NEURONS‘ ARRAYS AS NOISE-ADDRESSABLE MEMORY AND NOISE-CONTROLLED MULTI-STATES SWITCHES

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ABSTRACT: We discuss new aspects of the collective behavior of coupled neurons receiving noise as input, and in particular new cooperative effects between noise and an ensemble of integrate-and-fire neurons with feedback coupling. Applications of these effects as noise-addressable memory arrays and noise-controlled multi-states arrays are demonstrated. In a noise addressable neurons‘ array, the memory content is recalled by giving to each neuron an input signal corresponding to an independent Wiener process. The output of a multi-states arrays is determined by the noise level. Noise-induced, percolation-like ordering corresponding to the creation of long-range correlation is observed in two-dimensional arrays. This effect is exploited to create memory arrays that respond coherently only if the right amount of noise is furnished to the system.

KEYWORDS: neurons, noise processing, cooperative effects, stochastic resonance

INTRODUCTION

The study of cooperative effects between noise and nonlinear systems has permitted the identification of two important phenomena. The first is stochastic resonance, first proposed by Benzi (1981) as a model for climatic changes. Stochastic resonance corresponds to a large enhancement of the signal to noise ratio of a signal at a finite noise level. The effect has some practical importance as evidences for stochastic resonance have been found in real neurons. Furthermore, the effect is also related to the delta-sigma method that permits to enhance the resolution of AD converters. The second important effect between noise and a nonlinear system corresponds to the noise-induced synchronization of coupled neurons (Kurrer (1995), Lindner (1995)). This area of research has been much stimulated by the discovery of synchronous neurons firing patterns in the visual system of cats (Gray (1989), Malsburg (1986)), followed by the suggestion that synchronous neurons firing is used in the brain to bind related informations. The degree of synchronization between oscillators may be characterized by the mean correlation value between two adjacent oscillators. This offers a possibility of connecting synchronization effects in arrays of neurons to the study of the correlation between neurons (Rappel (1996), Chialvo (1997)).

In this paper, we study synchronization effects in a prototype system consisting of an array of coupled integrate-and-fire neurons. The input to each neuron is a white noise signal. We show that the information on the noise level may be extracted from the average correlation value between nearest-neighbors. Further we explain how such arrays of integrate-and-fire neurons can be used governed by such equations. The novelty of this approach is that both the recall of the memory and the multi-states switches are controlled by the level of input noise. This work shows the possibility to carry out complex processing tasks on white noise signals using very simple processing units.

The paper is organized as followed. After a short introduction on the model neuron, we present the main properties of two-coupled neurons. We discuss then two-dimensional arrays of coupled neurons and show the similarities and differences to the simpler system composed of two neurons.

THE MODEL NEURON

We consider a special case of an integrate-and-fire neuron defined by the equations

$$z(n+1) = \max(0, z(n) - 1 + c\, W(n)) \quad (1)$$

$$S(n+1) = - S(n) \qquad \text{if } z(n+1) = 0 \quad (2)$$
$$S(n+1) = S(n) \qquad \text{otherwise}$$

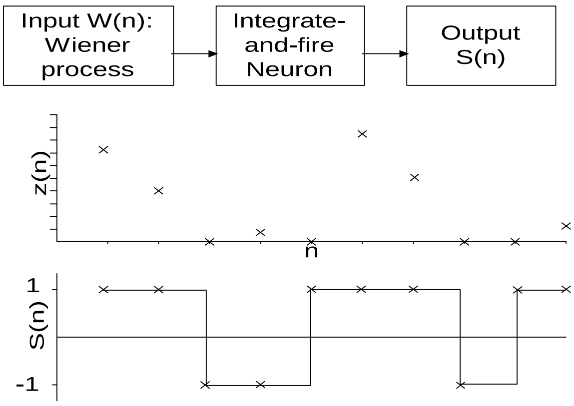

Figure 1: The integrate-and-fire neuron described by (1-2) receives as input a signal W(n) corresponding to a Wiener process of zero mean. The binary output S(n) changes value as z(n) reaches the barrier at z=0.

The input to the system W(n) is a signal corresponding to a Wiener process, that is a signal with a normal distribution N(0,1) of zero mean and variance 1. The neurons' output is the two states variable S. The discrete stochastic equation (1) defines a bounded random walk with a barrier at zero. As depicted in figure 1, the state S changes value each time, the random walk reaches zero. In the formalism of neural networks, (1-2) is a special case of an integrate-and-fire neuron with both reset and firing thresholds set to zero. The model is also physically motivated, as such a fire-and-integrate neuron is a good approximation at large forcing and in a slow time scale of the dynamics of a Mathieu-van der Pol oscillator with multiplicative noise (or equivalently a parametrically forced van der Pol oscillator) as shown by Thuillard (1998).

As sketched in figure 1, the lifetime of a state of S corresponds to the return time of a random walk process. The density function g(n) of the lifetime n of a state of S is given therefore by the density function of a random walk process (Bulsara (1994)):

$$g(n) \propto 1/\sqrt{2 \cdot p \cdot c} \cdot n^{-3/2} \cdot \exp(-n \;\; / \, 2 \cdot c^2 \;\;) \quad (3)$$

RESULTS AND DISCUSSION

TWO COUPLED NEURONS AS A MEMORY CELL

Consider two coupled integrate -and-fire neurons defined by the following set of equations:

$$z_i(n+1) = \max(0, z_i(n) - 1 + c\, W_i(n) + \lambda\, S(n)) \qquad (i=1,2) \qquad (4)$$

$z_i(0)=0$

The function S(n) is defined by

$$S(n) = S_1(n)\ S_2(n) \qquad (5)$$

$$S_i(n+1) = - S_i(n) \qquad \text{if } z_i(n+1) = 0 \qquad (6)$$

$$S_i(n+1) = S_i(n) \qquad \text{otherwise}$$

$W_{1,2}(n)$ are two indepedent Wiener processes and $S_i(n=0)$= 1 or -1 with a probability 0.5 each.

Figure 2 shows the mean correlation value between the outputs S_1 and S_2 which corresponds also to the mean value of the function S(n).

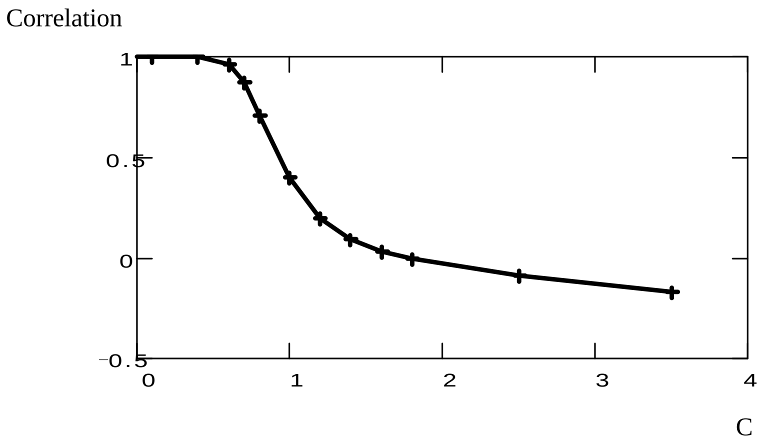

Figure 2. Mean correlation value between the output S_1 and S_2 of two integrate-and-fire neurons with a feedback coupling linearly proportional to $S(n) = S_1(n)\ S_2(n)$. Coupling constante λ= -0.8.

Figure 2 shows a new cooperative effect between noise and a non-linear system. The correlation changes sign as the noise increases. Normally, noise leads either to a more organized system or on the contrary destroys order. Several systems, such as spin neurons or oscillators models, have different ordered states (Crisanti (1987), Nagashino (1992), Kalitzin (1997)). In none of these systems, the correlation value between oscillators or spins does change its sign with increasing noise. Let us remark, that this new effect is also found in a 2-dimensional array of coupled integrate-and-fire neurons (see figure 3). We want to furnish here an explanation for this effect. Three different regions may be described:

<u>Low noise regime:</u> For a negative coupling and at very low noise, the expected correlation tends to one. According to (3), the first return density function g(n) decreases exponentially with increasing n. It follows that, at low noise, z_i (n) is seldom different from zero and states with n>2 can be neglected. Further the probability of simultaneous occurence of

two states with n=2 is negligeable. At low noise, the escape probability determines therefore the expected correlation value. For a negative value of lambda, the ratio of the escape probability out of the state S=1 to the state S=-1 tends to zero since

$\lim_{c\to 0}$ (Prob ((-1+λ) + c W(n)) >0) / Prob ((-1-λ) + c W(n))>0)= 0 (λ <0)

Generally, one obtains:
Mean correlation factor 1 if λ <0
Mean correlation factor -1 if λ >0.

Intermediate to large noise regime: In this regime, the escape probability is close to ½ The escape probability is therefore not the determinant factor as in the low noise case. The lifetime of a state of S depends on those of S_1 and S_2, which are difficult to compute. Nevertheless, the conditional lifetime density function g_c , assuming S_1 (resp. S_2) does not change state, is easy to compute. It corresponds to a random walk with a return term (-1 + λ S): $g_c(n|S_1 = const) \propto 1/\sqrt{2\cdot p\cdot c}\cdot n^{-3/2}\cdot\exp(-(-1+lS)^2 n \;/2\cdot c^2\;)$. For a negative λ, a state with S=1 has on average a shorter lifetime than a state with S=-1 and the expected correlation factor is negative. Consequently, one expects
Mean correlation factor < 0 if λ <0
Mean correlation factor > 0 if λ >0.

Very large noise regime: The expected value of S decreases for very large noise levels (typically c>200 in the conditions of figure 5). In the limit of large noise, the first return density function becomes independent on the return term in (-1 +λS) and the expected value of S tends to zero.

We believe that coupled integrate-and-fire neurons represent prototype equations for various neurons' arrays. The escape rate out of a limit cycle or the jump rate between two attractors are given very often by an exponential function of the noise variance (Arecchi (1984)), similar in its form to the escape rate of a random walk with a return term. This suggests that a sign reversal of the mean correlation value between nearest-neighbors with increasing noise is an effect that occurs in various other neurons with a white noise signals as inputs, a binary output and feedback coupling proportional to the instantaneous correlation value between nearest-neighbors, provided the system is close to a bifurcation point. As an example, we have verified this suggestion with coupled Mathieu-van der Pol neurons at large forcing.

MEMORY ARRAY OF COUPLED INTEGRATE-AND-FIRE NEURONS

In order to study cooperative effects between noise and an array of coupled integrate-and-fire neurons, we consider the neurons' array (k,m) defined by the following set of equations:

$$z_{k,m}(n+1) = \max(0, z_{k,m}(n) -1 + c\, W_{k,m}(n) + \lambda_{k,m} Si_{k,m}(n)) \qquad (7)$$

with $W_{k,m}(n)$ independent Wiener processes. The function Si(n) is defined as

$$Si_{k,m}(n) = \tfrac{1}{4} S_{k,m}(n) \; (S_{k+1,m}(n) + S_{k-1,m}(n) + S_{k,m+1}(n) + S_{k,m-1}(n)) \qquad (8)$$

$$S_{k,m}(n+1) = - S_{k,m}(n) \quad \text{if } z_{k,m}(n+1) = 0 \qquad (9)$$
$$S_{k,m}(n+1) = S_{k,m}(n) \quad \text{otherwise}$$

At equilibrium (n$\to\infty$), the curve giving the correlation value between two nearest-neighbors is qualitatively the same as in the two-neurons case and we will not discuss it further. The main difference between a 2-dimensional array of coupled neurons and two coupled neurons is the transient behavior of a neurons' array with randomly initialized output $S_{k,m}$ ($S_{k,m}$(n=0) = 1 or -1 with a probability 0.5). Figure 3 shows the mean value of Si at n=2000. Several plateaus are observed. Each plateau corresponds to particularly stable configurations of the near-range order. Figure 4 shows this also very clearly. The evolution of the correlation value is given as a function of the number of iterations steps for two different noise levels. Plateaus are observed with similar values to the plateaus in figure 4. The main difference to the memory cell composed of two coupled neurons is the establishment of long range order in a percolation-like transition. A high value of the mean correlation between nearest-neighbors is associated to an infinite cluster. Transient percolation transitions are known from other neuronal models, based on random boolean networks or spins models (Blatt (1997), van Hemmen (1991), Gravner (1996); Parrondo (1996)). Nevertheless the application, as shown below, of such a system as a noise-addressable memory array with several coherent responses depending on the noise level is new to the best of our knowledge.

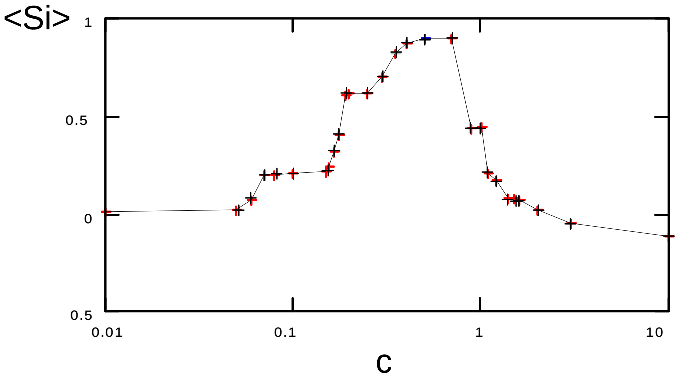

Figure 3: Mean correlation value <Si> between nearest-neighbors at n=2000 for a large two-dimensional neurons' array (200 x200) defined by (7-9) as a function of the noise level c (cyclic boundary conditions, λ= -0.8).

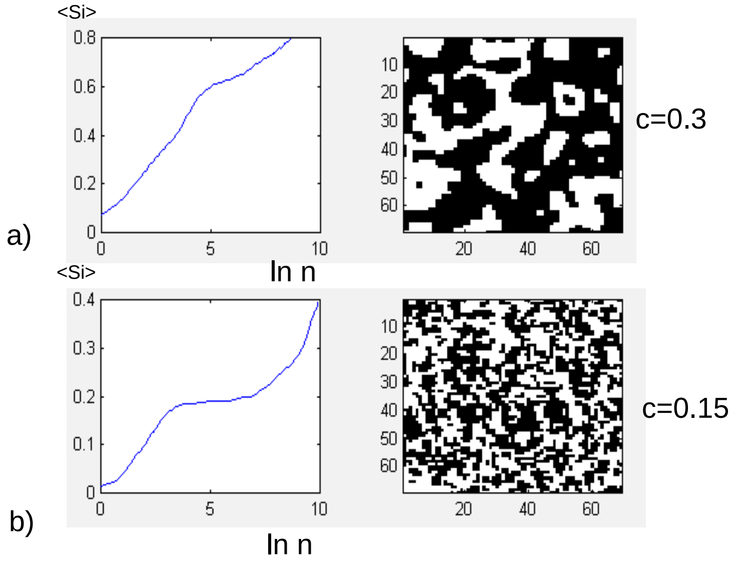

Figure 4: Left: evolution of <Si> the mean correlation between nearest neighbors, averaged on the whole lattice, as a function of the number of iteration steps. (Start values are chosen randomly 1 or -1 with probability 0.5 for each state, λ = -0.8). Right: Examples showing the state $S_{k,m}$ of the neuron array neuron (S=1: white; S=-1:black) just below the percolation threshold and well above the percolation level (c=0.3, n=10000) and just below it (c=0.15, n=20000).

In order to make a noise-addressable memory array, a second layer of neurons is added to the first layer. The second layer of neurons processes the output $Si_{k,m}$ of the first layer neurons. The output of a neuron in the second layer is given by:

$$Oi_{k,m} = 1 \quad \text{if } Si_{k,m} > T$$

$$Oi_{k,m} = 0 \quad \text{otherwise} \qquad (10)$$

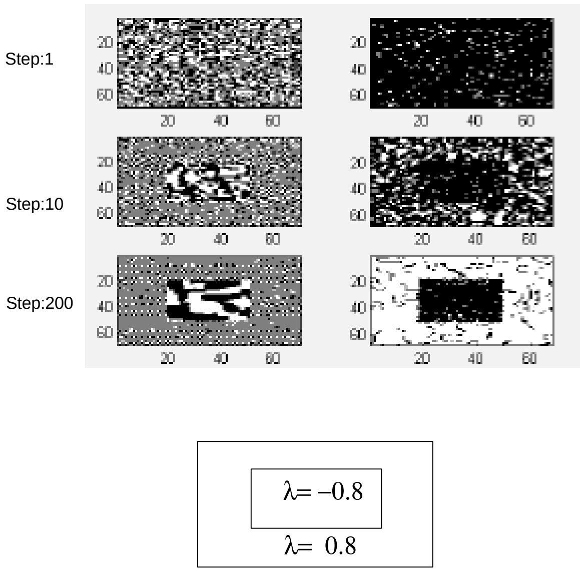

Figure 5: Response of a noise-addressable memory array after different numbers of steps (c=0.5). The left images show the output of the first layer. The right images show the output of the second layer with T=1.5 in eq. (10).

Figure 5 shows an example of a noise-addressable memory. The information is stored in the coupling constantes λ's. Depending on the noise value, the number of steps to recall the information may be very large or the information may also not be even recalled. Information is efficiently recalled only in a small range of values corresponding to large correlations values in figure 3.

Figure 6 shows a second application of a noise-addressable memory array using 3 different values of lambda. Depending on the noise level, very different images are recalled. At low noise level, a black square on a white background is observed, while at a high noise level, a white rectangle on a black background appears at a different location.

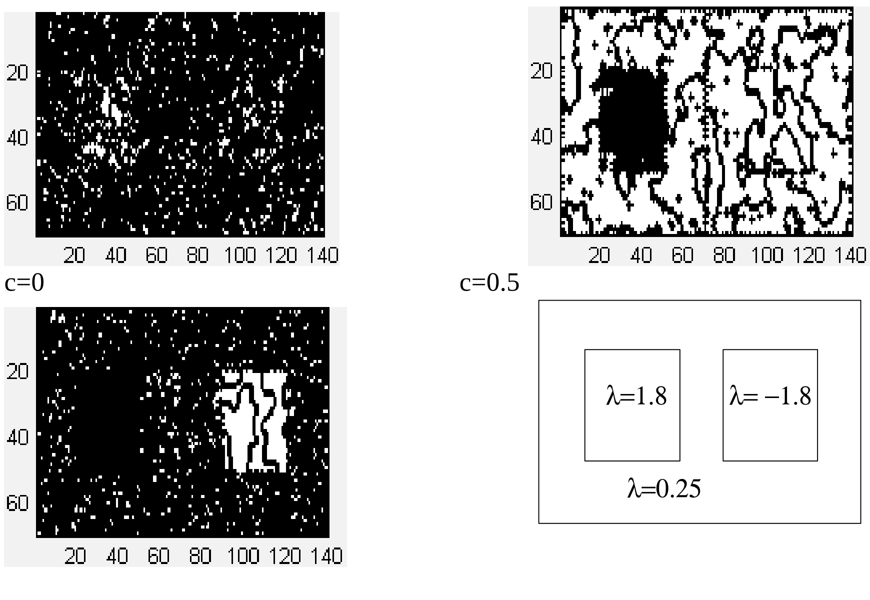

Figure 6: Response of a noise-addressable memory array for 3 different input noise levels at the 2000th iteration step. The bottom right schematics shows the coding of the memory array, which is the same for the 3 images.

In a complex system, such as the nervous system of an animal or a robot, a symbol such as square may code for a given action or task. In this context a modification of the input noise level may result in a different but still coherent response as shown in figure 6. With increasing noise the neurons’ array switches to other basins of attraction corresponding to different ordered states.

CONCLUSIONS

From the fundamental point of view of the study of cooperative effects between noise and nonlinear system, a new collective effect was identified. The correlation between coupled integrate-and-fire neurons changes sign with increasing noise. In order to visualize this effect, one should consider a spin system governed by such equations in an alternating magnetic field. In this system, the preferred configuration of the spins would change with increasing temperature from a ferroelectric to an anti-ferroelectric state, a quite interesting effect indeed. As explained in the appendix, coupled integrate-and-fire neurons represent prototype equations for various neurons’ arrays. We show possible applications of such systems as noise-driven multi-states switches or memory arrays. We would like to conclude with a prospective remark. As the processing of noise in real neurons is far from beeing understood, the possibility that real neurons use some of the presented mechanisms should be considered.

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