PROBLEMS OF THE DECISIONS STABILITY IN FUZZY PRODUCTIONS SYSTEMS

SERGEY V. ASTANIN, TATYANA G. KALASHNIKOVA
TAGANROG STATE UNIVERSITY OF RADIOENGINEERING, 44, NEKRASOVSKIY STR., 347928, TAGANROG, RUSSIA
FAX: +7-86344-61787, E-MAIL: SAIT@TSURE.RU

Abstract: The situational approach to processing of fuzzy productions in expert system is considered. The algorithms based on using of possibility functions and similarity operations are proposed for exception of ambiguity at decision making by using compositional conclusion. The cluster analysis is applied to learning of expert system with incomplete knowledge bases.

Keywords: compositional conclusion, situational conclusion, lexicographic estimations of plausibility, reasoning by analogy, cluster analysis, similarity operations, possibility functions, plausible reasoning.

INTRODUCTION

The theory of fuzzy sets offered by Zadeh allows to describe natural language concepts. These concepts are characterized by linguistic uncertainty connected to ambiguity of their interpretation by the different people in various situations. The following modelling schemes of linguistic uncertainty are applied usually:

1. $\tilde{A} \rightarrow H$, where $\tilde{A}$ is a fuzzy premise, Í is a precise conclusion. For example, "**If** the pressure is high **then** open the valve on 40°";
2. $\tilde{A} \rightarrow \tilde{H}$, where $\tilde{A}$ is a fuzzy premise and $\tilde{H}$ is a fuzzy conclusion. For example, "**If** the pressure is high **then** open the valve a little ";
3. $\tilde{A} \rightarrow \tilde{H}, \alpha(\tilde{H})$, where $\tilde{A}$ is a fuzzy premise, $\tilde{H}$ is a fuzzy conclusion, $\alpha(\tilde{H})$ is a conclusion plausibility;
4. $A \xrightarrow{\alpha} H$, where α is a rule uncertainty estimation which defines a conclusion plausibility degree under the given set of signs.

Such reasoning mechanisms as a classical compositional conclusion by Zadeh [1], compositional conclusion by Watanabe [2] and a situational conclusion [3] can be applied to realize the schemes 1 and 2. Lexicographic plausibility estimations offered by Batyrshin [4], reasoning by analogy [5], and cluster analysis [1] are used in the scheme 3. The scheme 4 is applied in fuzzy and multiple-valued logic, and during construction of fuzzy grammars. The first three schemes are used often in productions rules knowledge bases. However, in this case the ambiguous conclusions not allow to realize the reasoning correctly.

In the present work the disadvantages of the most traditional reasoning mechanisms in production knowledge bases are analyzed, and the approaches to the solution of the allocated contradictions are offered.

1. ANALYSIS OF MECHANISMS OF PLAUSIBLE REASONING

The basic peculiarities of the linguistic uncertainty modelling schemes are considered on the base of various plausible reasoning mechanisms.

A situational conclusion. In production systems the reasoning procedure based on the analysis of fuzzy rules (productions) such as "**If** $A_{i,1}$ and $A_{i,2}$ and ... and $A_{i,m}$ **then** H_i". The reasoning are realized as following: since the similarity between an entrance premise A' and a some premise of knowledge base A_i is not bigger than value $\alpha_{A'/Ai}$ then the conclusion plausibility H_i can not be bigger than value $\alpha_{A'/Ai}$ also. This assumption is similar to the decision making in reasoning by analogy and cluster analysis. Obviously, the correctness and stability of the decisions is defined by selected similarity operation between fuzzy or linguistic premises in this case. For example, in [3] the operation of fuzzy equality $\mu(s_i, s_k)$ is offered as similarity operation of fuzzy situations. This operation is defined as follows:

$$\mu(s_i, s_k) = \upsilon(s_i, s_k) \& \upsilon(s_k, s_i), \quad (1)$$

where $s_i=\{<\mu_{Si}(P_{i,j})/ P_{i,j}>\}$, $s_k=\{<\mu_{Sk}(P_{i,j})/ P_{i,j}>\}$, $P_{i,j}\in P$ are fuzzy situations; $\upsilon(s_i, s_k)$ and $\upsilon(s_k, s_i)$ define the fuzzy inclusion degrees of a fuzzy situation s_i in a fuzzy situation s_k and a fuzzy situation s_k in a fuzzy situation s_i. The fuzzy inclusion degree is defined as follows: $\upsilon(s_i, s_k) = \underset{P_{i,j}\in P}{\&} (\mu_{s_i}(P_{i,j}) \rightarrow \mu_{s_k}(P_{i,j}))$. The left part of production in knowledge base (s_i) and current set of signs (s_k) are compared by (1) during the analysis of fuzzy productions base.

The correctness condition is entered for estimation of the used similarity operation: the similarity operation of a linguistic signs set in knowledge base (s_i) and a current linguistic signs set is correct if for any current sets s_k and s_l which are similar by $\mu(s_k, s_l)$ the conclusion plausibilities $\mu(H_i)/s_k$ and $\mu(H_i)/s_l$ are differ by value equal to a similarity index $\mu(s_k, s_l)$. Consequently, it is impossible to produce the same plausible conclusion $\mu(H_i)/s_k=\mu(H_i)/s_l >0.5$ for any current sets s_k and s_l are dissimilar by $\mu(s_k, s_l)$.

The following example by using a correctness condition of similarity operation is considered as illustration of instability and contradictions of the decisions received by formula (1). Let's consider the fuzzy production i in knowledge base: $s_i=X_1\&X_2\&X_3\rightarrow H$. Let s_k and s_l are two current linguistic signs sets, where $s_k=<1/ X_1, 1/ X_2, 0.2/ X_3>$, $s_l=<1/ X_1, 0.2/ X_2, 0.2/ X_3>$. Thus we have: $\mu(s_i, s_k) = 0.2$, $\mu(s_i, s_l) =0.2$. In other words, both current sets are undistinguished concerning the conclusion H. Let's define similarity of the current sets s_k and s_l: $\mu(s_k, s_l) =0.2$. Therefore, the considered signs sets are dissimilar and should be characterized by the different conclusions. In this case the correctnesses condition of the similarity operation is not satisfied because the different premises are undistinguished in accordance with restrictions on the validities of coincident linguistic terms. This fact is demonstrated as follows example. Let $s_i =X_1\& X_2\& X_3\rightarrow H$, $s_k=<1/ X_1, 1/ X_2, 0.7/ X_3>$, $s_l=<1/ X_1, 0.7/ X_2, 0.7/ X_3>$, and $s_w=<0.7/ X_1, 0.7/ X_2, 0.7/ X_3>$. Applying (1) we have: $\mu(s_i, s_k)=0.7$, $\mu(s_i, s_l)=0.7$, $\mu(s_i, s_w)=0.7$. Thus, any considered current premise determines one and the same conclusion with one and the same restriction on its truth. Such procedure is not natural for men's reasoning. Obviously, the premise s_k is more preferred at the conclusion H and for s_k the conclusion plausibility H must be more than the conclusion plausibility H for s_l and s_w.

Therefore, the application of fuzzy equality operation $\mu(s_i, s_k)$ for definition of fuzzy situations similarity can give incorrect result. It occurs because of only one (minimal or maximal) operand influences on operation result by using of fuzzy logic operations (conjunction or disjunction). Thus, one and the same conclusion can be received for various situations during the fuzzy reasoning.

A classical compositional conclusion by Zadeh. In [1] the linguistic approach is used for description of membership degree dependence of object p to some fuzzy set F from linguistic values of an object signs in the objects space U ($\mu(p)$). It is supposed that the mathematical object $M(p)=\{ M_1(p), M_2(p),..., M_n(p)\}$ has n components $x_1= M_1(p),..., x_n= M_n(p)$, where x_i, i=1,..., n accept the meanings from U_i. Dependence $\mu_F(p)$ from $x_1,..., x_n$ can be defined as the (n+1)-dimensional fuzzy relation $\tilde{R}$ in $U_1\times\ldots\times U_n\times V$, where V=[0,1]. $\tilde{R}$ is presented as follows:

$$\tilde{R} = r_{11}\times\ldots\times r_{1n}\times r_1+...+ r_{m1}\times\ldots\times r_{mn}\times r_m,$$

where $r_{11}\times\ldots\times r_{1n}+...+ r_{m1}\times\ldots\times r_{mn}$ is the n-dimensional fuzzy relation in $U_1\times\ldots\times U_n$ submitting to restrictions $R(Q_1, ..., Q_n)$ on values of variables $Q_1, ..., Q_n$. The $q_1, ..., q_n$ are values $Q_1, ..., Q_n$, where $q_1, ..., q_n$ are given fuzzy subsets of sets $U_1,\ldots,U_n$ respectively. It is required to calculate value Q for an entrance premise given as $G= g_1\times\ldots\times g_n$. The result of substitution $R(g_1,\ldots,g_n)$ (and consequently required value Q) is defined as the following rule of representation: $R(g_1,\ldots,g_n)= \tilde{R} \circ G$, where "$\circ$" is a composition of the (n+1)-dimensional fuzzy relation $\tilde{R}$ with the n-dimensional fuzzy relation G.

The disadvantages of this approach are considered. When the fuzzy relation $\tilde{R}$ is filled with the misses, i.e. some $Q_1,..., Q_n$ are omitted in the table, the interpolation R can give defective approximation to the answer Q. Besides the result of substitution $g= r_{i1}\times\ldots\times r_{in}$ in $\tilde{R}$ will not be equal r_i exactly in consequence of mutual influence of lines from R. It is caused by the fuzzy sets, which forming a column in R are not shared generally, i.e. their crossing is not empty. Also at number productions increasing the size of the relations table is increased and computing complexity is increased too.

Using of *compositional conclusion by Watanabe* is considered as example. Let there is a production rule of a type "**If** premise **then** conclusion": "**If** the level of water high **then** open the valve". Premise and conclusion are described as fuzzy sets. For example, the expert knowledge concerning the high water level: "The high water level is approximately 2ì" are interpreted by the fuzzy set: HIGH={0.1/1.5, 0.3/1.6, 0.7/1.7, 0.8/1.8, 0.9/1.9, 1.0/2, 1.0/2.1, 1.0/2.2}. Similarly if "The Corner of turn = 90° is a complete opening of the valve" then the corner of the valve turn can be described by the fuzzy set: TO OPEN={0.1/30°, 0.2/40°, 0.3/50°, 0.5/60°, 0.8/70°, 1.0/80°, 1.0/90°}. Let "The water level is rather high" is the current water level then RATHER HIGH={0.5/1.6, 1.0/1.7, 0.8/1.8, 0.2/1.9}.

Fuzzy production rule and current observing are represented in figure 1a. Process of classical fuzzy conclusion is represented in figure 1b. Considered fuzzy conclusion can be described mathematically as follows. The fuzzy causal relation between the premise and conclusion is designated through R: R=A→B, where R is a fuzzy set in direct product X×Y. The fuzzy conclusion reception B' with using of the observing A' and the rule A→B can be presented as the formula $\hat{A}'=\grave{A}'\circ R=\grave{A}'\circ(A\to B)$, where ”∘” is a compositional rule of fuzzy conclusion, ”→” is a fuzzy implication. The fuzzy conclusion in figure 1 is the result of a maxmin composition of operation min as a fuzzy implication:

$$\mu_{\hat{A}'}=\bigvee_{x\in X}(\mu_{\grave{A}'}(x)\wedge\mu_R(x,y))=\bigvee_{x\in X}(\mu_{\grave{A}'}(x)\wedge(\mu_{\grave{A}}(x)\wedge\mu_B(y)))=(\bigvee_{x\in X}(\mu_{\grave{A}'}(x)\wedge\mu_{\grave{A}}(x)))\wedge\mu_B(y)=$$

$$=\bigvee_{x\in X}\mu_{\grave{A}'\cap\grave{A}}(x)\wedge\mu_B(y)=\alpha\wedge\mu_B(y)=\mu_{\alpha Y\cap B}(y).$$

So, for current data of observing A' as a result of a production rule application À→Â we have:

If HIGH **then** TO OPEN
RATHER HIGH .
SLIGHTLY TO OPEN

Here the conclusion B' is a fuzzy set in Y (see figure 1b) that does not allow to do of any concrete operations. It is necessary to calculate values for each point in Y based on the membership function $\mu_{\hat{A}'}(y)$. This process is called defuzzyfication. In this case the method of a gravity centre (GC) is used for defuzzyfication:

$$GC=\int_Y y\cdot\mu_{B'}(y)dy\,/\int_Y\mu_{B'}(y)dy\,.$$

As result of calculations GC=70°. Consequently, we have “to turn the valve by 70°”.

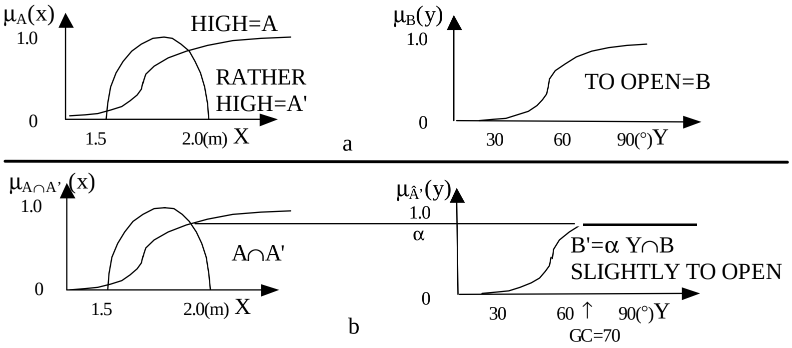

Figure 1: Example of the fuzzy conclusion by production rules.

Disadvantage of this method is ambiguity by reception of the conclusion B' with data of observing A' in case when one α-level and consequently one conclusion corresponds to different entrance values x_1 and x_2 (figure 2a).

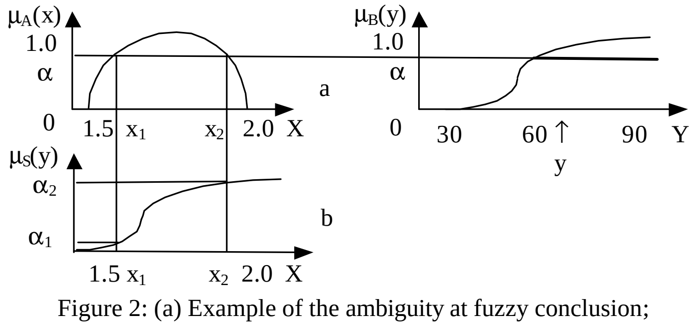

Figure 2: (a) Example of the ambiguity at fuzzy conclusion;
(b) function of possibility for premise A.

Lexicographic estimations of plausibility with universal borders. Offered in [4] approach to processing uncertainties based on concept of a plausibility lexicographic estimation. In algebra of a plausibility lexicographic estimation a hypotheses comparison is realized by comparison of premises plausibilities chains, which bring to these hypotheses. Premises plausibilities in these chains are ordered definitely. A comparison of these chains is realized lexicographically. The advantage of this approach is processing of plausibility qualitative estimations measured in ordinal scales, which most adequately reflect man’s judgements. The results of operations are the lists of operands with ordered by appropriate way. The algebra of plausibility ∧-estimations is used in the case when only one rule or one chain of premises brings to one conclusion. The algebra of plausibility ∨-estimations is used in a case when the same conclusion can be deduced from several rules, and each rule contains of one premise. However, these classes of estimations use only part of information about the premises and rules plausibility values, which bring to conclusions. The more complete information contains in ($\vee$, $\wedge$) – estimations. Its using requires more memory to storage premises and rules plausibility values in expert system. Disadvantage of this method is that at the knowledge base creation if productions rules number increasing then the plausibility estimations number which are defined by expert for each fact of premise and for each conclusion of rule is increased greatly. The lists of all plausibility estimations used by conclusion are stored in a computer memory. It slows the speed of calculations.

The analysis of plausible reasoning mechanisms allows making the following conclusions. The modern approaches of uncertainty representation and processing in models of plausible reasoning, in decision-making systems and in expert systems have its peculiarities and application restrictions. The advantage of the described mechanisms is that fuzzy logic allows describing judgements and ways of decision making by man rather adequately. The disadvantages of the existing approaches are instability of the decisions. It is caused by large changes of uncertainty estimations in the system exit by small changes of uncertainty estimations in the system entrance by some approaches. Also it is small sensitivity of results by significant change of uncertainty estimations in the entrance by other approaches. Thus, it is necessary to define more correct operations for comparison of current and standard situations in reasoning by analogy. Also it is necessary to use the approach allowing to avoid uncertainty by decision making with help of a compositional conclusion by Watanabe in a case when the one conclusion corresponds to various entrance data.

2. MODELS OF PLAUSIBLE REASONING ON THE BASE OF CORRECT SIMILARITY OPERATIONS

The procedures of plausible reasoning based on the using of various similarity operations between entrance premise A' and premises from knowledge base A_i. Result of comparison is a similarity index $\alpha_{A'/Ai}$. This index is used for truth restriction of a conclusion H_i frequently.

As mentioned above that result of situations comparison depends on an operand with the minimal value by using fuzzy equality operation as similarity operation of fuzzy situations [3], and both current sets are indistinguishable concerning of the conclusion. However, it is necessary to accept the various decisions in these situations in the man viewpoint. Let's consider the approaches to define similarity between fuzzy premises based on the more correct operations used in reasoning by analogy and in cluster analysis.

The majority of plausible reasoning samples could be presented as follow [5]:

$$\begin{array}{c}(s, q)\\ s^1\\ \hline q^1\end{array} \quad , \tag{2}$$

where (s, q) represents the premise expressed by some connection (be or not to be causal relation) of two propositions s and q. At the same time the second premise s^1 is "parallel" s somewhat. Then proposition q^1 is deduced as "parallel" for q. Formula (2) contains the reasoning by analogy (q^1 for q same that s^1 for s).

Adequate operations allowing to realize the comparison s and s^1 are the geometrical operations of similarity (affinity) based on the using of some metrics. In the simple case such metrics is a standard deviation between fuzzy premises s and s^1 [6]:

$$\alpha(s, s^1) = 1 - \frac{1}{m}\sum_{j=1}^{m}\left(\left|\mu_{U_j}(s) - \mu_{U_j}(s^1)\right|\right) , \tag{3}$$

where $\mu_{U_j}(s)$ and $\mu_{U_j}(s^1)$ are the restrictions of the terms truth (values linguistic variable) included in the premises s and s^1, m is a number of the terms by which s and s^1 are compared. The formula (3) shows average relative size that premises s and s^1 are differed by. Also other metrics can be used as a measure of similarity.

Using of the formula (3) as similarity measures opens new opportunities by the description of fuzzy productions and by the organization of a plausible conclusion. These opportunities are determined by high sensitivity of measures to changes of the current premises elements truth. Application of an abductive conclusion in fuzzy productions knowledge bases is considered as variant of a plausible conclusion by formula (3).

Let knowledge base consists of rules set. Each rule determines connection between set of linguistic signs and diagnosis H_i. The estimation of rule uncertainty is known. The uncertainty estimations can have various interpretations [4]. They accept values in some set of plausibility estimations W determined in a scale [0,1]. The value $w(S_i) \in W$ is the estimation of the rule uncertainty S_i, and it determines a plausibility degree of the conclusion Í$_i$ by the given signs set. Thus the fuzzy productions knowledge base S can be defined as follows

$$
\begin{aligned}
&S_1\text{: if } A_{1,1} \text{ and } \grave{A}_{1,2} \text{ and ... and } A_{1,m} \text{ then } H_1, w(S_1)\\
&S_2\text{: if } A_{2,1} \text{ and } \grave{A}_{2,2} \text{ and ... and } A_{2,m} \text{ then } H_2, w(S_2)\\
&\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\\
&S_n\text{: if } A_{n,1} \text{ and } \grave{A}_{n,2} \text{ and ... and } A_{n,m} \text{ then } H_n, w(S_n).
\end{aligned}
\qquad (4)
$$

In this case everyone of Í$_i$ can be defined as function Í$_i$: $W \rightarrow L$, where L is the linguistic values set determining an opportunity of the conclusion Í$_i$. For example L={"impossible", "possible", "probably", "absolute"} can define a doctor reliance degree that a patient have some disease. Obviously, Í$_i$ is a fuzzy set determined in a scale [0,1] with values from an ordinal scale L.

A degree of distinction between objects is defined as follows:

$$\beta(s, s^1) = \frac{1}{m}\sum_{j=1}^{m}\left(\left|\mu_{U_j}(s) - \mu_{U_j}(s^1)\right|\right) \qquad (5)$$

Therefore, the formula (5) defines the uncertainty estimation displacement of the conclusion Í$_i$ by the facts set s^1 concerning of the known uncertainty estimation of the conclusion Í$_i$ by the facts set s. If operation of getting the absolute value is excluded from this formula then the sign of expression (5) will define displacement on a scale W concerning of the known uncertainty estimation. In other words, the displacement shows as far as is increased (or is reduced) the probability of the conclusion Í$_i$ by the current object signs set s^1.

The following expression is used for definition of linguistic probability value of the conclusion Í$_i$ by the current facts set s^1 based on the size of displacement $\beta(s, s^1)$:

$$w(s^1) = \begin{cases} w(s) + \beta(s, s^1), \text{åñëè } \sum_{i=1}^{m}(\mu_{U_i}(s) - \mu_{U_i}(s^1)) < 0 \\ w(s) - \beta(s, s^1), \text{åñëè } \sum_{i=1}^{m}(\mu_{U_i}(s) - \mu_{U_i}(s^1)) > 0 \end{cases},$$

where w(s) and $w(s^1)$ are the uncertainty estimations of the conclusion Í$_i$ by signs (facts) sets s and s^1 respectively. Thus, new value of the probability of Í$_i$ is l= Í$_i$ ($w(s^1)$), where $l \in L$ is received on the base of the description Í$_i$.

The considered plausible conclusion is abductive conclusion since each production of base defines private law between set of signs s and conclusion Í$_i$. This private law is expressed by the uncertainty estimation. The new private law is deduced on the base of this private law and analysis of analogy between the current signs set s^1 and signs set s.

Let's consider a problem of ambiguity at the conclusion reception by using of a compositional conclusion by Watanabe (figure 2à). In figure for various values of a premise A $x_1<x_2$ their membership functions to the given fuzzy set are equal, i.e. $\mu_{\grave{A}}(x_1)= \mu_{\grave{A}}(x_2)$. Consequently, one α-level and one conclusion correspond them. For exception of this ambiguity we shall enter possibility function for each sign. This function describes sign informationality concerning of the conclusion (figure 2b). The following operations $\min\{1, \mu_1(x)+ \mu_2(x)\}$ or $\mu_1(x)+\mu_2(x)-(\mu_1(x)\cdot\mu_2(x))$ were used for union of possibility and uncertainty functions at a plausible conclusion. Advantage of these operations application is that the both operands influence to the received result. Consequently, the sign informationality is taken into account in greater degree than its uncertainty. The result of this union is the generalized function determining the weights of signs values concerning of the conclusion, and one a-level corresponds for each value. Thus, the various a-levels and hence various values of the conclusion will be received for various entrance values (figure 3).

So, geometrical similarities operations based on the metrics are offered by definition of similarity between fuzzy premises. In this case the standard deviation between fuzzy premises is considering. It shows on what average relative size the premises are differed. The function describing sign informationality is entered along with the function describing sign uncertainty for exception of ambiguity by using of a compositional conclusion. Such approach allows to receive the various conclusions for the various entrance data.

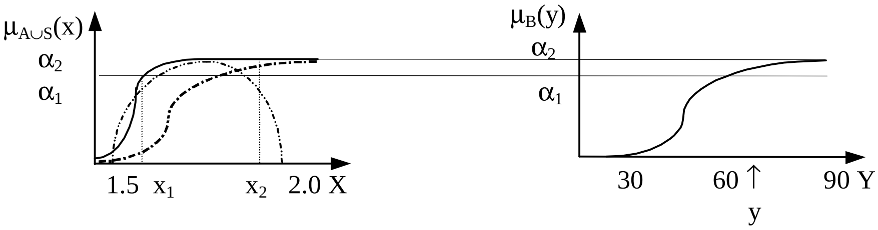

Figure 3: Fuzzy conclusion by using fuzzyness and informationality of sign.

3. INFERRING NEW PRODUCTION RULES BY SYSTEM LEARNING

By creation of expert system knowledge base the most typical situations are taken into account because it is impossible to expect all probable variants. Therefore, system learning is necessary, i.e. update of knowledge base by new rules on the operation phase. The reception conclusion of new rules can be realized by cluster analysis [7]. All space of fuzzy situations, which describe an object state, is divided into classes (fuzzy clusters), where the decision (conclusion) corresponds to each situation. The clusters centres are set originally, i.e. the standard situations are accepted as the clusters centres. The clusters centres are displaced after receipt of new information (the current situations set). New conclusions are defined according to the described above method concerning the new clusters centres. The new clusters centres are new the rules which are inserted in knowledge base and used by further fuzzy conclusion.

The approach essence is a representation of each fuzzy situation s_i by a point in the n-dimensional space of signs U, i.e. each situation s_i is described by an fuzzy subset in U. Let $\{s_1, s_2, \ldots, s_k\}$ is a set of fuzzy subsets U with the corresponding membership functions $\mu_1, \mu_2, \ldots, \mu_k$. Then S_r $(r=\overline{1,k})$ are the fuzzy clusters produced by the relation of affinity P if they have property of fuzzy affinity.

The algorithm of Bezdek [7] is used for formation of fuzzy situations standard classes and is formulated as follows. Let $\mu_1, \mu_2, \ldots, \mu_k$ are the membership functions of $F_1, F_2, \ldots, F_k$, where F_r are the fuzzy subsets (clusters) of points X. $X=\{x_1, x_2, \ldots, x_n\}$ is a signs values set describing object. Fuzzy clusters $F_1, F_2, \ldots, F_k$ form fuzzy k-splitting of X if and only if $\mu_1(x)+\mu_2(x)+\ldots+\mu_k(x)=1$, $x\in X$, where "+" means arithmetic summation. According to [1] qualities of fuzzy k-splitting is defined by characteristic function

$$J(\mu)=\min_{v}\sum_{i=1}^{k}\sum_{x\in X}(\mu_i(x))^2\left\|x-v_i\right\|^2,$$

where $\mu=(\mu_1, \mu_2, \ldots, \mu_k)$, $v=(v_1, v_2, \ldots, v_k)$, $v_i\in L$, $j=\overline{1,k}$ and L is a vector space with norm $\|\ \|$ caused by scalar product.

The calculations in the algorithm of Bezdek are simplified by applying distance of Heming. Let $S_1, S_2, \ldots, S_k$ is a standard fuzzy situations set. Each situation is described by a signs set $Y=\{y_1, y_2, \ldots, y_n\}$. The standard situations are nominated as the fuzzy clusters centres. Let $s_i, \ldots, s_l$ is a current (controllable) situations set. The membership functions s_i $(i=1, \ldots, l)$ to fuzzy clusters s_r are calculated by using the following formula:

$$\mu_r(s_i)=\frac{\dfrac{1}{\sum_{p=1}^{n}\left|s_i(y_p^i)-s_r(y_p^r)\right|}}{\sum_{r=1}^{k}\dfrac{1}{\sum_{p=1}^{n}\left|s_i(y_p^i)-s_r(y_p^r)\right|}}, \qquad (6)$$

where $s_i(y_p^i)$, $s_r(y_p^r)$ are the membership functions of signs y_p to situations s_i and s_r respectively. In this case $s_r(y_p^r)$ is the centre of r-cluster, and $s_i(y_p^i)$ is an element of the cluster. The clusters centres are specified with taking into account the description of the current situation s_i with membership functions to clusters $S_1, S_2, \ldots, S_k$ – $\mu_r(s_i)$ by the formula:

$$s_r^1(y_p)=\frac{\sum_{s\in S}(\mu_r(s_i))^2 s_i(y_p)}{\sum_{s\in S}(\mu_r(s_i))^2}. \qquad (7)$$

As well as in the algorithm of Bezdek the deviation measure δ of value s_r from s_r^1 is calculated. If δ≤ε, where ε is the threshold value, then we consider that the fuzzy splitting is generated. Otherwise continue using of the formulas (6, 7).

Using of the cluster analysis by the reasoning process brings to the reduction of time spent on comparison of the current situation with standard situations, because for recognition of a situation it is enough to compare it to the centre of the each cluster and there is no necessity to analyze all situations from knowledge base. Application of the cluster analysis is effectively enough for a class of tasks which are characterized by a large space of objects states description signs, and in case when the automatic allocation of standard situations is necessary. The offered approach for fuzzy situations processing allows to take off restrictions to completeness of the standard situations description and expands system probabilities during plausible reasoning under incomplete and indefinite information.

4. EXAMPLES

In the case when very many sign characterize the states and the signs are independence then the cluster analysis methods are used for the states identification and for new rules conclusion taking account the current data.

Since the size of the paper is limited then only one example is considered. Let's consider the case when the fuzzy productions knowledge base S (4) is used. Let the signs set $Y=\{y_1, y_2\}$ characterize the controlled object. The signs are given by linguistic variables, where y_1 is "Temperature" and y_2 is "Level of the pressure". In the knowledge base there are two standard situations, which at the same time accepted as the clusters centres:

S_1: "If temperature is high/1 And pressure is high /1, Í$_1$, w_1=1" and

S_2: "If temperature is low/1 And pressure is low/1, Í$_2$, w_2=0.8".

Let for first current situation Temperature=37 and Pressure=120. Therefore input fuzzy situation is described in the manner of S_k: "Temperature is high/0.4 And pressure is high/0.8" and S_k: "Temperature is low/0 And pressure is low/0". By using (6) and (7) will calculate for S_1: b_{Sk}=0.4 and w_{Sk}=0.6; for S_2: b_{Sk}=1 and w_{Sk}=0. Thus the conclusion Í$_1$ corresponds for first current situation ("possible") and the conclusion Í$_2$ not corresponds for first current situation ("impossible").

Let for second current situation Temperature=35 and Pressure=120. Therefore input fuzzy situation is described in the manner of S_l: "Temperature is high/0 And pressure is high/0.8" and S_l: "Temperature is low/0.7 And pressure is low/0". By using (6) and (7) will calculate for S_1: b_{Sl}=0.6 and w_{Sl}=0.4; for S_2: b_{Sl}=0.65 and w_{Sl}=0.15. Thus the conclusion Í$_1$ corresponds for first current situation ("probably") and the conclusion Í$_2$ not corresponds for first current situation ("impossible").

The membership functions S_k and S_l to fuzzy clusters S_1 and S_2 are calculated by using (6) on the first iteration of the Bezdek's algorithm: $\mu_1(S_k)$=0.71; $\mu_2(S_k)$=0.28; $\mu_1(S_l)$=0.52; $\mu_2(S_l)$=0.48. Also the clusters centres are specified by using (7): $S_1^1(y_1)$=0.68; $S_1^1(y_2)$=0.91, δ_1=0.21; $S_2^1(y_1)$=0.88; $S_2^1(y_2)$=0.76, δ_2=0.18. New rules are got on this step:

S_1^1: "If temperature is high/0.68 And pressure is high /0.91, Í$_1$, w_1=0.79" and

S_2^1: "If temperature is low/0.88 And pressure is low/0.76, Í$_2$, w_2=0.82"

but since $\delta_2 \le \varepsilon$ is not executed by ε=0.2 consequently continue the algorithm performing: $\mu_1^1(S_k)$=0.78; $\mu_2^1(S_k)$=0.2; $\mu_1^1(S_l)$=0.54; $\mu_2^1(S_l)$=0.46 and the clusters centres are specified: $S_1^2(y_1)$=0.48; $S_1^2(y_2)$=0.86, δ_1=0.13; $S_2^2(y_1)$=0.82; $S_2^2(y_2)$=0.63, δ_2=0.1. New rules are got on the second step:

S_1^2: "If temperature is high/0.48 And pressure is high /0.86, Í$_1$, w_1=0.66" and

S_2^2: "If temperature is low/0.88 And pressure is low/0.76, Í$_2$, w_2=0.72".

Since $\delta_1 \le \varepsilon$ and $\delta_2 \le \varepsilon$ we consider that the fuzzy splitting is formed. Thus the clusters centres are specified with taking into account the current situations set (S_1^2 and S_2^2 are the new clusters centres). Two new rules are inserted in the knowledge base.

CONCLUSION

The considered methods of plausible reasoning allow to solve the following problems arising at construction and analysis of fuzzy productions knowledge base:

– to compare current and standard situations at fuzzy conclusion more correctly;

– to take into account at conclusions by the displacement $\beta(s, s^1)$ influence of fuzzy premises values on conclusion;
– to reduce dimension of knowledge base at its analysis concerning current data set by using for the analysis not all set of rules but only standard situations – clusters centres;
– to construct learning procedure of fuzzy knowledge base by using cluster analysis and compositional conclusion.

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