

Graph Method For Criteria Evaluation Of The Fuzzy Sub-Relation

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ABSTRACT: This paper proposes a new way to obtain the criteria dependence relations for a decision problem. The concept of criteria dependence is well known in the decision theory and used for solving multi-criteria decision problems. These problems can be put in fuzzy graphs form. The maximum value flows in fuzzy transportation networks gives the dependence affinity for power sets fuzzy relation $\tilde{\mathfrak{R}}_{\mathcal{P}(C) \times \mathcal{P}(V)}$. The advantage of this method, comparative with other methods (Moore family method, Galois lattices method so.), is given by their facility to take in consideration the criterion coefficients, experts' competence and other specific parameters. The method will be demonstrated with a significant case study.

KEYWORDS: multi-criteria decision problem, dependence affinity, fuzzy relation, fuzzy graph, ∞ -cuts, experts' competence, criterion coefficients

INTRODUCTION

Let be a set of variants $V = \{V_1, V_2, \dots, V_m\}$ and a set of criteria $C = \{C_1, C_2, \dots, C_n\}$. For each criterion C_j , $j = 1, 2, \dots, n$, it is associate to variants V_i , $i = 1, 2, \dots, m$, a vector representing the result of variants evaluation in conjunction with the criterion C_j . These vectors forms the rows of a matrix $A = (a_{ij})$, called consequences matrix.

The problem characterised by a matrix $A = (a_{ij})$, $i=1, 2, \dots, m$, $j=1, 2, \dots, n$, when the value a_{ij} represents evaluation of V_i variant in C_j criterion, is called *cardinal multi-criteria decision problem*. If hierarchies of variants set are direct given for each criterion separately, it is said that we have an *ordinal multi-criteria decision problem*.

The decisional criterions can be of maxim or minim, quantitative or qualitative and, also, they can differ as importance. The criterion importance is evaluated with *criteria coefficients vector* \mathbf{P} , $\mathbf{P} = (p_1, p_2, \dots, p_n)$. In case that we don't have $a_{ij} \in [0, 1]$, using specific *scaling techniques* (vectorial, linear, fuzzy and so.) it is obtained from A matrix the normalised matrix $R = (r_{ij})$, $r_{ij} \in [0, 1]$, $i=1,2,\dots,m$, $j=1,2,\dots,n$, where criteria appreciation are *standardised*, but with the price of the input dates *distortion*.

A decision algorithm applied for a practical decisional situation gives an "optimal" action variant, V_i^* , generally giving a *final classification*, V^* , of the action variants, **final-place**: $V \rightarrow N$.

Using the Fuzzy Transportation Network (FTN) in fuzzy information processing and decisions making is discussed in Negoita (1987), Chanas (1990), Teodorescu (1991), Kaufmann (1992), Aluja (1995) so.

For a multi-attribute decisional problem follow fuzzy sets are used (see Bizon(1996)):

$$\diamond \tilde{A}_{is} = \overbrace{\text{Appreciation}} (V_i, D_s) = \left\{ \frac{a_{1is}}{C_1}, \dots, \frac{a_{nis}}{C_n} \right\}$$

where $a_{jis} \in [0,1]$ is the appreciation of V_i variant, given by D_s expert in conjunction with C_j criterion. For a_{jis} can be used the values: 1 – great; 0,9 - very good; 0,7 – good; 0,5 – mediocre; 0,3 – bad; 0,1 - very bad; 0 - unacceptable.

$$\diamond \tilde{A}_i = \overbrace{\text{Final - Appreciation}} (V_i) = \left\{ \frac{a_{1i}}{C_1}, \dots, \frac{a_{ni}}{C_n} \right\}$$

where $a_{ji} \in [0,1]$ represents the appreciation of the V_i variant in conjunction with C_j criterion, given by experts.

The criteria can have different importance level, and experts' experience can also be different. For that reason follow fuzzy sets are defined:

$$\diamond \tilde{P} = \left\{ \frac{p_1}{C_1}, \dots, \frac{p_n}{C_n} \right\} \quad - \text{criteria importance};$$

$$\diamond \tilde{I} = \left\{ \frac{i_1}{D_1}, \dots, \frac{i_n}{D_n} \right\} \quad - \text{criteria competence}.$$

If q_1 and q_2 are the input and output special node from FTN, the others nodes of the FTN belong to L, V, C, D sets, depending on the type and structure of existing dates for decisional problem. In the same manner the functions \acute{o} , \grave{i} and k_{xy} are defined (see Chanas (1992)):

$$\acute{o} : X \rightarrow [0,1] \quad - \text{membership level of each node to FTN,}$$

$$\grave{i} : X \times X \rightarrow [0,1] \quad - \text{membership level of each arc to FTN,}$$

$k_{xy} : [0, \acute{i}(x,y)] \rightarrow H$ - the capacity associated to each arc (x,y) , $(x,y) \in \tilde{A} = \{(x,y) \mid \acute{i}(x,y) > 0\}$, where X is the nodes sets, \tilde{A} is the support sets of the arcs and H is a particularised set in function of the concrete problem.

The maximum value of the flux in FTN represents a fuzzy subset of the admissible fluxes, that explain, in fact, the decisional sub-relations from the sets $V \times L$, $C \times V$ so. The fuzzy sets and fuzzy relations are studied by the valuable researchers: Zadeh (1987) (similarity relation, fuzzy partial order relation so.), Kaufman (1985) (perfect fuzzy order relation, relational matrixes so.), Zimmermann (1991, 1992) so.

PROBLEM STATEMENT

In case of decisional situation evaluation by an expert, follow sets are obtained: $V, C, \tilde{A}_i, \tilde{P}$. Fuzzy graph will be considerably simplified (see figure 1) if we assume that for each criterion there is at least one optimal variant for that the experts' appreciation is maximum (that means $\forall C_j, \exists V_i$ so we have $1 = a_{ji}^* = \max_i a_{ji}$).

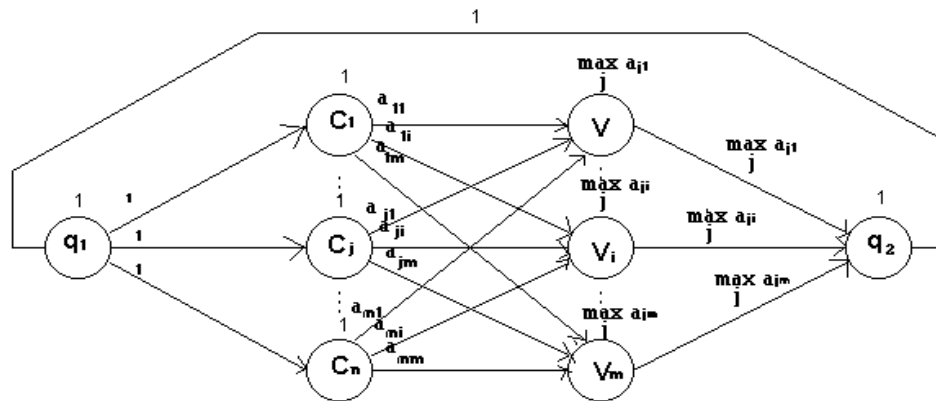


Figure 1: Fuzzy graph for a decisional problem

CASE STUDY

An example of decisional situation, described in Kaufmann (1992), will illustrate the way to obtain the dependence sub-relations defined in $C \times V$ set, in comparison with the method using Galois' lattices.

Let be:

$$A = \begin{pmatrix} 0,8 & 0,5 & 0,0 & 0,7 & 0,8 & 0,5 \\ 0,2 & 0,6 & 0,8 & 1,0 & 0,6 & 1,0 \\ 0,5 & 0,5 & 0,9 & 1,0 & 0,5 & 0,9 \\ 0,0 & 0,4 & 0,8 & 0,7 & 0,6 & 1,0 \end{pmatrix}, P = (1 \ 1 \ 1 \ 1)$$

Matrix **A** allows defining the fuzzy relationship on $C \times V$, called criteria evaluation set:

$$\tilde{\mathfrak{R}}_{C \times V} = \left\{ \begin{array}{cccccccccccc} 1 & 1 & 1 & 1 & 0,9 & 0,9 & 0,8 & 0,8 & 0,8 & 0,8 & 0,7 & 0,7 \\ (C_2; V_4), (C_2; V_6), (C_3; V_4), (C_4; V_6), (C_3; V_3), (C_3; V_6), (C_1; V_1), (C_1; V_5), (C_2; V_3), (C_4; V_3), (C_1; V_4), (C_4; V_4), \\ 0,6 & 0,6 & 0,6 & 0,5 & 0,5 & 0,5 & 0,5 & 0,5 & 0,4 & 0,2 & 0 & 0 \\ (C_2; V_{24}), (C_2; V_{56}), (C_4; V_5), (C_1; V_2), (C_1; V_6), (C_3; V_1), (C_3; V_2), (C_3; V_5), (C_4; V_2), (C_2; V_1), (C_1; V_3), (C_4; V_1) \end{array} \right\}$$

Also results:

$$\tilde{A}_1 = \left\{ \frac{0,8}{C_1}, \frac{0,2}{C_2}, \frac{0,5}{C_3} \right\}, \tilde{A}_2 = \left\{ \frac{0,5}{C_1}, \frac{0,2}{C_2}, \frac{0,5}{C_3}, \frac{0,4}{C_4} \right\}, \tilde{A}_3 = \left\{ \frac{0,8}{C_2}, \frac{0,9}{C_3}, \frac{0,8}{C_4} \right\}, \tilde{A}_4 = \left\{ \frac{0,7}{C_1}, \frac{1}{C_2}, \frac{1}{C_3}, \frac{0,7}{C_4} \right\}$$

$$\tilde{A}_5 = \left\{ \frac{0,8}{C_1}, \frac{0,6}{C_2}, \frac{0,5}{C_3}, \frac{0,6}{C_4} \right\}, \tilde{A}_6 = \left\{ \frac{0,5}{C_1}, \frac{1}{C_2}, \frac{0,9}{C_3}, \frac{1}{C_4} \right\}$$

The fuzzy graph is shown in figure 2.

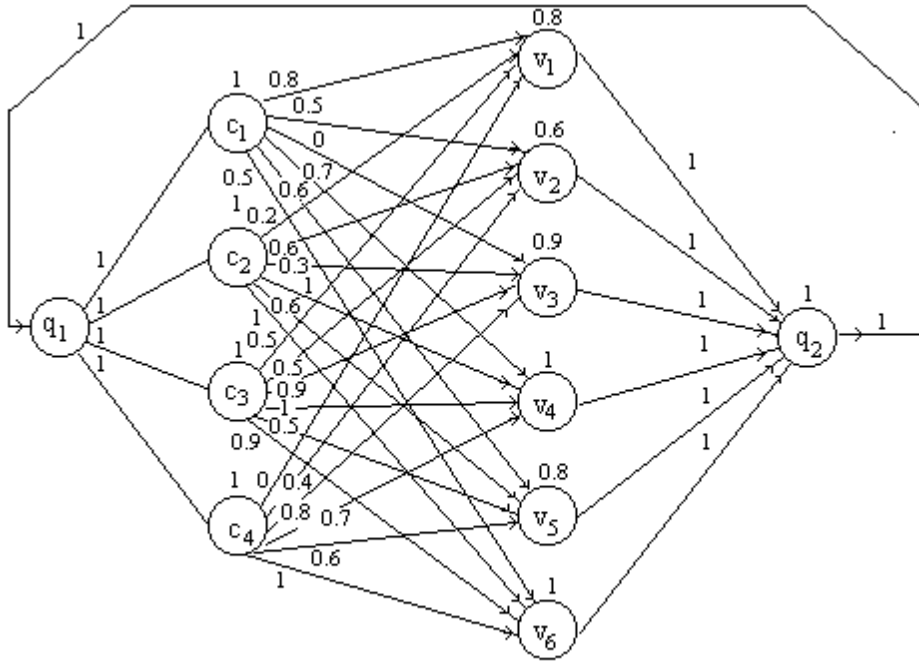


Figure 2: Fuzzy graph for the case study

Applying α -cuts to determine maximum flux, we obtain follow sub-relations:

$\alpha = 1$			$\alpha = 0,9$			$\alpha = 0,8$			
C_2	C_3	C_4	C_2	C_3	C_4	C_1	C_2	C_3	C_4
$V_4; V_6$	V_4	V_6	$V_4; V_6$	$V_3; V_4; V_6$	V_6	$V_1; V_5$	$V_3; V_4; V_6$	$V_3; V_4; V_6$	$V_3; V_6$
$\alpha = 0,7$					$\alpha = 0,6$				
C_1	C_2	C_3	C_4	C_1	C_2	C_3	C_4		
$V_1; V_4; V_5$	$V_3; V_4; V_6$	$V_3; V_4; V_6$	$V_3; V_4; V_6$	$V_1; V_4; V_5$	$V_2; V_3; V_4; V_5; V_6$	$V_3; V_4; V_6$	$V_3; V_4; V_5; V_6$		
$\alpha = 0,5$				$\alpha = 0,4$					
C_1	C_2	C_3	C_4	C_1	C_2	C_3	C_4		
$V_1; V_2;$ $V_4; V_5$	$V_2; V_3; V_4;$ $V_5; V_6$	$V_1; V_2; V_3;$ $V_4; V_5; V_6$	$V_3; V_4;$ $V_5; V_6$	$V_1; V_2;$ $V_4; V_5; V_6$	$V_2; V_3;$ $V_4; V_5; V_6$	$V_1; V_2; V_3;$ $V_4; V_5; V_6$	$V_2; V_3;$ $V_4; V_5; V_6$		

Table I: Relation for different α -cuts

These relations allow extracting of the correlation information between variant groups from the set power of V, P(V), and criterion groups from P(C). Maximal sub-relationship in $\tilde{\mathfrak{R}}_{P(C) \times P(V)}$ is:

$$\left\{ \begin{array}{cccccccc} 0,9 & 0,8 & 0,8 & 0,8 & 0,8 & 0,8 & 0,8 & 0,8 \\ (V_3 \times V_4 \times V_6; C_3) & (V_4 \times V_4; C_1) & (V_5 \times V_4 \times V_6; C_2) & (V_5 \times V_6; C_4) & (V_5 \times V_4 \times V_6; C_2 \times C_3) & (V_5 \times V_6; C_2 \times C_4) & (V_5 \times V_6; C_2 \times C_3 \times C_4) & \end{array} \right\}$$

It can be observed that this represents, in fact, Galois lattice obtained in Kaufmann (1992) (see figure 3). The extraction method of the decisional affinity with fuzzy graphs allows, in addition, obtaining the evaluation information of these in case of some differing criterion coefficients, too.

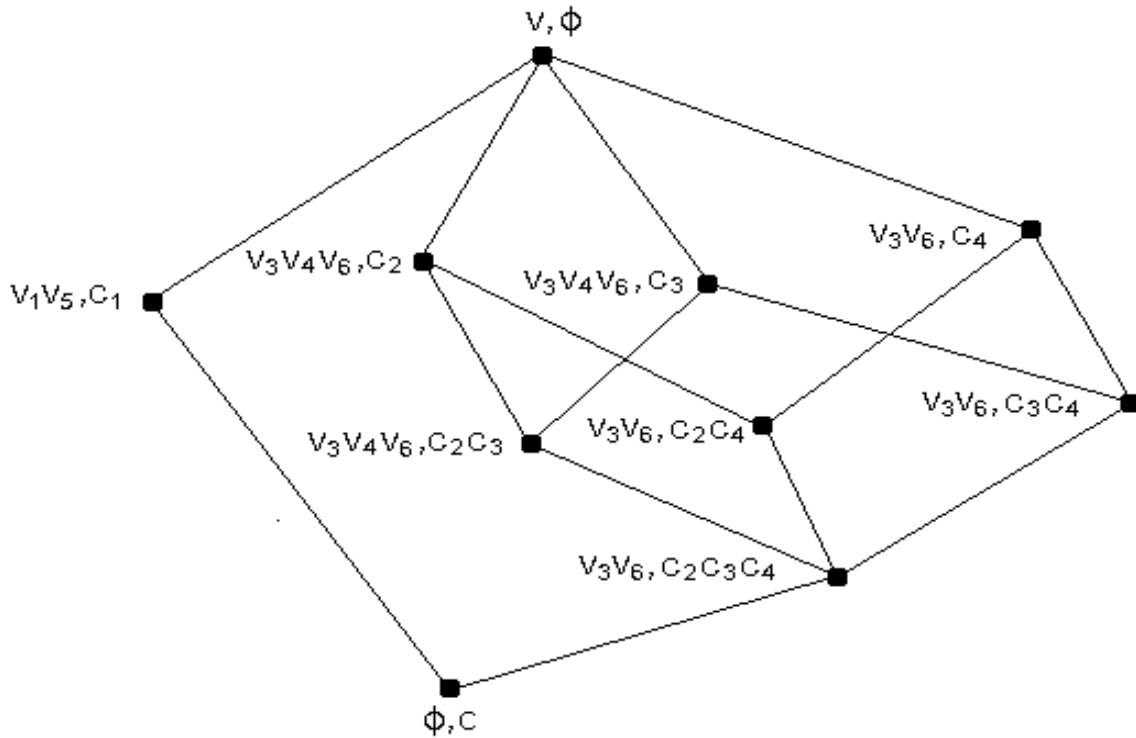


Figure 3: Galois lattice for the case study

GROUP MULTI-CRITERIA DECISIONS

The graph method that can resolve group decisional problem , too.

In condition in which at the decisional process take share more than one experts, the input dates of the problem are: V, C, D, A_{is} and P. Fuzzy graph associated to this decisional problem is presented in figure 4, when:

$$\gamma_{ji} = a_{ji} + 1 - \max_j \max_i a_{ji}, \text{ were } a_{ji} = \sum_{s=1}^h \frac{1}{h} a_{jis} \text{ and } h = \text{cardinal}(D)$$

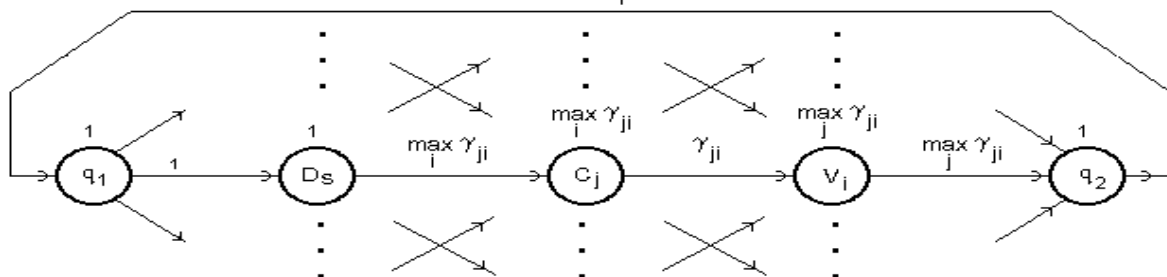


Figure 4: Fuzzy graph for the group decisional problem

Finding fuzzy maximum flux set allows us to establish preferential decisional sub-relation, defined in $C \times V$ set or $D \times C \times V$ set.

CONCLUSIONS

For complex multi-criteria decision problems the proposed method has some advantages in comparison with the other decisional methods:

- The capacity function associated to each arc in fuzzy graph, function of decisional problem specific, can be a real number, a fuzzy number, language labels so;
- Specific parameters of the multi-criteria decisional problem (criterion coefficients, experts' competence so.) are easily integrated in structure of the fuzzy graph;
- Graph structure can be adapted to any kind of decisional problem (see Bizon (1998));
- The decisional affinity can be easily selected;
- The obtained results are the same with other obtained results by decisional methods.

This paper shows a way to apply the graph theory in the decisional problem's domain.

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