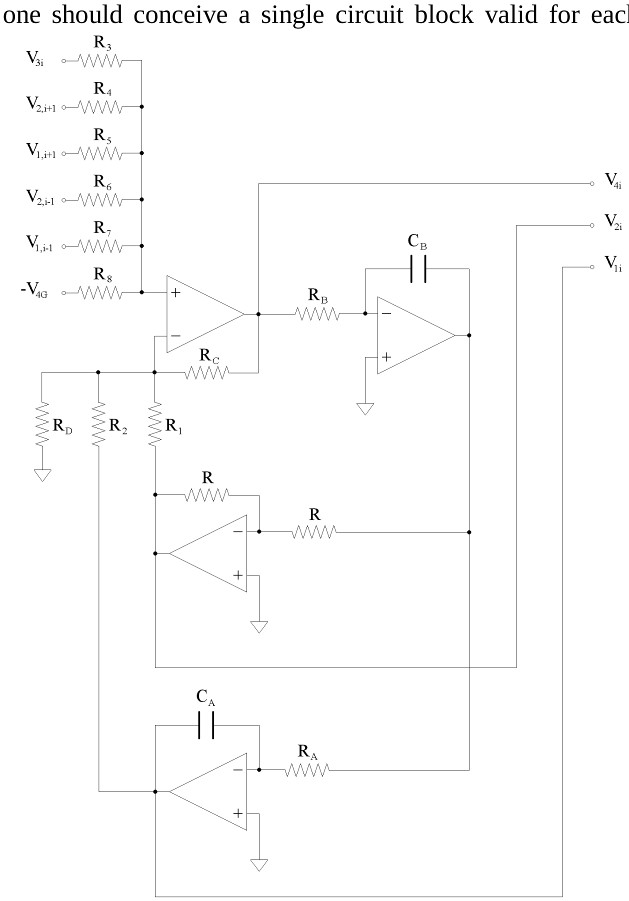


Figure 2: Electronic circuit used for real-time emulation of the frame. By connecting three of these blocks, the frequency response of the three-degree-of-freedom structure can be closely approximated.

explained before, the basic equation to be used is (4). In the appendix, the equivalent electronic model for the structure is derived following an approach which allows the second requirement to be satisfied. The resulting electronic circuit is shown in Fig. 2.

It is worth pointing out that all resistors in the circuit were implemented as trimmers, which allows the response parameters to be easily adjusted.

The implementation described above differs from the one in (Casciati et al., 1998) in that a multi-degree-of-freedom circuit is designed in this paper. Also, this circuit can be used with $x_0$ as an input, thus making the mechanical and electronic tests fully equivalent. In fact, except for a scaling factor, the same $x_0$ signal can be used for both the shaking table and the circuit.

## PARAMETER IDENTIFICATION

When three electronic blocks as the one described above are set up and connected, the resulting circuit behaves like a three-story frame. The mechanical parameters were extracted from the frame by shaking it with white-noise ground acceleration and measuring the frequency acceleration response of each story. In particular, the three peaks occur at 1.3 Hz, 3.6 Hz and 5.2 Hz, respectively (Battaini et al., 1997). The electronic parameters were then determined following the procedure described in the appendix.

The first three plots in Fig. 3 show the good agreement between the frequency responses of the mechanical structure and those measured on the electronic circuit. The results for the electronic circuit were computed as the ratio between the output story displacement and the input displacement. To check the accuracy of the channel providing the acceleration, the frequency response function was also derived for the third story as the ratio between output and input acceleration. As the fourth plot in Fig. 3 shows, the correspondence between the mechanical response and the electronic one is very good.

## SURVEY OF THE RESULTS

To investigate the controlling capability of the fuzzy board and better understand how the choice of the feedback variables affects the controller effectiveness, the electronic emulator was stimulated with a white-spectrum acceleration input signal. Several feedback variable combinations were tested.

The first one, which will be used as a reference throughout this section, is the uncontrolled response, which is plotted in Fig. 4. The system displays the maximum oscillations on the top story, reaching a maximum displacement of about 5 cm and a maximum velocity of 30 cm/s.

Two fuzzy controllers were developed for the tests presented in this paper. Both of them take velocities as input variables. The first one has a single input variable, while the second one has two.

The first controlled experiment was carried out with the single-variable fuzzy controller. The system appears to be stable only when $v_3$ is the controller feeding variable (Fig. 5). However, since one-variable control does not seem very attractive, no attempt was made to tune the fuzzy project in order to improve the stability and the control effectiveness. Nevertheless, a great improvement over the uncontrolled behavior can be seen, as the maximum displacements and velocities measured on the top story drop to 3 cm and 6 cm/s, respectively.

Using two feedback variables, even better results are expected. In fact, a 70-story structure was successfully controlled using only two velocities as feedback variables (Battaini et al., 1999). The problem of choosing the two most representative velocities and their optimum relative weights was faced as follows. First, a numerical simulation of the structure with a Linear Quadratic Gaussian (LQG) controller previously implemented (Battaini, 1994) was run in order to sample displacements, velocities, accelerations and the control variable for a significant amount of time. Next, a linear regression technique was employed to correlate the control variable with any given combination of the input variables. Although a perfect regression law can be obtained when using all velocities and accelerations, our efforts were aimed at finding the two most representative velocities. We discovered that these two velocities are $v_2$ and $v_3$. This result could be expected because they are closest to the actuator, which minimizes the propagation delay around the control loop.

Furthermore, the regression coefficients show that $13v_2 + 21v_3 \cong u$, where $u$ is the control variable. This means that $v_2$ should be weighted by a factor of $13/21 \cong 0.6$ with respect to $v_3$. The resulting controller behavior is outlined in Fig. 6.

The system is not perfectly stable, as oscillations can be seen even in the absence of any input excitation. To address this issue, the fuzzy project was modified by enlarging the zero fuzzy set, so as to reduce the control action when no seismic excitation is present. In this way, the controller displays very good stability while it still shows its ability to keep the system oscillations under control. Measured results are shown in Fig. 7.

A comparison between one-variable control (Fig. 5) and two-variable control (Fig. 7) shows a remarkable reduction of story velocities but no significant improvement on displacements.
To fully appreciate the controller effectiveness under resonance conditions, in Fig. 8 the system response in the case of a single-frequency signal is reported. At about $t$ = 60 seconds, the controller is switched off, so the controlled response can be compared to the uncontrolled one. At $t$ = 120 seconds also the input excitation is removed.
Finally, in Fig. 9, displacements relative to ground are compared in the uncontrolled, single-variable-controlled and two-variable-controlled cases.

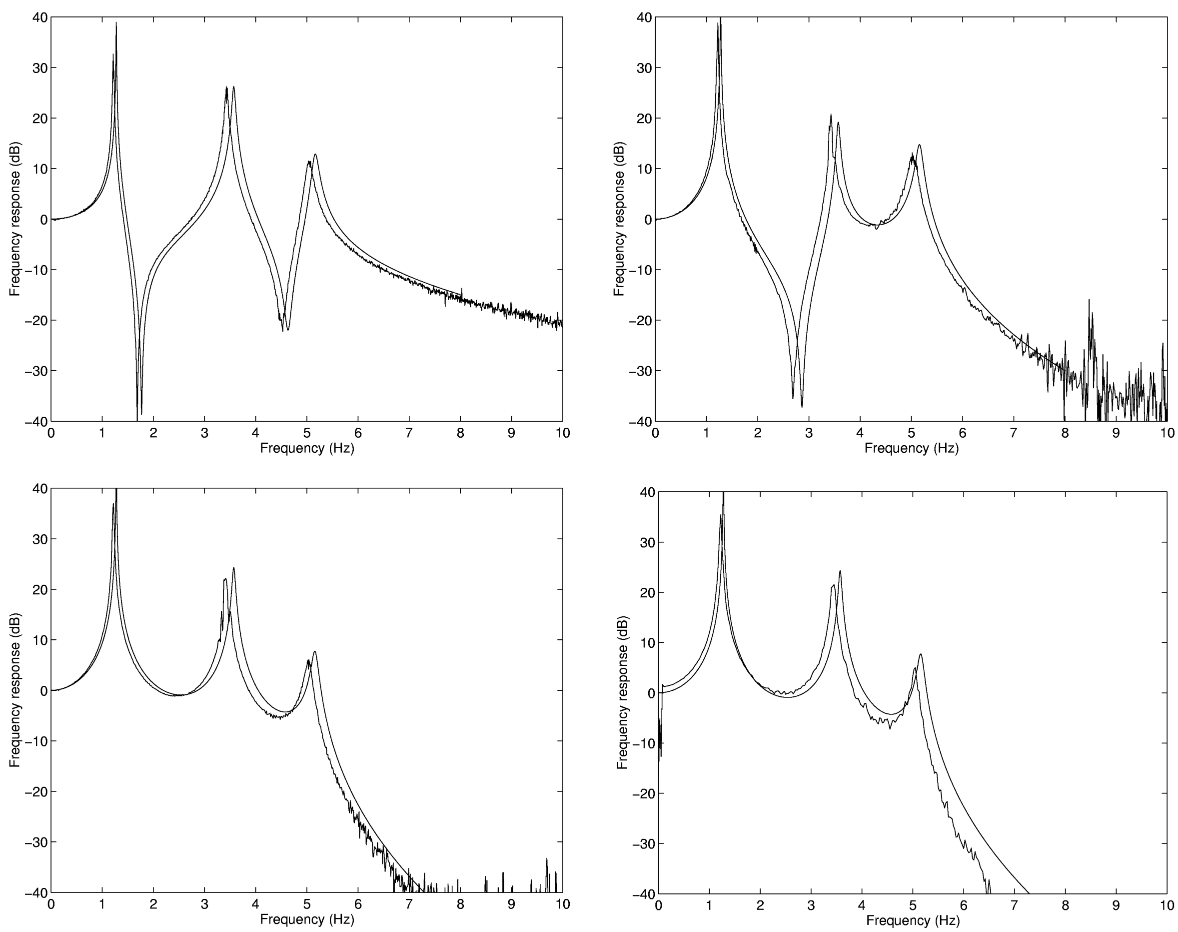


Figure 3: Displacement transfer functions for the first, the second and the third story (non-noisy curves), compared with their electronic counterparts (noisy curves). The fourth plot shows the third story transfer function in terms of acceleration.

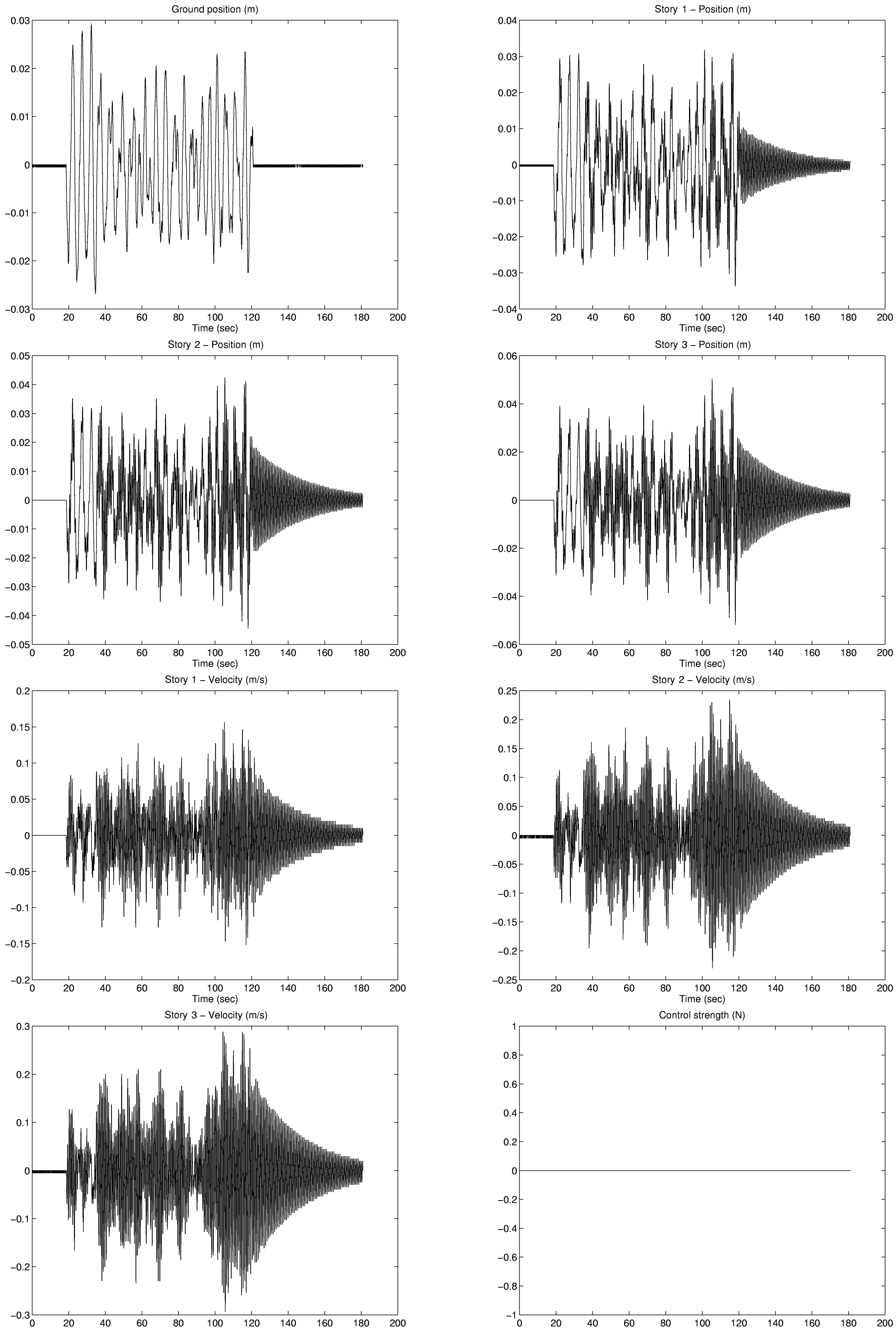


Figure 4: Uncontrolled system response.

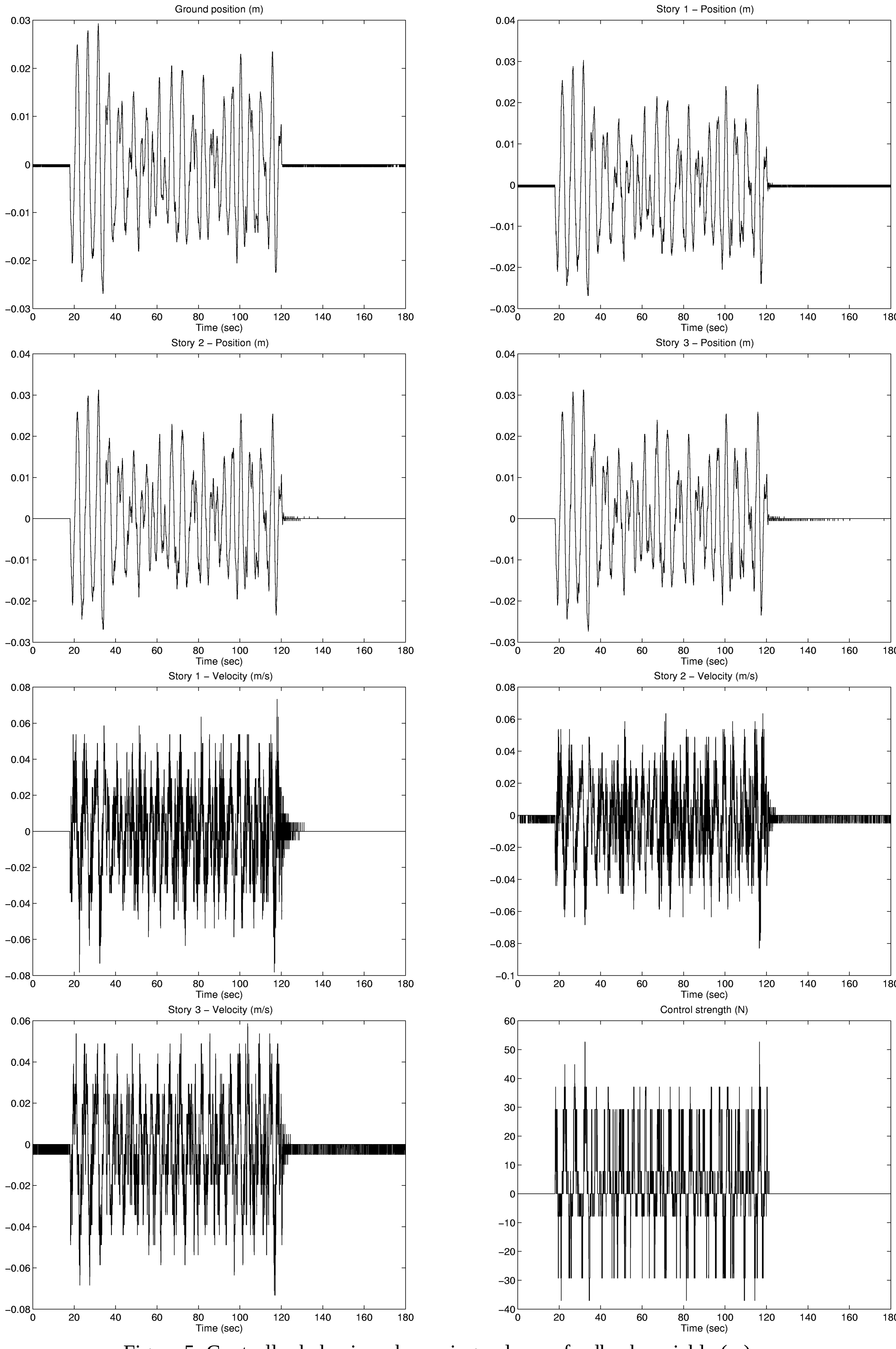


Figure 5: Controller behavior when using only one feedback variable ($v_3$).

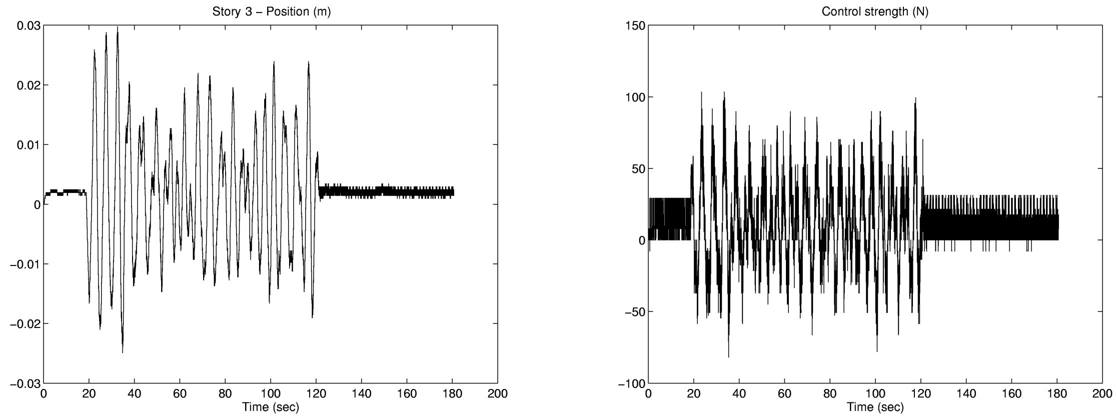

Figure 6: Controller behavior when using two feedback variables. The system is not very stable, as control-induced oscillations are present even when the ground acceleration is zero.

## CONCLUSIONS

In this paper, the design of a fuzzy controller for active structural control is carried out by using an electronic emulation circuit. This greatly helps the controller development process, as it makes lengthy and time-consuming numerical simulations unnecessary.
Future improvements include optimizing the controller to fully exploit the greater amount of information provided by the use of two feedback variables and testing on the real specimen frame.

## ACKNOWLEDGEMENT

This research was supported by grant ARS-98-207 from the Italian Space Agency (ASI).

## APPENDIX

This appendix starts from the mechanical model described by equation (4) and shows how an equivalent electronic model can be developed. Mapping from the mechanical model to the electronic model is achieved as follows. First of all, to have accelerations output from the equivalent circuit, two sets of auxiliary variables are introduced: velocity $v_i$ and acceleration $a_i$. They are defined as:

$$\begin{cases} v_i = \dot{x}_i \\ a_i = \dot{v}_i \end{cases} \tag{5}$$

So, the mechanical model for the $i$-th story becomes:

$$m_i a_i + c_{i,i-1}(v_i - v_{i-1}) + c_{i,i+1}(v_i - v_{i+1}) + k_{i,i-1}(x_i - x_{i-1}) + k_{i,i+1}(x_i - x_{i+1}) = F_i - m_i a_G \tag{6}$$

Subsequently, four sets of voltage variables are introduced, which are the electronic counterparts of the mechanical ones:

$$\begin{cases} x_i = AV_{1i} \\ v_i = BV_{2i} \\ F_i = CV_{3i} \\ a_i = DV_{4i} \qquad (\text{and } a_G = DV_{4G}) \end{cases} \tag{7}$$

where $A$, $B$, $C$ and $D$ can be chosen to properly scale the circuit voltages. Substitution in (4) yields:

$$\begin{aligned} V_{4i} + \frac{B(c_{i,i+1} + c_{i,i-1})}{m_i D} \cdot V_{2i} + \frac{A(k_{i,i+1} + k_{i,i-1})}{m_i D} \cdot V_{1i} = \frac{C}{m_i D} \cdot V_{3i} + \left( \frac{c_{i,i+1}B}{m_i D} \cdot V_{2,i+1} + \frac{k_{i,i+1}A}{m_i D} \cdot V_{1,i+1} \right) + \\ + \left( \frac{c_{i,i-1}B}{m_i D} \cdot V_{2,i-1} + \frac{k_{i,i-1}A}{m_i D} \cdot V_{1,i-1} \right) - V_{4G} \end{aligned} \tag{8}$$

This equation can be rearranged in the following form:

$$V_{4i} = -\alpha_{1i}V_{2i} - \alpha_{2i}V_{1i} + \alpha_{3i}V_{3i} + \left(\alpha_{4i}V_{2,i+1} + \alpha_{5i}V_{1,i+1}\right) + \left(\alpha_{6i}V_{2,i-1} + \alpha_{7i}V_{1,i-1}\right) - \alpha_8 V_{4G} \quad (9)$$

where $\alpha_{1i} = \frac{B(c_{i,i+1} + c_{i,i-1})}{m_i D}$, $\alpha_{2i} = \frac{A(k_{i,i+1} + k_{i,i-1})}{m_i D}$, $\alpha_{3i} = \frac{C}{m_i D}$, $\alpha_{4i} = \frac{c_{i,i+1}B}{m_i D}$, $\alpha_{5i} = \frac{k_{i,i+1}A}{m_i D}$, $\alpha_{6i} = \frac{c_{i,i-1}B}{m_i D}$, $\alpha_{7i} = \frac{k_{i,i-1}A}{m_i D}$ and $\alpha_8 = 1$.

By setting $\tau_1 = A/B$, $\tau_2 = B/D$ and passing to the Laplace domain, the electronic model of the structural system can be finally formulated:

$$\begin{cases} V_{1i} = \frac{V_{2i}}{s\tau_1} \\ V_{2i} = \frac{V_{4i}}{s\tau_2} \\ V_{4i} = -\alpha_{1i}V_{2i} - \alpha_{2i}V_{1i} + \alpha_{3i}V_{3i} + \left(\alpha_{4i}V_{2,i+1} + \alpha_{5i}V_{1,i+1}\right) + \left(\alpha_{6i}V_{2,i-1} + \alpha_{7i}V_{1,i-1}\right) - \alpha_8 V_{4G} \end{cases} \quad (10)$$

This set of equations leads to the well-known Kerwin-Huelsman-Newcomb (KHN) biquad loop (Sedra-Smith, 1991), which can be implemented with four operational amplifiers (Fig. 2). Other structures were investigated to see if the number of amplifiers could be reduced, but this search gave no results.

It can easily be shown that the following relations between the mechanical parameters and the electronic ones hold:

$$\alpha_{1i} = \frac{R_C}{R_1} \quad (11)$$

$$\alpha_{2i} = \frac{R_C}{R_2} \quad (12)$$

and, for every *j* in the range from 3 to 8:

$$\alpha_{ji}R_j = \left(\sum_{p=3}^{8}\frac{1}{R_p}\right)^{-1} \cdot \left(1 + \frac{R_C}{R_1} + \frac{R_C}{R_2} + \frac{R_C}{R_D}\right) \quad (13)$$

The latter equation implies that

$$\sum_{j=3}^{8}\alpha_{ji} = 1 + \frac{R_C}{R_1} + \frac{R_C}{R_2} + \frac{R_C}{R_D} \quad (14)$$

Recalling the mechanical definitions of the $\alpha_{ji}$ coefficients, it can be shown that

$$\sum_{j=3}^{8}\alpha_{ji} = 1 + \alpha_{1i} + \alpha_{2i} + \alpha_{3i} \quad (15)$$

Hence, it must be:

$$\alpha_{3i} = \frac{R_C}{R_D} \quad (16)$$

In addition, the two time constants of the integrators are:

$$\begin{cases} \tau_1 = R_A C_A \\ \tau_2 = R_B C_B \end{cases} \quad (17)$$

Using this set of equations, all the resistance values can be determined. The key point is that $\alpha_{ji}R_j$ is constant in the range $3 \le j \le 8$. So, $\min_{3 \le j \le 8} R_j$ is associated to $\max_{3 \le j \le 8} \alpha_{ji}$: the minimum $R_j$ can be chosen arbitrarily, though it should not be too small to avoid unnecessary high currents.

## REFERENCES


Battaini M., 1994, *Sistemi strutturali controllati: progettazione e affidabilità*, Ph.D. Thesis., Dept. of Structural Mechanics, University of Pavia.

Battaini M., Casciati F. and Faravelli L., 1998, *Fuzzy Control of Structural Vibration. An Active Mass System Driven by a Fuzzy Controller*, Earthquake Engineering and Structural Dynamics, 27, 1267-1276 (1998).

Battaini M., Casciati F. and Faravelli L., 1999, *Control Algorithm and Sensor Location*, Proceedings of the Second World Conference on Structural Control, Kyoto, Japan.

Battaini M., Dyke S., 1997, *Controllo attivo di una struttura mediante un dispositivo a massa vibrante*, Atti 8° Convegno Nazionale ANIDIS, Taormina, Italia.

Casciati F., Faravelli L. and Giorgi F., 1997, *Laboratory Validation of a Fuzzy-Chip Controller*, *Proceedings of EUFIT'97*, Aachen.

Casciati F., Faravelli L. and Yao T., 1996, *Control of Nonlinear Structures Using the Fuzzy Control Approach. Nonlinear Dynamics*, 11, 171-187.

Casciati F. and Giorgi F., 1996, *Fuzzy Controller Implementation. Proceedings of 2nd International Workshop on Structural Control. IASC*, Hong Kong, 119-125.

Casciati F. and Yao T., 1995, *Comparison of Strategies for the Active Control of Civil Structures, Proceedings of 1st world Conference on Structural Control, IASC*, Los Angeles, Vol.I, WA1-3.

Casciati F., Faravelli L. and Torelli G., *A Fuzzy Chip Controller for NonLinear Vibrations* (submitted for publication).

Cobelli A., 1998 *Design and Implementation of a Fuzzy Logic System for Active Structural Control (in Italian)*, Master Thesis, Dept. of Electronics – University of Pavia.

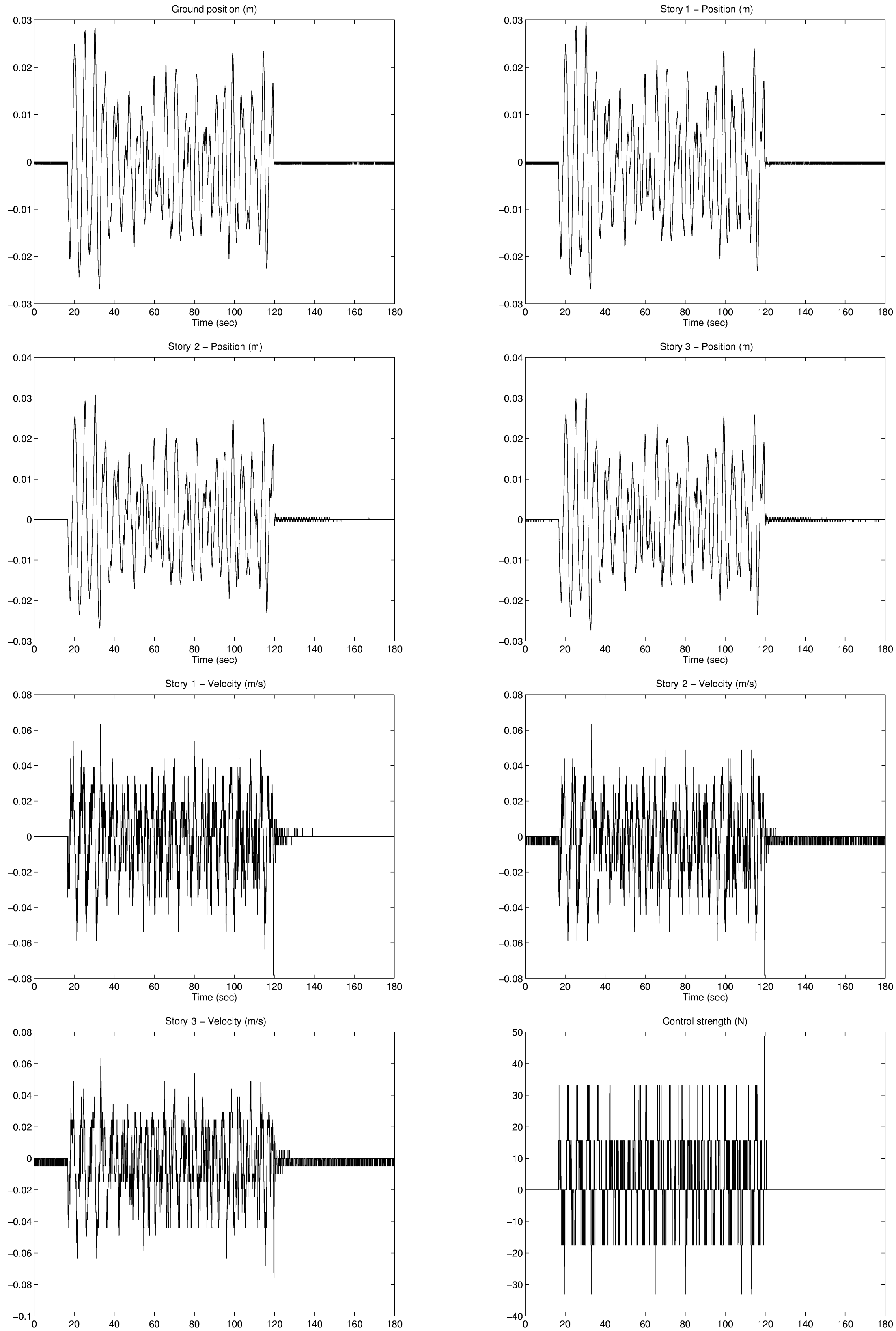


Figure 7: Controller response with two feedback variables after enlarging the zero fuzzy set to ensure better stability.

Faravelli L. and Yao T., 1996, *Use of Adaptive Network in Fuzzy Control of Civil Structures*, *Microcomputers in Civil Engineering*, 11 (1), 67-76.

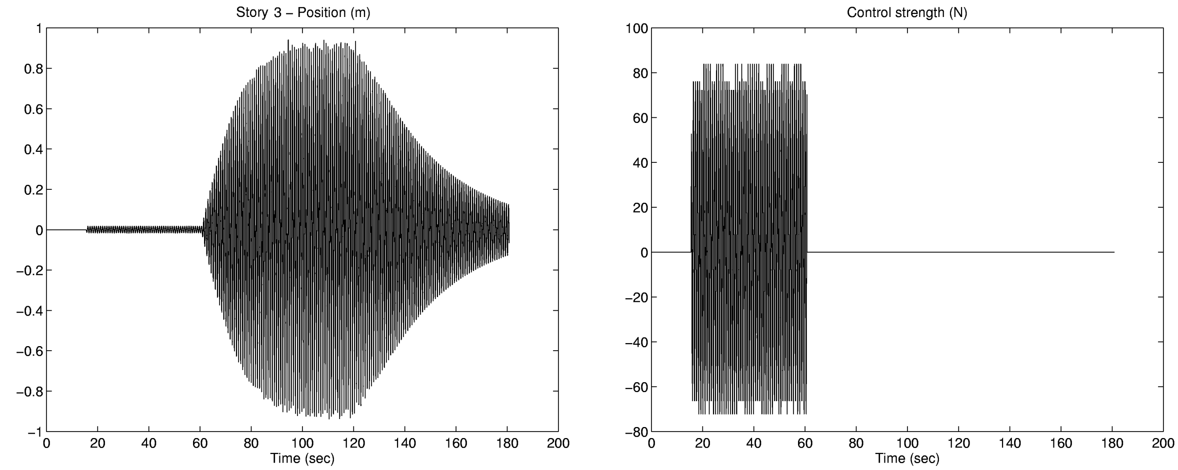


Figure 8: Controlled behavior for a sinusoidal input (case of two feedback variables). At $t$ = 60 seconds, the controller is switched off. At $t$ = 120 seconds, the input signal is removed.

SGS-THOMPSON Microelectronics, 1996, *Fuzzystudio™ 2.0 User Manual.*

Sedra A.S. and Smith K.C. , 1991, *Microelectronic Circuits*, Third Edition, Saunders College Publishing, Philadelphia.

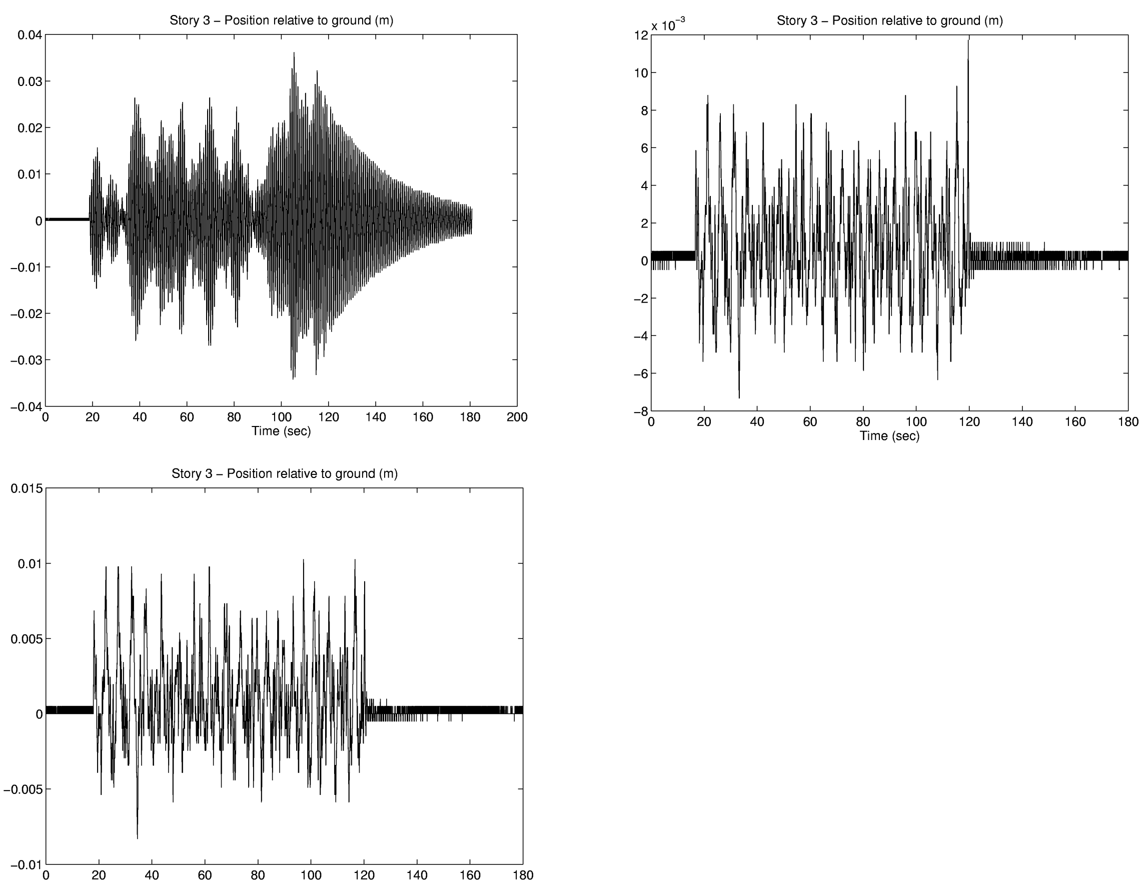


Figure 9: Relative displacements (relative to ground) measured on the third story in the uncontrolled, single-variable and two-variable control case, respectively.