An Approach to Linguistic Negation of Fuzzy Property Combinations

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ABSTRACT: In this paper, we present the new developments of a model dealing with affirmative or negative fuzzy information, and more particularly information referring to linguistic negation. We propose an extension of the linguistic negation concept allowing us to deny more general information, that is to say, combinations of nuanced properties based on the conjunction and disjunction connectors. This new model seems to be in better accordance with linguistic analysis. Moreover, this extended linguistic negation can be viewed as a generalization of the logical one.

KEYWORDS: Fuzzy sets, Artificial Intelligence, Linguistic Negation, Nuanced Information, Natural Language

1 INTRODUCTION

In this paper, we present a new model dealing with nuanced information within a fuzzy context. We refer to the methodology proposed in (Pacholczyk (1997), Pacholczyk (1998a), Pacholczyk (1998b)) to represent negative information referring to linguistic negation. Previous models allow us to represent a rule like "if the wage *is not high,* then the summer holidays *are not very long*" and a fact like "the wage *is really low*". But facts and rules are only based upon nuances of *one property*. So, they do not accept facts or rules based upon *combinations of properties* as they may appear in knowledge bases including a rule like, "if the man *is not visible* in the crowd, *he is not medium or tall*", or a fact like "the man *is rather small or very small*". Our main idea has been to extend the definitions of linguistic negation proposed in (Pacholczyk (1997), Pacholczyk (1998a), Pacholczyk (1998b)) in order to take into account *combinations of nuanced properties* based on conjunction or disjunction connectors. Section 2 briefly presents a previous model (Desmontils and Pacholczyk (1997)) allowing the representation of nuanced properties. In Section 3, we present the main ideas leading to the previous methodology. More precisely, we point out that it is necessary to leave a representation of linguistic negation in terms of a one-to-one correspondence and to turn towards a one-to-many correspondence, called here a *multiset function*. We recall previous results leading to the *standard forms of negation*. The definitions of the linguistic negation and of the *intended meaning* of a denied property are based upon the ones proposed in (Pacholczyk (1998b)). Some modifications have been made to obtain better accordance with the linguistic analysis proposed in (Muller (1991), Culioli (1991), Horn (1989), Ducrot et al (1995), Ladusaw (1996)). Section 4 is devoted to new developments which improve the abilities of the previous model. We *extend the definition of linguistic negation* in order to be able to deny previous combinations of nuanced properties. Then, we can define the set of intended meanings of "x is not A". Finally, we study the particular *contraposition law*. It appears clearly that this more powerful concept of linguistic negation can be viewed as a generalization of the logical one.

2 THE REPRESENTATION OF DISCOURSE UNIVERSE

We suppose that our discourse universe is characterised by a finite number of *concepts* C_i. A set of properties P_{ik} is associated with each C_i, whose description *domain* is denoted as D_i. The properties P_{ik} are said to be the *basic properties* connected with concept C_i. For example, the concepts of "height", "wage" and "appearance" should be understood as qualifying individuals of the human type. The concept "wage" can be characterised by the basic fuzzy properties "low", "medium" and "high". *Linguistic modifiers* bearing on these basic properties permit us to express *nuanced knowledge*. This work uses the model proposed in (Desmontils and Pacholczyk (1997)) to represent affirmative information expressed in the form « x *is* $f_\alpha m_\beta P_{ik}$ » or « x *is not* $f_\alpha m_\beta P_{ik}$ » in the case of negation. In this context, expressing a property like "$f_\alpha m_\beta P_{ik}$" called here *nuanced property*, requires a list of linguistic terms.

Two ordered sets of modifiers are selected depending on their modifying effects. The first one groups *translation modifiers* resulting somehow in both a *translation and a possible precision variation* of the basic property*:* For example, the set of translation modifiers could be M_7={extremely little, very little, rather little, moderately (∅), rather, very, extremely} totally ordered by the relation: $m_\alpha < m_\beta \Leftrightarrow \alpha < \beta$ (Figure 1). The second one consists of *precision modifiers* which make it possible to increase (or decrease) the precision of the previous properties. For example, F_6={vaguely, neighbouring, more or less, moderately, really, exactly} totally ordered by $f_\alpha < f_\beta \Leftrightarrow \alpha < \beta$ (Figure 2).

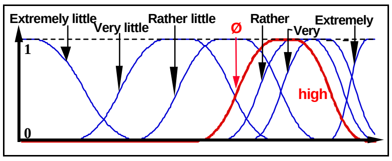

Figure 1: Fuzzy Modifiers

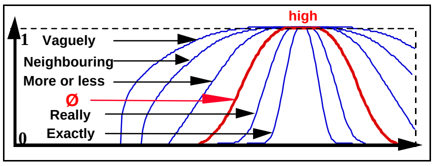

Figure 2: Fuzzy Operators

3 SOME IMPROVEMENTS OF PREVIOUS MODELISATION

It has been pointed out (Pacholczyk (1997), Pacholczyk (1998a)) that the fuzzy set theory (Zadeh (1965)) is hard put to propose an adequate representation of negative information. Indeed, saying that "Smith is not tall", one can implicitly refer to "medium" or "extremely small" and not to "¬tall". Within a *qualitative context*, one can refer to a set of linguistic labels denoted as $L=\{u_0, \ldots, u_n\}$ totally ordered: $u_i < u_j \Leftrightarrow i < j$ (Pacholczyk (1992)). The linguistic negation, denoted as Neg, verifies: $Neg(u_i)=u_{n-i}$. But, when the speaker asserts that "the wage of Smith is not high", he can only say that the intended meaning belongs to the set {"medium", "low"}. So, the representation of linguistic negation should be made by using a correspondence between an element of L and a subset of L. More generally, any function associating with each nuanced property A, a subset of nuanced properties defined in the same domain as A, will be called a *multiset function*. Since the models presented in (Pacholczyk (1997), Pacholczyk (1998a), Pacholczyk (1998b)) alleviate the difficulties of Torra's approach (Torra (1996)) based upon a multiset function, we refer to this satisfactory methodology. The model proposed in (Pacholczyk (1998b)) improves the previous ones (Pacholczyk (1997), Pacholczyk (1998a)) by explicitly taking into account the different *scopes of the linguistic negation*. This permits us to make clearer the different interpretations of "x is not A" resulting from scoping effects in accordance with linguistic theories (Muller (1991), Culioli (1991), Horn (1989), Ducrot et al (1995), Ladusaw (1996)).

Within the discourse universe, let us denote as: C the set of distinct concepts C_i, D_i the domain associated with the concept C_i, M the set of modifier combinations, B_i the set of associated basic properties P_{ik} defined on D_i, N_{ik} the set of all nuances of the basic property P_{ik}, N_i the set of all nuanced properties associated with C_i. Then let: $\mathsf{D}=\cup_i \mathsf{D}_i$, $\mathsf{B}=\cup_i \mathsf{B}_i$, $\mathsf{N}=\cup_i \mathsf{N}_i$, $\mathsf{E}=\mathsf{M}\cup\mathsf{D}\cup\mathsf{B}\cup\mathsf{N}$. The *reference frame of the linguistic negation* is defined as follows.

Definition 3.1. Let Neg a multiset function Neg : $\mathsf{E} \to \mathsf{P}(\mathsf{E})$ verifying the conditions:
L1: $\forall n_\gamma \in \mathsf{M}$, $Neg(n_\gamma)=\mathsf{M}\setminus\{n_\gamma\}$, **L2**: $\forall P_{ik} \in \mathsf{B}_i$, **L21:** $\exists m$, $Neg(P_{ik})=\{P_{im}\}$ *or* **L22**: $Neg(P_{ik})=\mathsf{B}_i\setminus\{P_{ik}\}$,
L3: $\forall x \in \mathsf{D}_i$, $Neg(x)=\mathsf{D}_i\setminus\{x\}$, **L4**: $\forall n_\gamma \in \mathsf{M}$, $\forall P_{ik} \in \mathsf{B}_i$, $Neg(n_\gamma P_{ik})=\mathsf{N}_i\setminus\{n_\gamma P_{ik}\}$,
L5: $\forall n_\gamma \in \mathsf{M}$, $\forall P_{ik} \in \mathsf{B}_i$, $Neg(n_\gamma (P_{ik}))=\mathsf{N}_{ik}\setminus\{n_\gamma P_{ik}\}$, and
L6: $\forall n_\gamma \in \mathsf{M}$, **L61:** $n_\gamma(Neg(P_{ik}))= \mathsf{N}_{im}$ *or* **L62:** $\forall Neg(P_{ik})$, $n_\gamma(Neg(P_{ik}))= \mathsf{N}_i\setminus\mathsf{N}_{ik}$.

From a linguistic point of view "x *is not* $n_\gamma P_{ik}$" can generally express something corresponding to an affirmative assertion "y *is* $n_\delta P_{ij}$". Following are the different *standard forms* of the nuanced property $n_\delta P_{ij}$ resulting from each possible scope. Saying "x is not $n_\gamma P_{ik}$", for this x, the speaker may express *or* refer to:

- *A rejection without reference to an affirmative translation of the negation* **[F₀]**

"Smith is not guilty" without reference to affirmative property, may occur in a context where the only thing about his culpability is that his alibi is confirmed.

- *Another object instead of x belonging to the same domain and satisfying the nuanced property*. **[F₁]**

So, "x is not $n_\gamma P_{ik}$" means "not(x) is $n_\gamma P_{ik}$" or in other words, not(x)=y∈Neg(x) (cond. **L3**). As an example, "Jack is not guilty" since it is John who is guilty.

- *Another nuance of the same property*. **[F₂]**

So, "x is not $n_\gamma P_{ik}$"⇔"x is not($n_\gamma (P_{ik})$)"⇔"x is $n_\delta P_{ik}$". So, $not(n_\gamma (P_{ik}))=n_\delta P_{ik} \in Neg(n_\gamma (P_{ik}))$ (cond. **L5**). As an example, "Jack is not small" since "Jack is extremely small".

- *A nuanced property except $n_g P_{ik}$, which is nonetheless associated with the same concept*. **[F₃]**

So, "x is not $n_\gamma P_{ik}$" means "x is not($n_\gamma P_{ik}$)" which is equivalent to "x is $n_\delta P_{ij}$". So, $not(n_\gamma P_{ik})=n_\delta P_{ij} \in Neg(n_\gamma P_{ik})$ (condition **L4**). For example, "the wage being not very high" can be "really low", "medium" or "rather little high".

- *A nuance of another basic property associated with the same concept.* **[F₄]**

So, "x is not $n_\gamma P_{ik}$" means "x is $n_\gamma(not(P_{ik}))$" or "x is $n_\delta P_{ij}$". Here, $n_\gamma(not(P_{ik}))=n_\delta P_{ij} \in n_\gamma(Neg(P_{ik}))$ (cond. **L62**). For example, "John is not small" since he is at least "medium".

- *A nuance of another precise basic property associated with the same concept.* **[F_5]**
So, "x is not $n_\gamma P_{ik}$" means "x is $n_\gamma(not(P_{ik}))$" or "x is $n_\delta P_{im}$". Here, $n_\delta P_{im} \in n_\gamma(Neg(P_{ik}))=N_{im}$ (Cond. **L61**). For example, "Jack is not very big", since "he is rather thin".
- *A new basic property of the same concept* **[F_6]**
In this case "x is not A" means "x is not-A": a new basic property denoted as "not-A" is associated with the same concept. As an example, "this wine is not bad" can induce the new basic property "not-bad".

- *Remark.* In the following, we no longer refer to the first standard form F_1 which seldom occurs in knowledge-bases and suppose that the new property introduced in the last form F_6 appears among other basic properties. Moreover, for any $A \in N$ and F_t (t=2, …,5) the set Neg(A) is exactly defined.

Definition 3.2 If standard form F_t (t=2, …,5) is applied to nuance $A \in N$, the set Neg(A) will be denoted $Neg_t(A)$. It defines *the basic reference frame from which the intended meanings of linguistic negation of* A *will be extracted*.

-*Remark:* Since F_0 implies a rejection of A without reference to an affirmative assertion, we can put: $Neg_0(A)=\emptyset$.

In this work, we refer to the definition proposed in (Pacholczyk (1998b)). As a result, a comparison of different definitions proposed in (Pacholczyk (1997), Pacholczyk (1998a), Pacholczyk (1998b)) has been done in (Pacholczyk (1999)): It is proved that the definition proposed in (Pacholczyk (1998b)) can induce previous ones. In accordance with previous linguistic analysis of linguistic negation, it has been pointed out in (Pacholczyk (1997), Pacholczyk (1998a)) that when one asserts that « x is not A » then, *(1)* one *rejects* a reference to « x is A », and *(2)* if necessary, *one refers either* to the *logical negation of A*, or to *another property* P different from A but defined in the same domain, or sometimes to a *nuance* $f_\alpha m_\beta A$ of A, or finally to *a new basic property* denoted as not-A. In the following, the judgement of rejection receives as an interpretation: $\mu_A(x)<\varepsilon$, which differs from the one proposed in (Pacholczyk (1998a), Pacholczyk (1998b)). Indeed, linguistically speaking, one rejects that "x is A" receives a significant truth degree for x. In other words, its value is close to false, or the corresponding membership degree is close to 0. So, this value is not lower than a value close to 1, as is the case in the initial definition. Moreover, asserting that "x is not A", *if necessary the user refers* to "x is P" as the intended meaning of his negation. The previous analysis only defines the standard forms of the linguistic negation. But, it is obvious that any element of $Neg_t(u)$ cannot lead to the intended meaning of "x is not A". Intuitively, the speaker understands a real difference between the membership degrees belonging to A and P for their significant values, that is to say: $\mu_A(x)$ *(resp.* $\mu_P(x))\geq\rho \Rightarrow \mu_P(x)$ (resp. $\mu_A(x))\leq\varepsilon$.

Definition 3.3 Let ρ, ε such that: $0\leq\varepsilon\leq\rho\leq 1$. Given a standard form F_t, the multiset function $Neg^t_{\rho,\varepsilon}: N \to P(N)$ defines for any $A \in N$, a set $Neg^t_{\rho,\varepsilon}(A) \subset Neg_t(A)$. More precisely, $P \in Neg^t_{\rho,\varepsilon}(A)$ if and only if P satisfies the following conditions: **CD0**: $P \in Neg_t(A)$, **CD1:** $\forall x$, $\{\mu_A(x)\geq\rho \Rightarrow \mu_P(x)\leq\varepsilon\}$, and **CD2**: $\forall x$, $\{\mu_P(x)\geq\rho \Rightarrow \mu_A(x)\leq\varepsilon\}$. Then, $Neg^t_{\rho,\varepsilon}(A)$ is said to *be the linguistic negation r-compatible with* A *with the tolerance threshold e, given the standard form* F_t. Moreover, any $P \in Neg^t_{\rho,\varepsilon}(A)$ is said to be *a linguistic negation r–compatible with A with the tolerance threshold e, given the standard form* F_t.

Example. We have collected in Figure 3, $Neg^3_{0.75,\ 0.35}$ ("low") a set of negations 0.75–compatible with "low" with tolerance threshold 0.35, given the standard form F_3.

The previous definition constructs the *effective reference frame* $Neg^t_{\rho,\varepsilon}(A)$ from which we have to extract the intended meanings of the linguistic negation. Then, we have to define a subset of $Neg^t_{\rho,\varepsilon}(A)$, denoted as $neg^t_{\rho,\varepsilon}(x, A)$, which consists of the intended meaning of "x is not A". Since ε is the tolerance threshold from which the membership degrees are significant, we accept as intended meanings only the solutions satisfying this condition for x.

Definition 3.4 Put $neg^t_{\rho,\varepsilon}(x, A)=\{P \in Neg^t_{\rho,\varepsilon}(A) \mid \mu_P(x)\geq\varepsilon\}$. Any $P \in neg^t_{\rho,\varepsilon}(x, A)$ is called an *intended meaning for x of the linguistic negation r–compatible with A with tolerance threshold e, given the standard form* F_t. We say also that "x is P" is an intended meaning with the tolerance threshold ε of the linguistic negation ρ–compatible of "x is A". If no confusion is possible, we simply say that *P is an intended meaning of the linguistic negation of A for x*.

Example. By using previous solutions (*Cf.* Figure 3), we have collected in Figure 4 the intended meanings of "x is not low" for the values a (2 solutions based upon "low") and b (7 solutions based upon "medium" and "high").

Remark. This definition also differs from the one proposed in (Pacholczyk (1997), Pacholczyk (1998a), Pacholczyk (1998b), Pacholczyk (1999)). Linguistically speaking, "x is P" the intended meaning of "x is not A" should not be false (or not close to false) for this x. So, it seems more natural to reject a truth degree close to false and not a truth degree lower that a value close to true, as is the case in the previous definitions. So, in terms of membership degrees, we have to take a linguistic negation P at x for which $\mu_P(x)\geq\varepsilon$, instead of P for which $\mu_P(x)\geq\rho$.

If the user wishes only one interpretation of the linguistic negation, it is possible to use the *default choice strategy* proposed in (Pacholczyk (1999)). A particular choice can be made among the plausible solutions leading to the most significant membership degree and having the weakest complexity.

Definition 3.5 The *complexity* of the nuanced property A, denoted as comp(A), is equal to the number of nuances (different from $\emptyset$) required in its definition.

Definition 3.6 A choice of a nuanced property P satisfying the following conditions: **I1 :** $P\in neg^t_{\rho,\varepsilon}(x, A)$, **I2 :** $\mu_P(x)=Max\{\mu_P(x) | P\in neg^t_{\rho,\varepsilon}(x, A)\}$, and **I3:** $\forall Q\in neg^t_{\rho,\varepsilon}(x, A)$, $\{\mu_Q(x)=\xi(x, A)\Rightarrow comp(P)\leq comp(Q)$, defines "x is P" as intended meaning of "x is not A".

Example. By using the solutions collected in Figure 4, "a is not low" receives as intended meaning "a is really rather little high".

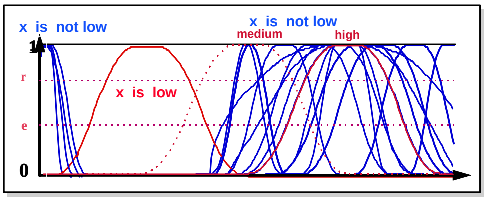

Figure 3: Negations 0.75–compatible with "low" with tolerance threshold 0.35 given $\mathbf{F_3}$.

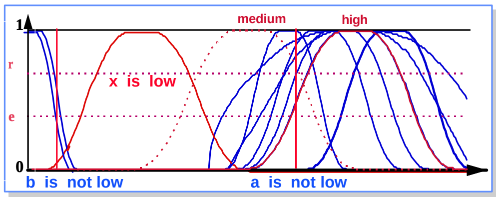

Figure 4: Intended meanings of negations 0.75–compatible for a or b with "low" with tolerance threshold 0.35 given $\mathbf{F_3}$.

We can *recall* basic properties of linguistic negation (Pacholczyk (1997), Pacholczyk (1998a), Pacholczyk (1998b)).

Proposition 3.1 The knowledge about "x is A" doesn't automatically define the knowledge about "x is not A".

Proposition 3.2 $\{\rho\leq\rho', \varepsilon'\leq\varepsilon\leq\rho\}\Rightarrow Neg^t_{\rho',\varepsilon'}(A)\subseteq Neg^t_{\rho,\varepsilon}(A)$.

Proposition 3.3 There exists ρ and ε such that the negation ρ-compatible with the tolerance threshold ε takes into account all previous interpretations of "x is not A".

Proposition 3.4 $P\in Neg^t_{\rho,\varepsilon}(A) \Rightarrow A\in Neg^t_{\rho,\varepsilon}(P)$. But, $P\in neg^t_{\rho,\varepsilon}(x, A)$ does not imply $A\in neg^t_{\rho,\varepsilon}(x, P)$

Proposition 3.5 $neg^t_{\rho,\varepsilon}(x, A)$ can be an empty set. Even if N_i, the set of all nuanced properties associated with C_i, can be totally ordered, for any $A\in N_i$ the set $neg^t_{\rho,\varepsilon}(x, A)$ can be a non-convex set.

Proposition 3.6 The model can deal with boolean basic properties without nuances by choosing ρ=1; ε=0. In this case, the standard forms correspond with the notion of marked (or not) property (Muller (1991), Ducrot et al (1995)).

4 TOWARDS AN EXTENDED LINGUISTIC NEGATION

The previous approach to linguistic negation does not allow the use of assertions referring to a combination of nuances. It can pointed out that this possibility of combinations implicitly appears in standard forms $\mathbf{F_2}$ to $\mathbf{F_5}$.. When the user asserts that "x is not P", he can also refer to a combination of nuances based upon a conjunction (*and* denoted here $\wedge$) or a disjunction (*or* denoted here $\vee$). For example, "John is not tall" since "John is small *or* extremely tall", "Jack is not tall *or* small" since "Jack is really medium" or "My hat is not black *and* white" since "it is red *and* blue". So, we have to modify the previous definition of linguistic negation in order to take into account this natural extension of the linguistic negation. Moreover, the previous results do not establish an explicit link between an implication and its contraposition, as is the case in the boolean context. In the following, we introduce the more general notion of *extended linguistic negation* in order to improve the abilities of previous model. For any linguistic nuance *A*, we suppose that A is its associated fuzzy set (defined by its membership function μ_A). We suppose that the linguistic connectors $\vee$ and $\wedge$ are defined with the fuzzy set operators $\cup$ and $\cap$: $\mu_{A\cup B}(x)= max(\mu_A(x), \mu_B(x))$ and $\mu_{A\cap B}(x)=min(\mu_A(x), \mu_B(x))$. In order to extend the concept of linguistic negation, we introduce the notion of complex nuance.

Definition 4.1 Given a concept C_i, a linguistic expression $U(A_1, \ldots, A_h, \ldots, A_p)$ with $A_h\in N_i$, h=1,…,p, resulting from the application of $\vee$ and $\wedge$ and to nuances A_h, h=1,…,p, is said to be a *complex nuance* induced from N_i. If no confusion is possible, U stands for $U(A_1, \ldots, A_h, \ldots, A_p)$.

Example. The properties "low", "medium" and "high" being associated with C_i="wage", then U=U(extremely low, medium, rather little high)=extremely low$\vee$(medium$\wedge$ rather little high) is a complex nuance induced from N_i.

Definition 4.2 Let us suppose that C_i is a concept, F_t a standard form, and $U(A_1, \ldots, A_h, \ldots, A_p)$ (or U) is a *complex nuance* induced from N_i. The *extended linguistic negation* ρ-compatible with U with the tolerance threshold ε, given the standard form $\mathbf{F_t}$, denoted as $C\_Neg^t_{\rho,\varepsilon}(U)$, is a set of complex nuances induced from N_i. More precisely, $V(B_1, \ldots, B_m, \ldots, B_q)$ (or V) being a complex nuance induced from N_i, then $V\in C\_Neg^t_{\rho,\varepsilon}(U)$ iff:
CD0': $\forall B_m, \exists A_h$ such that $B_m\in Neg^t_{\rho,\varepsilon}(A_h)$, **CD1':** $\forall x, \{\mu_U(x)\geq\rho\Rightarrow\mu_V(x)\leq\varepsilon\}$, and **CD2'**: $\forall x, \{\mu_V(x)\geq\rho\Rightarrow\mu_U(x)\leq\varepsilon\}$.

Then, any $V\in C\_Neg^t_{\rho,\varepsilon}(U)$ is said to be *an extended linguistic negation* ρ–compatible with U with the tolerance threshold ε, given the standard form $\mathbf{F_t}$.

Remark. It is obvious that $Neg^t_{\rho,\varepsilon}(A)\subset C\_Neg^t_{\rho,\varepsilon}(A)$, for any $A\in N_i$. Moreover, this extended definition can be viewed as a generalization of the previous one.

The following theorem will be at the basis of general properties of this extended linguistic negation.

Proposition 4.1 A complex nuance $V\in C\_Neg^t_{\rho,\varepsilon}(U)$ if :

(1): $U\in N_i$ and :

(1a): $V=\vee_{l\in L}Q^l$ with $\forall l\in L$, $Q^l\in Neg^t_{\rho,\varepsilon}(U)$, or

(1b): $V=\wedge_{l\in L}R^l$ with $\forall l\in L$, $R^l\in Neg^t_{\rho,\varepsilon}(U)$, or

(1c): $V=Q^0\vee(\wedge_{l\in L}R^l)$ with $Q^0\in Neg^t_{\rho,\varepsilon}(U)$ and $\forall l\in L$, $R^l\in Neg^t_{\rho,\varepsilon}(U)$, or

(1d): $V=R^0\wedge(\vee_{l\in L}Q^l)$ with $R^0\in Neg^t_{\rho,\varepsilon}(U)$ and $\forall l\in L$, $Q^l\in Neg^t_{\rho,\varepsilon}(U)$ or

(1e): V=U with $V\in N_i$ and $U\in Neg^t_{\rho,\varepsilon}(V)$.

(2): $U=A\wedge B$ with $A\in N_i$ and $B\in N_i$, $V=P\vee Q$ with $P\in Neg^t_{\rho,\varepsilon}(A)$ and $Q\in Neg^t_{\rho,\varepsilon}(B)$,

(3): $U=A\vee B$ with $A\in N_i$ and $B\in N_i$, $V=P\wedge Q$ with $P\in Neg^t_{\rho,\varepsilon}(A)$, and $Q\in Neg^t_{\rho,\varepsilon}(B)$.

Proof. It is obvious to verify condition **CD0'**. Let us now examine conditions **CD1'** and **CD2'**. The following elements of proof are based on definition 3.4.

(1a): Put $V=\vee_{l\in L}Q^l$ with $\forall l\in L$, $Q^l\in Neg^t_{\rho,\varepsilon}(U)$. Then, $\mu_U(x)\geq\rho\Rightarrow\{\forall l\in L,\ \mu_Ql(x)\leq\varepsilon\}\Rightarrow\mu_V(x)\leq\varepsilon$. Conversely we have: $\mu_V(x)\geq\rho\Rightarrow\{\exists l\in L,\ \mu_Ql(x)\geq\rho\}\Rightarrow\mu_U(x)\leq\varepsilon$.

(1b): Put $V=\wedge_{l\in L}R^l$ with $\forall l\in L$, $R^l\in Neg^t_{\rho,\varepsilon}(U)$. Then, $\mu_U(x)\geq\rho\Rightarrow\{\forall l\in L,\ \mu_Rl(x)\leq\varepsilon\}\Rightarrow\mu_V(x)\leq\varepsilon$. Conversely, we have: $\mu_V(x)\geq\rho\Rightarrow\{\forall l\in L,\ \mu_Rl(x)\geq\rho\}\Rightarrow\mu_U(x)\leq\varepsilon$.

(1c): Put: $V=Q^0\vee(\wedge_{l\in L}R^l)$ with $Q^0\in Neg^t_{\rho,\varepsilon}(U)$ and $\forall l\in L$, $R^l\in Neg^t_{\rho,\varepsilon}(U)$. Then, we have: $\mu_U(x)\geq\rho\Rightarrow\{\mu_Q0(x)\leq\varepsilon$ *and* $\forall l\in L,\ \mu_Rl(x)\leq\varepsilon\}\Rightarrow\mu_V(x)\leq\varepsilon$. Conversely, we have: $\mu_V(x)\geq\rho\Rightarrow\{\mu_Q0(x)\geq\rho$ *or* $\forall l\in L,\ \mu_Rl(x)\geq\rho\}\Rightarrow\mu_U(x)\leq\varepsilon$.

(1d): Put: $V=R^0\wedge(\vee_{l\in L}Q^l)$ with $R^0\in Neg^t_{\rho,\varepsilon}(U)$.and $\forall l\in L$, $Q^l\in Neg^t_{\rho,\varepsilon}(A)$. Then, we have: $\mu_U(x)\geq\rho\Rightarrow\{\mu_R0(x)\leq\varepsilon$ *and* $\forall l\in L,\ \mu_Ql(x)\leq\varepsilon\}\Rightarrow\mu_V(x)\leq\varepsilon$. Conversely, we obtain: $\mu_V(x)\geq\rho\Rightarrow\{\mu_R0(x)\geq\rho$ *and* $\exists l\in L,\ \mu_Rl(x)\geq\rho\}\Rightarrow\mu_U(x)\leq\varepsilon$.

(1e): If $P\in Neg^t_{\rho,\varepsilon}(U)$, then $U\in Neg^t_{\rho,\varepsilon}(P)$. So, the double linguistic negation of A can be a linguistic negation of A.

A and B being associated with the same concept, let us suppose that $P\in Neg^t_{\rho,\varepsilon}(A)$ and $Q\in Neg^t_{\rho,\varepsilon}(B)$. Then:

(2): $\mu_{A\wedge B}(x)\geq\rho\Rightarrow\{\mu_A(x)\geq\rho$ *and* $\mu_B(x)\geq\rho\}\Rightarrow\{\mu_P(x)\leq\varepsilon$ *and* $\mu_Q(x)\leq\varepsilon\}\Rightarrow\mu_{P\vee Q}(x)\leq\varepsilon$. Conversely, we have: $\mu_{P\vee Q}(x)\geq\rho\Rightarrow\{\mu_P(x)\geq\rho$ *or* $\mu_Q(x)\geq\rho\}\Rightarrow\{\mu_A(x)\leq\varepsilon$ *or* $\mu_B(x)\leq\varepsilon\}\Rightarrow\mu_{A\wedge B}(x)\leq\varepsilon$.

(3): A similar analysis gives us :$\mu_{A\vee B}(x)\geq\rho\Rightarrow\mu_{P\wedge Q}(x)\leq\varepsilon$ and $\mu_{P\wedge Q}(x)\geq\rho\Rightarrow\mu_{A\vee B}(x)\leq\varepsilon$.

Example. Given the concept "wage" and the standard form $\mathbf{F_3}$, we can obtain: extremely high∨really medium∈ $C\_Neg^3_{0.75,0.35}$(low), and very high∨extremely low∈ $C\_Neg^3_{0.75,0.35}$(low∧medium).

In order to obtain the extended intended meanings of "x is not A", we modify as follows definition 3.4. We can distinguish two cases:

(a): Any complex property belonging to $C\_Neg^t_{\rho,\varepsilon}(A)$ can be an intended meaning for any value of x. So, we can put : $\forall x$, $C\_neg^t_{\rho,\varepsilon}(x, A)=C\_Neg^t_{\rho,\varepsilon}(A)$.

(b): If it is not the case, we can only accept for this x, like in modified definition 3.4, the significant combinations. Put: $C\_neg^t_{\rho,\varepsilon}(x, A)=\{P\in C\_Neg^t_{\rho,\varepsilon}(A)\mid\mu_P(x)\geq\varepsilon\}$

Definition 4.3 The set of intended meanings of "x is not A", can be defined as follows: (a): *either*, $\forall x$, $C\_neg^t_{\rho,\varepsilon}(x, A)=C\_Neg^t_{\rho,\varepsilon}(A)$, (b): *or* $C\_neg^t_{\rho,\varepsilon}(x, A)=\{P\in C\_Neg^t_{\rho,\varepsilon}(A)\mid\mu_P(x)\geq\varepsilon\}$. Any $P\in C\_neg^t_{\rho,\varepsilon}(x, A)$ is called an *extended intended meaning* for x of the linguistic negation ρ–compatible with A with tolerance threshold ε, given the form $\mathbf{F_t}$.

Remark. It is also possible to propose a choice strategy of an extended intended meaning of "x is not A" by working in $C\_Neg^t_{\rho,\varepsilon}(A)$ and by choosing as a complexity the sum of the complexities of the components of A.

In the following, we suppose that the rule: if "x is *A*" then "y is *B*" receives as a translation $\mu_A(x)\rightarrow\mu_B(y)$ where A and B are the fuzzy sets associated with *A* and *B*. Then, the following result extends the classical property: $u\rightarrow v=\neg v\rightarrow\neg u$ to the extended linguistic negation.

Proposition 4.2 Let us suppose that the implication → and its associated T-norm T satisfy the properties: $u\rightarrow v=1$ iff $u\leq v$; $T(u\rightarrow v, v\rightarrow w)\leq u\rightarrow w$ (weak transitivity law); $u\rightarrow v=\neg v\rightarrow\neg u$ (contraposition law). Moreover, we suppose that: $\forall x$, $C\_Neg^t_{\rho,\varepsilon}(A)=C\_neg^t_{\rho,\varepsilon}(x, A)$ and $C\_Neg^r_{\rho,\varepsilon}(B)=C\_neg^r_{\rho,\varepsilon}(x, B)$. Then, the extended linguistic negation possesses the following properties:

(i) If there exists $Q\in C\_Neg^r_{\rho,\varepsilon}(B)$ and $P\in C\_Neg^t_{\rho,\varepsilon}(A)$ such that $Q\subset\neg B$ and $\neg A\subset P$, then: $\mu_A(x)\rightarrow\mu_B(y)\leq\mu_Q(y)\rightarrow\mu_P(x)$. In other words, the rule if "x is *A*" then "y is *B*" implies the rule if "y is not *B*" then "x is not *A*".

(ii) If there exists $P\in C\_Neg^t_{\rho,\varepsilon}(A)$ and $Q\in C\_Neg^t_{\rho,\varepsilon}(B)$ such that $P\subset\neg A$ and $\neg B\subset Q$, then: $\mu_Q(y)\rightarrow\mu_P(x)\leq\mu_A(x)\rightarrow\mu_B(y)$. In other words, the rule if "y is not *B*" then "x is not *A*" implies the rule if "x is *A*" then "y is *B*".

(iii) If there exists $P\in C\_Neg^{t}_{\rho,\varepsilon}(A)$ and $Q\in C\_Neg^{t}_{\rho,\varepsilon}(B)$ such that $\neg A=P$ and $\neg B=Q$, then $\mu_A(x)\to\mu_B(y)=\mu_Q(y)\to\mu_P(x)$. In this case, the rules if "y is not *B*" then "x is not *A*", and if "x is *A*" then "y is *B*" are equivalent.

Proof. It can be noted that Lukasiewicz's implication: $u\to v=1$ if $u\le v$ else $1-u+v$ satisfies the previous properties. Let us suppose that "y is *Q*" is an intended meaning of "y is not *B*" such that the associated fuzzy set satisfies $Q\subset\neg B$. So, we have: $\mu_Q(y)\to\mu_{\neg B}(y)=1$. Then, transitivity gives us: $\mu_A(x)\to\mu_B(y)=\mu_{\neg B}(y)\to\mu_{\neg A}(x)\le\mu_Q(y)\to\mu_{\neg A}(x)$. Let P be a fuzzy set such that: $\neg A\subset P$. Then, $\mu_{\neg A}(x)\to\mu_P(x)=1$. By transitivity, we obtain the following result: $\mu_A(x)\to\mu_B(y)\le\mu_Q(y)\to\mu_P(x)$. We search to define P (or the corresponding property *P*) in terms of linguistic negation of *B*, knowing that generally $\neg A$ is not an extended linguistic negation of A. But, such a solution, satisfying $\neg A\subset P$, can exist among the extended linguistic negations of A. In this case, P has the following form: $P=\vee_{l\in L}P^l$ with $\forall l\in L$, $P^l\in Neg^{t}_{\rho,\varepsilon}(A)$. Conversely, Let us suppose that "x is *P*" is an intended meaning of "x is not *A*" such that $P\subset\neg A$. A similar analysis leads to $\mu_A(x)\to\mu_B(y)\ge\mu_Q(y)\to\mu_P(x)$ with Q, an extended linguistic negation of B satisfying $\neg B\subset Q$ having the following form: $Q=\vee_{l\in L}Q^l$ with $\forall l\in L$, $Q^l\in Neg^{r}_{\rho,\varepsilon}(B)$. Finally, if $\neg A$ and $\neg B$ can be viewed as extended linguistic negations of A and B, we obtain the classical result.

Example. Let us suppose that: P=rather small∨very small∨ extremely small $\in C\_Neg^{3}_{\rho,\varepsilon}$(medium∨tall), rather small∨very small∨extremely small⊂¬(medium ∨ tall), Q=invisible=¬visible. The rule "If the man *is not visible* in the crowd, he *is not medium or tall*" receives as translation "If the man *is invisible* in the crowd, *he is rather or very or extremely small*". The property 4.2(ii) *implies* the rule "If the man *is medium or tall*, he *is visible* in the crowd".

5 CONCLUSION

We have presented an extension of a concept of linguistic negation, which can now be viewed as a generalization of the logical one, and this in satisfactory accordance with linguistic analysis. This new model improves the abilities of the previous ones, in that, the extended linguistic negation possesses a more powerful set of properties. In particular, the user can now deny complex combinations of nuanced information.

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