Toward Models of Case-Based Decision Making

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Abstract

The idea of *case-based decision making*, which combines principles from decision theory and case-based reasoning, has recently emerged as a new paradigm for decision making under uncertainty. This paper briefly surveys proposals to formalize this idea and outlines a new approach which is based on possibility theory and makes use of approximate reasoning techniques as well as generalizations of expected utility theory. This is followed by a discussion concerning some general aspects of case-based decision making, with a special emphasis on differences of formalizations thereof.

1 Introduction

The mental notions of *preference* and *belief* are the main "ingredients" of classical (statistical) decision theories. Corresponding mathematical approaches are based on the formalization of these concepts in the form of, e.g., utility functions and probability distributions. Recently, Gilboa and Schmeidler [7] have proposed a case-based decision theory (CBDT) which combines principles from decision theory and case-based reasoning, and in which the cognitive concept of *similarity* plays a predominant role. The main idea underlying their approach is the assumption that an agent faced with a certain decision problem, exploits his experience from similar situations he encountered in the past: he chooses an act based on the performance of (potential) acts in previous problems similar to the current one. Alternative formalizations of case-based decision making (CBDM) have been presented in [1, 2] and [10, 12].

Incorporating the concept of similarity into formal approaches to decision making raises some interesting questions. Particularly, it has to be clarified which part similarity plays and, hence, what the relation between this and other concepts such as preference and belief could be. Clearly, the latter concerns basic assumptions about a decision theoretic model or set-up. Therefore, we cannot expect to find definite answers. Classical works in the field of decision theory as well as recent developments such as, e.g., qualitative approaches to decision making, show that notions such as preference and belief can be formalized and related in various ways. Moreover, a consensus concerning the actual meaning of the concept itself seems to exist even less in the case of similarity than in the case of utility or uncertainty. Indeed, the above mentioned approaches to CBDM do not only differ with respect to the mathematical formalization, but seem to be based on different principles and ideas for incorporating similarity and case-based reasoning into decision making. Having a closer look at such differences might be helpful in clarifying the relation between case-based reasoning and decision making, and in deepening the understanding of CBDM. We hope that Section 3 of this paper, which (albeit briefly) discusses some important aspects of formalizations of CBDM, contributes in this direction. Such formalizations are presented in Section 2, where we also outline a new approach which is based on possibility theory and which makes use of approximate reasoning techniques and generalized decision theories.

2 Formalizing Case-Based Decision Making

Consider the problem of CBDM in the following form: Let $\mathcal{Q}$ be a set of problems, $\mathcal{A}$ a set of possible acts, and denote by $\mathcal{R}$ a set of results or outcomes. We assume that choosing act $a \in \mathcal{A}$ for solving problem $p \in \mathcal{Q}$ leads to the outcome $r = r(p,a)$. The utility function $u : \mathcal{R} \rightarrow U$ assigns utility values to such outcomes. Let $sim_{\mathcal{Q}} : \mathcal{Q} \times \mathcal{Q} \rightarrow [0,1]$ and $sim_{\mathcal{R}} : \mathcal{R} \times \mathcal{R} \rightarrow [0,1]$ be similarity functions measuring the similarity of problems and results, respectively. Depending on the approach, these functions may be interpreted in different ways. Particularly, the interval $[0, 1]$ needs not necessarily be considered as a ratio scale, it can also encode some underlying ordinal scale of similarity. The same remark applies to the utility function u. Suppose the decision making agent to have a memory of cases $\mathcal{M} = \{\langle (p_1, a_1), r_1\rangle, \ldots, \langle (p_n, a_n), r_n\rangle\}$ at his disposal, where $(p_k, a_k) \in \mathcal{Q} \times \mathcal{A}$, $r_k = r(p_k, a_k)$ $(k = 1, \ldots, n)$, and that he has to choose an act for a new problem p_0. If a certain act $a \in \mathcal{A}$ has not been applied to the problem p_0 so far (i.e., there is no case $\langle (p_0, a), r\rangle \in \mathcal{M}$), the agent will generally be uncertain about the result $r(p_0, a)$ and, hence, about the utility $u(r(p_0, a))$. According to the assumption underlying the paradigm of CBDM, he then evaluates an act based on its performance in similar problems in the past, as represented by (parts of) the memory $\mathcal{M}$.

2.1 The approach of Gilboa and Schmeidler

In their original approach to CBDT, Gilboa and Schmeidler propose the following evaluation of an act $a \in \mathcal{A}$:

$$V(a) = V_{p_0,\mathcal{M}}(a) := \sum_{\langle (p,a),r\rangle \in \mathcal{M}} sim_{\mathcal{Q}}(p, p_0) \cdot u(r). \tag{1}$$

The summation over an empty set yields the "default value" 0 which plays the role of an "aspiration level" (see Section 3).

As in expected utility theory (EUT) the decision maker is supposed to choose an act maximizing (1). Alternatively, a "normalized version" of the linear functional (1) has been proposed, which results from replacing $sim_{\mathcal{Q}}$ in (1) by the "normalized similarity" $sim(p, p_0) := sim_{\mathcal{Q}}(p, p_0)/\sum_{\langle (p',a),r\rangle \in \mathcal{M}} sim_{\mathcal{Q}}(p', p_0)$. Theoretical details of CBDT, including special assumptions concerning the memory $\mathcal{M}$ and an axiomatic characterization of decision principle (1), are presented in [7]. This paper also contains a thorough discussion of the relation between CBDT and EUT.

2.2 Fuzzy modelling of CBDM

Case-based decision making has been realized in [2] as some kind of similarity-based approximate reasoning. Let $\rightarrow$ be a multiple-valued implication connective. The value

$$V_{*p_0,\mathcal{M}}(a) := \min_{\langle (p,a),r\rangle \in \mathcal{M}} sim(p, p_0) \rightarrow u(r) \tag{2}$$

can be interpreted as a generalized truth value of the proposition that "choosing the act a for problems similar to p_0 has *always* resulted in good outcomes." As a special realization of (2) the evaluation

$$V(a) = V_{*p_0,\mathcal{M}}(a) := \min_{\langle (p,a),r\rangle \in \mathcal{M}} \max\{1 - sim(p, p_0), u(r)\} \tag{3}$$

is proposed. As an *optimistic* counterpart of (3) the criterium

$$V(a) = V^*_{p_0,\mathcal{M}}(a) := \max_{\langle (p,a),r\rangle \in \mathcal{M}} \min\{sim(p, p_0), u(r)\} \tag{4}$$

is introduced, which can be seen as a generalized truth degree of the proposition that "there is at least one problem similar to p_0 for which the act a has led to a good result." As noted in [2], (3) only makes sense if the memory contains at least one problem p such that $sim(p, p_0) = 1$ and where a has been chosen for solving p. Otherwise, it may happen that (3) is very high even though none of the problems contained in the memory is similar to the current problem p_0. Modifications of (3) and its optimistic counterpart have been proposed in order to cope with these difficulties. The modified measures are based on some kind of *normalization* of the similarity function for each act a, and a discounting of (3) and (4) which takes the absence of problems similar to p_0 into account. It should be mentioned that expressions (3) and (4) are closely related with decision criteria which have recently been derived in [4] in connection with an axiomatic approach to qualitative decision making under uncertainty.

2.3 A possibilistic formalization

The formalization of CBDM proposed in this section, details of which can be found in [12], can be seen as a possibilistic counterpart of the probabilistic approach presented in [10]. Define a *situation* as a problem-act tuple $s = (p, a)$ and denote by $\mathcal{S} := \mathcal{Q} \times \mathcal{A}$ the set of all situations. From $sim_{\mathcal{Q}}$ we may derive a similarity measure $sim_{\mathcal{S}}$ on $\mathcal{S}$ as follows: $sim_{\mathcal{S}}((p,a),(p',a')) := sim_{\mathcal{Q}}(p,p')$ if $a = a'$ and 0 otherwise. This definition has the same effect as the summation in (1): in order to estimate the utility of an act a given the problem p, only cases with the same act are taken into account. Observe, however, that $sim_{\mathcal{S}}$ can be defined in a more general way in order to allow for modelling the assumption of *act similarity* [8], i.e., the assumption that "similar acts lead to similar results for similar problems."

Now, consider the following formulation of the CBR principle: "The more similar two situations are, the more *certain* it is that the associated outcomes are similar." An adequate formalization is then obtained by utilizing the concept of a *certainty rule*, which is a special kind of a fuzzy rule. Such a rule corresponds to statements of the form "the more X is $\widetilde{A}$, the more certain Y lies in $\widetilde{B}$," where $\widetilde{A}$ and $\widetilde{B}$ are fuzzy sets. According to Zadeh's possibility theory-based approach to approximate reasoning [18], such a rule imposes a constraint on the possibility of tuples (x, y) in form of an upper bound: $\pi(y|x) \leq \max\{1 - \widetilde{A}(x), \widetilde{B}(y)\}$. Applying this approach to our problem of CBDM, we observe that each case of the memory $\mathcal{M}$ imposes a constraint on the possibility of an unknown result $r_0 = r(p_0, a)$ to be realized by some outcome $r \in \mathcal{R}$ if act $a \in \mathcal{A}$ is chosen. Since the "certainty-semantics" corresponds to the implication-based approach to fuzzy rules, the rules associated with these cases should be combined conjunctively. Thus, modelling the (generalized) conjunction by means of the min-operator, associating fuzzy sets with corresponding similarity functions, and applying a minimal specificity principle, we obtain the following possiblity distribution which characterizes the uncertainty concerning the outcome r_0 if act a is chosen:

$$(\forall\, r \in \mathcal{R}) \,:\, \pi_{a,\mathcal{M}}(r) := \min_{\langle (p',a'),r'\rangle \in \mathcal{M}} \max\{1 - sim_{\mathcal{S}}((p_0,a),(p',a')), sim_{\mathcal{R}}(r,r')\} \tag{5}$$

For taking account of the degree to which the CBR hypothesis actually holds true for a certain application, (5) is generalized to

$$(\forall\, r \in \mathcal{R}) \,:\, \pi_{a,\mathcal{M}}(r) := \min_{\langle (p',a'),r'\rangle \in \mathcal{M}} \max\{1 - m(sim_{\mathcal{S}}((p_0,a),(p',a'))), sim_{\mathcal{R}}(r,r')\} \tag{6}$$

with a function $m : [0,1] \to [0,1]$. Then, (6) corresponds to a formalization of the *generic* "CBR knowledge" of the decision maker, and m allows for an adequate adaptation of this knowledge. For instance, $m \equiv 0$ means that the similarity of situations does not constrain the similarity of results, i.e., that the CBR hypothesis does not apply at all.[1]

The approach (6) is a special (possibilistic) realization of

$$(\forall\, r \in \mathcal{R}) \,:\, \overline{\mu}_{a,\mathcal{M}}(r) := \min_{\langle (p',a'),r'\rangle \in \mathcal{M}} \eta_{sim_{\mathcal{S}}((p_0,a),(p',a'))}(sim_{\mathcal{R}}(r,r')), \tag{7}$$

where $\{\eta_\alpha \,|\, 0 \leq \alpha \leq 1\}$ is a parametrized class of fuzzy measures. This evaluation has been derived in [13] as an upper approximation of a likelihood function in connection with a probabilistic formalization of CBR.

The problem of choosing an act $a \in \mathcal{A}$ now turns out as one of choosing among the set $\{\pi_{a,\mathcal{M}} \,|\, a \in \mathcal{A}\}$ of possibility distributions, a situation quite similar to classical decision theories. There are different ways of realizing a corresponding selection. We can, for instance, adopt a probabilistic point of view and interprete the derived possibility distributions as upper probabilities. Then, an act $a \in \mathcal{A}$ is evaluted according to the *Choquet-expected utility*, which defines a generalized expected utility resp. an upper bound of an expected utility:

$$V(a) := \mathrm{CEU}(\pi_{a,\mathcal{M}}) = (\mathrm{C}) \int_{\mathcal{R}} (u \circ r)\, d\pi_{a,\mathcal{M}}.$$

It is, however, also possible to interprete the approach presented so far in a purely qualitative way if similarity, plausibility and preference are measured by means of linearly ordered (ordinal) scales (embedded into $[0,1]$) and if $1 - (\cdot)$ in (6) is replaced by the order-reversing function of the similarity scale (which is assumed to be "commensurable" with the preference scale.) If we also assure the commensuration of the plausibility scale and the preference scale via some order-preserving function h, then the qualitative decision theory of [4] can be applied. This leads, for instance, to the following (pessimistic) evaluation of an act $a \in \mathcal{A}$:

$$V(a) := \mathrm{QU}(\pi_{a,\mathcal{M}}) = \min_{r \in \mathcal{R}} \max\{n(h(\pi_{a,\mathcal{M}}(r))), u(r)\}, \tag{8}$$

with n an order-reversing function. In [3] it has been proposed to apply (8) to the following counterpart of (6):

$$(\forall\, r \in \mathcal{R}) \,:\, \pi_{a,\mathcal{M}}(r) := \max_{\langle (p,a),r\rangle \in \mathcal{M}} sim_{\mathcal{Q}}(p_0, p). \tag{9}$$

[1]The problem of determining or learning an adequate function m is not addressed in this paper.

3 Some Aspects of Case-Based Decision Making

3.1 The decision-theoretic set-up

As a main difference between the possibilistic approach to CBDM presented in Section 2.3 and the approaches of Section 2.1 and Section 2.2 it has to be mentioned that the latter make use of a decision theoretic set-up which is based on the concepts of similarity and utility. As opposed to this, the former approach combines the concept of similarity with those of belief and utility, and can be seen as an extension of classical (statistical) decision-theoretic models. This point has already been emphasized in [11]. In fact, case-based reasoning is not used for selecting an act directly. Rather, the cases of a memory $\mathcal{M}$ are treated as observations, and the similarity-based information associated with $\mathcal{M}$ has influence on the *belief* of the decision maker, represented here as a possibility distribution over $\mathcal{R}$. Observing that an act a has led to a good result for a similar problem, for instance, will increase the belief of the agent that a is also a good choice for the problem at hand.[2] In fact, the approach of Section 2.3 realizes a two-stage process, in which the actual decision problem is only solved in the second stage by means of (more or less) classical techniques from decision theory.

There are different motivations for assuming a connection of the form *similarity* $\rightarrow$ *belief* instead of *similarity* $\rightarrow$ *utility*. Firstly, it allows for making uncertainty concerning the outcome of decisions explicite, which seems very natural in connection with CBR if we realize the heuristic character of this reasoning method. Indeed, it is not reasonable to interprete the CBR hypothesis as a deterministic rule. Rather, it should be understood in such way that the similarity of two situations makes it more *likely* that the corresponding outcomes are similar, thus still allowing for "the exception to the rule."[3] A formalization utilizing certainty rules seems therefore meaningful, since a certainty rule is thought of as a rule which holds true in normal cases but still allows for exceptional situations [17].

Secondly, viewing the cases of a memory as an (additional) information source and, hence, utilizing case-based reasoning for decision making only indirectly leads to a more expressive approach which also avoids some technical difficulties. This becomes obvious, for instance, when considering the extreme example of a memory which does not contain any cases similar to the current problem, which means that the memory is actually empty. If, however, no cases exist, it seems strange that a *case-based* reasoning procedure can be used for estimating the utility of choosing some act for solving the problem. Instead of assigning a "default utility" it seems natural to expect the result of case-based reasoning to be *complete ignorance* about utilities, which is adequately reflected by the possibility distribution $\pi \equiv 1$ resulting in this situation from (6). More generally, an uncertainty measure is able to reproduce certain characteristics of a memory $\mathcal{M}$ better than a "point estimation." The evaluation (1), for instance, is unable to distinguish between a memory containing one situation with a large associated utility very similar to the current situation and a memory containing many situations also with large associated utility values only a little similar to the current situation [2].

Thirdly, the distinction between two "mental" levels, one for representing knowledge and one for making decisions, seems to have advantages in connection with the design of intelligent systems [6]. The integration of different information sources at the knowledge representation level, for instance, seems to be less problematic than doing the same at the decision level. Particularly, the latter would require a corresponding extension of the underlying decision theory. The approach of Section 2.3, for instance, allows the result of the CBR process, which takes place at the knowledge representation level, to be easily combined with general background knowledge represented by fuzzy rules [12].

3.2 Learning from cases

It is interesting to compare the different ways in which case-based information might influence the evaluation of acts. Such differences are obvious already for the *direct* evaluations (1), (3), (4). According to (3), an observed case can only decrease the evaluation of an act, which reflects the pessimistic or cautious character of this decision rule. As opposed to this, each observation can only have a positive influence on the evaluation according to (4). The decision criterion (1) lies somewhere in between, and it compensates between the "pros" and "cons" of an act. Particularly, the normalized version of (1) derives a weighted average and corresponds to Shephard's interpolation method, which is a special case of the k-NEAREST NEIGHBOR algorithm.

[2]The probabilistic formalization of case-based inference in [13] makes this explicite. Namely, it allows for treating the (similarity-based) information contained in a memory as some kind of statistical data.

[3]Interestingly enough, the connection *similarity* $\rightarrow$ *belief* is also supported by psychological evidence [16].

The opposite character of the evaluations (6) and (9), which conclude on the possibility of outcomes, is also worth mentioning. According to (9), the decision maker considers all outcomes as fully possible as long as he has not made any observations. Each new case serves as a constraint and decreases the possibility of certain results. This becomes especially clear if we consider the generalization (7) and let $\eta_x \equiv 1_{[x,1]}$ on $[0,1]$. Then $\overline{\mu}_{a,\mathcal{M}}(r) \in \{0,1\}$, and the set of possible outcomes for an act $a \in \mathcal{A}$ is given by the set

$$C_{a,\mathcal{M}} = \bigcap_{\langle (p',a'),r' \rangle \in \mathcal{M}} \{ r \in \mathcal{R} \,|\, sim_{\mathcal{S}}((p_0,a),(p',a')) \leq sim_{\mathcal{R}}(r,r') \}.$$

Thus, each new case completely excludes certain outcomes.[4] Observe, however, that the agent will often undo this exclusion or the reduction of possibility degrees in the course of a learning process: observed cases will make him revise his "CBR knowledge," represented by (6), by adapting the function m. The incorporation of this learning capability should be seen as an important aspect of the approach to CBDM based on (6).

According to (9), the decision maker starts with the possibility distribution $\pi \equiv 0$. Each new case serves as evidence for certain results and increases the possibility correspondingly. Loosly spoken, the agent learns what *can* happen, whereas he learns what *cannot* happen if he relies on (6). In this sense, (9) can be seen as a "data-driven" approach, whereas (6) is a more "knowledge-driven" method. The difference between (6) and (9) becomes also clear, if we realize that (6) is based on the idea of a certainty-rule, i.e., a constraint-based or implicative fuzzy rule, whereas (9) is closely related with the concept of a possibility-rule, i.e., an example-based or conjunctive fuzzy rule.

It is interesting to note that the *dependency* of cased-based information has been more or less ignored so far by approaches to CBDM.[5] Suppose, for instance, that we have two problems p and p', both of which are similar to the current problem p_0. The extend to which these observations increase the belief of a certain act a being a good solution for p_0, if it has been a good choice for p and p', should also depend on the similarity of p and p'. For instance, the observation that a has led to a good result for p', given the fact that it has done so in connection with p, should not be very astonishing if p and p' are more or less identical. The information that p and p' are similar to a smaller degree, however, could be taken as evidence for the act a to be a somehow generalizable solution. More generally, the observation that an act has led to good outcomes for two (or more) problems supports the CBR hypothesis stronger if these problems are similar rather than identical. The CBR hypothesis in turn supports the reusing of solutions for similar problems.

3.3 Behavioral implications

Since CBR is closely related with the idea of repeated problem solving and aspects of learning it seems natural to consider how a "CBDM-agent" acts over time in a certain environment. The approaches presented in this paper might already lead to a great variety of behaviors. As an interesting consequence of the decision principle (1), for instance, it has been pointed out in [7] that it can be seen as a formalization of Simon's idea of *satisficing decisions* [15]. Suppose, for example, that the selection of a certain act for a problem p has lead to some utility $v > 0$. When faced with the same problem again, the agent, at least as long as the "aspiration level" is defined as 0, will prefer this act to all other acts which have not been tried yet. More generally, he may try several acts until one results in a satisfying utility, but he will not attempt to maximize utility. If, however, the "aspiration level" is allowed to be adapted over time we might obtain an "experimenting" agent.

Qualitative decision rules such as (3) and (4) are generally not intended for maximizing some kind of average performance and, therefore, might lead to apparently paradoxical choice behaviors in connection with sequential or repeated decisions. Suppose, for instance, that all problems under consideration are very similar. According to the extremely pessimistic criterion (3), an agent might prefer an act with a poor performance for all problems to an act with a very high performance for almost all problems, simply because the latter has led to some low utility in a single case. Similar behaviors might also originate in connection with (8).

The study of behavioral implications might be helpful in evaluating and improving approaches to CBDM, regardless of whether these are thought of as descriptive theories or normative decision models. In connection with the idea of problem solving as a process of (sequential) decision making, advocated by the "agent-based" view of artificial intelligence [14], it is of special interest to investigate the potential of CBDM to support the

[4] The derivation of $C_{a,\mathcal{M}}$ is closely related with the concept of a *gradual inference rule*, which constrains outcomes according to the assumption that "the more similar (in the sense of $sim_{\mathcal{S}}$) two situations are, the more similar are the corresponding outcomes (in the sense of $sim_{\mathcal{R}}$)" [5].

[5] This kind of dependency has been considered, however, within the probabilistic model underlying the derivation of (6) [13].

design of intelligent (problem solving) agents is of special interest. This has to be done, of course, by means of concrete applications and simulation studies [9].

4 Summary and Conlusion

In this paper, we have briefly surveyed some formal approaches which incorporate the concept of similarity into models of decision making. Moreover, we have outlined a formalization based on possibility theory and approximate reasoning techniques which is realized as a two-stage process. In the first stage, the decision maker applies case-based reasoning for quantifying his uncertainty associated with different decisions in form of possibility distributions over the set of consequences. In the second stage, he utilizes generalizations of expected utility theory for choosing among acts resp. associated distributions. It has been argued that this approach provides an adequate model of the principle underlying case-based reasoning and may avoid some technical difficulties arising in previous approaches. Particularly, it seems to meet the heuristic character of CBR and CBDM in the sense that it explicitely allows for the modelling of uncertainty and emphasizes the ability of learning in the form of adapting the formalization of the CBR hypothesis.

We have also touched on some interesting topics in connection with the idea of case-based decision making. Of course, this discussion was intended to raise rather than answer questions, and a more thorough investigation of corresponding aspects has to follow. Moreover, there are further questions of basic importance which have not been addressed such as, e.g., the acquisition of similarity measures. Nevertheless, the discussion shows that CBDM can be approached in various ways, leading to models with different properties and implications for decision making behavior. Thus, it also demonstrates the need for comparing corresponding models and, hence, for (objective) criteria which may supply evidence for the model's usefulness and efficiency.

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