Conflict Analysis

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ABSTRACT: Historically, game theory has been mainly used to define and analyse conflicts Fraser (1984), Hipel (1993), Rosenheim (1994), Sandholm (1996). We propose rough sets and the Boolean reasoning methods to specify conflicts and transform the conflict analysis problem and the conflict-resolving problem into the Boolean-reasoning problem Pawlak (1991). Our model is an extension of that proposed by Pawlak e.g. in Pawlak (1993), Pawlak (1998). We discuss some basic conflict problems expressible in this new model as well as algorithms for solving them.

KEYWORDS: conflict (model, analysis, resolving), decision analysis, rough set theory, Boolean reasoning.

1 INTRODUCTION

Everyone is meeting and recognizing conflicts in everyday life. Nowadays more often we are negotiate, specify our views on discussed issues, and analyze the behavior in a given situation. In the paper, we explain the nature of conflict and we define the conflict situation model in a way to encapsulate the conflict components in a clear manner. We propose some methods to solve the most fundamental problems related to conflicts. The model introduced in this paper is an enhancement of the model proposed by Pawlak in papers e.g. Pawlak (1993), Pawlak (1998). In the Pawlak model, conflicts are presented at the outermost level. Some issues are chosen, and the agents are asked to specify their views: are they favorable, neutral or against. In the real world, views on the issues to vote are consequences of the decision taken, based on the local issues, the current state and some background knowledge using some strategy. Therefore, the Pawlak model is enhanced here by adding to the model some local aspects of conflicts. The introduced model also gives a possibility to check if the issues to vote are chosen correctly, i.e., if the local issues determine the decisions.

In the whole paper, one simple example will be analysed. The example is taken from the author's observation and refers to the conflict between an employer and an employee. Job attributes considered for the worker are compensation and work conditions. On the other hand, the employers are interested in the factory profit, good investment level and, maybe, worker's satisfaction. We can think about these attributes quite generally, for example, the compensation can consist of the worker’s salary and all his income but it also can include the repeated profit division like the social fund. Similarly the worker’s conditions includes a modern and safe work place and in addition a nice team and development possibilities. One can easily find that these affairs contradict each other in this example. We analyse this problem more deeply.

2 PAWLAK MODEL

The simple model introduced by Pawlak e.g. in Pawlak (1993), Pawlak (1998) forms the basis for the model presented in this paper. In Pawlak model, the relation of each agent to the specific issue is depicted in the form of the table in which agents are represented by rows and issues by columns. The value assigned to each agent and to each attribute (issue) is from the set {-1, 0, 1}, where -1 means, that the agent is *against*, 0 *neutral* and 1 - *favourable* toward the issue.

Formally, the table described above specifies itself as an *information system* defined as follows:
An information system is a pair S = (U, A), where
U - is nonempty, finite set called the universe; elements of U are called objects (here agents),
A - is nonempty, finite set of attributes (issues).

Every attribute $a \in A$ is a map, $a: U \rightarrow V_a$, where the set V_a is the *value set* of a; elements of V_a are referred to as *opinions*, i.e. a(x) is opinion of agent x about issue a. The domain of each attribute (for conflict analysis model) is restricted to three values only, i.e. $V_a = \{-1, 0, 1\}$, which means against, neutral and favourable respectively.

Example 1

Let's choose the issues to be voted:
a – increasing the employees' incomes,
b – improving the work conditions,
c – increase the factory profit by reducing the costs of work,
d – increase the level of investment.
Then, the following information table, where ag1 – is the employee and ag2 is the employer, can describe the situation.

	a	**b**	**c**	**d**
ag_1	1	1	-1	0
ag_2	-1	0	1	1

Table I:. The conflict situation in the Pawlak Model

The tension of the conflict Pawlak (1993), Pawlak (1998) in the described situation can be calculated as equal to ½.

Analysis of the conflicts described by Pawlak model is restricted to outermost conclusions like finding the most conflicting attributes or the coalitions of agents if more than two take part in the conflict Deja (1996).

Because in the Pawlak model the reason of the conflict cannot be determined, there is no way to specify the situation to avoid the conflict. Moreover, we cannot be sure that the issues the agents vote represent the issues each agent takes care of. In the next section, we will define a model allowing to answer the following basic questions.

- What are the conflict reasons?
- How the consensus can be found?
- Is it possible to satisfy all the agents?

3 NEW MODEL

We assume in the conflict are taking part at least two participants, called agents. Ag denotes the set of all the agents taking part in the conflict.

Example 2

The exemplary conflict of agents from the set Ag consists of two agents: ag_1- *employer* and ag_2- *employee*.

3.1 LOCAL STATES

The set of information about the local states U_{ag} of the agent ag can be presented in the form of an information table, thus creating the agent ag's information system $\mathbf{I}_{ag}=(U_{ag}, A(ag))$ where $a: U_{ag} \rightarrow V_a(ag)$ for any $a\in A(ag)$ and $V_a(ag)$ is the value set of attribute a. We assume

$$V(ag) = \bigcup_{a \in A(ag)} V_a(ag)$$

Any local state $s\in U_{ag}$ is explicitly described by its *information vector* $Inf_{A(ag)}(s)$, where $Inf_{A(ag)}(s)=\{(a, a(s)): a\in A(ag)\}$. Thus the local state is represented by the set of pairs: (*attribute*, *value*) (the issue and the view on it) which are often called *descriptors*. The set $\{Inf_{A(ag)}(s): s\in U_{ag}\}$ is denoted by $INF_A(ag)$ and is called the *information vector set* of ag. We assume that the sets A(ag) are pairwise disjoint, i.e., $A(ag)\cap A(ag')=\emptyset$ for $ag\neq ag'$. This condition emphases that any agent describes the situation in its own way. The manner of understanding the "same world" by each agent can be completely different. The relationships of some of attributes of different agents will be defined by constraints as shown in Section 3.3.

We denote by INF*(ag) the set

$$\mathrm{INF}^*(\mathrm{ag}) = \{ f : A(ag) \rightarrow \bigcup_{a \in A(ag)} V_a(ag) : f(a) \in V_a(ag) \text{ for } a \in A(ag) \}$$

Example 3

Table II: shows local issues (attributes) a, b of agent ag_1:
a - compensation,
b - the work conditions

local states	a	b
s1	2	2
s2	2	1
s3	1	2
s4	1	1
s5	2	0

Table II:. Agent ag_1's local states

Agent ag_2 describes its view on local issues k, l, m by
k - the factory profit,
l - the level of investment,
m - the worker's satisfaction.

The local states of agent ag_2 are presented in Table III:.

local states	k	l	m
s1	2	2	2
s2	1	2	2
s3	1	1	2
s4	1	1	1
s5	2	0	1

Table III:. Agent ag_2's local states

For simplicity let us assume that the attributes' domains for both agents are the same, and values belong to the set V={0, 1, 2}. One can interpret the values from set V as the *"small"*, *"medium"* and *"high"* level, respectively. For example, the state s1 of agent ag_1 expresses a "high" level of compensation and "high" level of work conditions. Similarly, the state s3 of agent ag_2 means "medium" profit level with a "medium" level of investment, while at the same time the worker satisfaction should be "high".

3.1.1 Subjective evaluation of local states (similarity of states)

Every agent has favourable (target) states among the set of local states, i.e. the states the agent wants to reach. In the information table of ag the states the agent ag cannot accept can also appear; being in such a state could mean a disaster for the agent. Actually, an agent evaluates each state. The *subjective evaluation* corresponds to an order (or partial order) in the states of agent information table. We assume the function e_{ag} called the *target function*, assigns an evaluation score to each state. An exemplary target function used in our example is defined by $e_{ag}:U_{ag} \rightarrow R_{[0,1]}$. The states with score 1 are preferred by the agent as target states, while the states with score 0 are not acceptable.

The current information vector of ag is a vector describing the current local state of agent ag.

The state evaluation can also help us to find the state similarity (see e.g. Polkowski (1998), Nguyen (1997) for references on similarity in rough set investigations). For any $\varepsilon>0$, $s \in U_{ag}$ we define ε-neighbourhood of s by:

$$\tau_{ag,\varepsilon}(s)=\{s' \in U_{ag}: |e_{ag}(s) - e_{ag}(s')| \leq \varepsilon\}$$

The family $\{\tau_{ag,\varepsilon}(s)\}_{s \in U_{ag}}$ defines a tolerance relation $\tau_{ag,\varepsilon}$ in $U_{ag} \times U_{ag}$ by $s\tau_{ag,\varepsilon}s'$ iff $s' \in \tau_{ag,\varepsilon}(s)$.

Example 4

The results of adding scores for local states are presented in Table IV: and Table V:.

local states	**a**	**b**	**evaluation e_{ag1}**
s1	2	2	1
s2 (current)	2	1	2/3
s3	1	2	1/3
s4	1	1	0
s5	2	0	0

Table IV:. Local states of agent ag_1 with subjective evaluation

local states	**k**	**l**	**m**	**evaluation e_{ag2}**
s1	2	2	2	1
s2	1	2	2	2/3
s3	1	1	2	1/3
s4	1	1	1	1/3
s5 (current)	2	0	1	0

Table V:. Local states of agent ag_2 with subjective evaluation

3.1.2 Distance function

The tolerance relation τ describes similarity of states according to the subjective evaluation. However, it is necessary to describe the states' similarity according to the differences on values of the attributes. The similarity of states from U_{ag} can be often defined as follows.

We assume that for any $a \in A(ag)$ there is a distance function d_a: $U_{ag} \times U_{ag} \to R_+$. For example $d_a(s, s')=|a(s) - a(s')|$ if $V_{ag}(a) \subseteq R$. Next we define the distance function d: $U_{ag} \times U_{ag} \to R_+$ by $d(s, s') = F(d_{a1}(s, s'),..., d_{am}(s, s'))$, where A(ag)={a1,...,am} and $F : R_+^m \to R_+$ is a suitable, chosen function e.g. $F(r_1,...,r_m) = \sqrt{r_1^2 + ... + r_m^2}$.

The crucial for the negotiation process results and solving any conflict is the agents' willingness of changing the current state (possibly resigning of some resources). This disposition is the basis to define the *closeness* of the states the agents are ready to accept. The closeness is defined by the distance function in the following manner: two states s and s' are close iff $d(s, s')<\varepsilon(ag)$, where $\varepsilon(ag)$ is a given threshold for ag. Consequently the closeness, neighbourhood of the state s with diameter ε is defined by $\{s': d(s, s')< \varepsilon(ag)\}$.

Example 5

Let $\varepsilon(ag_2)=2/3$. The example of closeness neighbourhood of the local state s2 with diameter 2/3 is presented in Table VI:; e.g. $d(s2, s2_1)=F(d_k(s2, s2_1), d_l(s2, s2_1), d_m(s2, s2_1))=1/3(1+0+0)=1/3$, where F(v1, v2, v3)=1/3(v1+v2+v3).

local states	**k**	**l**	**m**
s2	1	2	2
$s2_1$	2	2	2
$s2_2$	1	1	2
$s2_3$	1	2	1

Table VI:. The closeness of state s2 within the threshold 2/3.

3.1.3 Local set of goals (targets)

The target function introduces a partial order in the set of local states in the way that one can find the maximal element(s) (with the highest evaluation) and the minimal one(s). The maximal elements can be interpreted as those, which are targets of the agent, i.e. the agent wants to reach them e.g. in the negotiation process. The agent ag's *set of goals (targets)* denoted by T(ag) is defined as the set of target states of ag, that means $T(ag)=\{s\in U_{ag}: e_{ag}(s)>\mu_{ag}\}$, and μ_{ag} is the boundary level, chosen arbitrary - it is subjective which evaluation level is acceptable by the agent.

Example 6

In the considered example, the minimal acceptable level of evaluation by the both agents will be, e.g., a score greater than 1/3. Accordingly the set of goals of agents ag_1 and ag_2 are as follows: $T(ag_1)=\{s1, s2\}$ and $T(ag_2)=\{s1, s2\}$.

The set of goals can also be presented in the propositional form. The information table with scores is going to be converted to the decision table in which the decision 1 means that the state belongs to the set of goals, while 0 that it does not. Then the rules for decision 1 are found (for the way of rules generation see e.g. Pawlak (1993)).

The decision table of agent ag_1 with the threshold 1/3 is built and presented in Table VII:.

local states	**a**	**b**	**decision d**
s1	2	2	1
s2 (current)	2	1	1
s3	1	2	0
s4	1	1	0
s5	2	0	0

Table VII:. The local set of goals – the decision table

The way of rules generating is comprehensive (based on the equivalence classes) i.e. they can be used to specify the decisions for the states not included in the table.

Rule for d1: $(a2\wedge b2) \vee (a2\wedge b1) \rightarrow d1$
Rule for d0: $(a1\wedge b2) \vee (a1\wedge b1) \vee b0 \rightarrow d0$

In the rest of the paper, the parentheses are omitted in the Boolean expressions, according to the rule that conjunction operator binds strongly than disjunction. Thus, the expression of the form $\alpha\wedge\beta \vee \gamma\wedge\delta$ is understood as $(\alpha\wedge\beta) \vee (\gamma\wedge\delta)$.

3.2 SITUATION

Let us consider n agents $ag_1,...,ag_n$. A situation on Ag is any element of Cartesian product

$$S(Ag) = \prod_{i=1}^{n} INF^*(ag_i),$$

where $INF^*(ag_i)$ is the set of all possible information vectors of agent ag_i, defined in Section 3.1.

The *situation* S(Ag) corresponding to a global state $\bar{s}=(s1, ..., sn)\in U_{ag1}\times...\times U_{agn}$ is defined by $(Inf_{A(ag1)}(s1),..., Inf_{A(agn)}(sn))$.

Example 7

An example of current situation is the one presented in Table VIII:.

Situations	**a**	**b**	**k**	**l**	**m**
current	2	1	2	0	1

Table VIII:. Current situation

3.3 CONSTRAINTS

Any conflict (on local or global level, defined in Section 4) can appear only in the situation when local states of agents in this situation are correlated and depend on each other. Without any dependencies, any agent could take the next state freely. If there is no any influence on the states of other agents - there is no conflict at all. The dependencies between the local states of agents in a given situation come, e.g., from the bound of the number of resources (any kind of resource may be considered, e.g. a water on Golan Hills see Pawlak (1998) or an international position see Nęcki (1994), everything that is essential for the agents). Constraining relations are introduced to express which local states of agents' can coexist in the situation. More precisely, *constraints* are used to define a subset of S(Ag) of global situations.

Constraints restrict the set of possible situations to admissible situations satisfying constraints. We will consider only admissible situations (shortly, situations) in the rest of the paper.

Example 8

The following dependencies restrict the set of situations and are constraints in our example. The attributes' names here stand for the variables corresponding to attributes values. Constants here were taken experimentally to express relationships and allow comparison of two variables with another one.

- $a > 0$ (compensation must be middle at least)
- $1.5+k > a + l$ (division of the profit - very simple case, i.e., the company uses its current profit for all expenses)
- $2.5+m > a + b$ (the workers' satisfaction comes from a good level of salary and work conditions)

Constraints above can be converted to propositional formulas (φ1, φ2 and φ3) accordingly. The conjunction of formulas $\varphi=\varphi1\wedge\varphi2\wedge\varphi3$ defines all admissible situations in our example. Let us see how the formulas φ1, φ2, φ3 are created.

The equation $a>0$ yield to formula $\varphi1=a1\vee a2$. The next formula (from equation $1.5+k > a + l$) is much more complex:

$\varphi2=k0\wedge a0\wedge l0 \vee k0\wedge a0\wedge l1 \vee k0\wedge a1\wedge l0 \vee k1\wedge a0\wedge l0 \vee k1\wedge a0\wedge l1 \vee k1\wedge a0\wedge l2 \vee k1\wedge a1\wedge l0 \vee k1\wedge a1\wedge l1 \vee k1\wedge a2\wedge l0 \vee k2\wedge a0\wedge l0 \vee k2\wedge a0\wedge l1 \vee k2\wedge a0\wedge l2 \vee k2\wedge a1\wedge l0 \vee k2\wedge a1\wedge l1 \vee k2\wedge a1\wedge l2 \vee k2\wedge a2\wedge l0 \vee k2\wedge a2\wedge l1$

The formula φ3 is created in a similar way: $\varphi3=m0\wedge a0\wedge b0 \vee m0\wedge a0\wedge b1 \vee ... \vee m2\wedge a2\wedge b2$.

As it was mentioned, constraints describe the situations that are admissible in a way that all local states can coexist in the admissible situation. For example, the situation a=2, b=2, k=2, l=2, m=2 is not admissible because of the second and third constraints. The set of all admissible situations (presented in Table IX:) is described by the prime implicants of the Boolean formula $\varphi=\varphi1\wedge\varphi2\wedge\varphi3$.

3.4 OBJECTIVE EVALUATION OF SITUATIONS

We have noticed that changing the agents' states - their views on some issues - is crucial for resolving conflicts. However, changing the local states is the mechanism for changing the global situation, and the good global situation is the real goal when solving the conflict. It has been proven by negotiators Nęcki (1994) that only keeping in mind the common goal by all the participants, the real consensus can be found. This observations suggest to introduce the objective evaluation of situations, i.e. the function $q:S(Ag)\rightarrow R_{[0, 1]}$, which assigns the evaluation score to each situation, and will be called the *quality function of the situations*, where S(Ag) is the set of all admissible situations. An expert could give the score of each situation.

The set of situations satisfying a given level of quality t is defined by:

$$Score_{Ag}(t) = \{\bar{s} \in \prod_{ag\in Ag} U_{ag} : q(\bar{s}) \geq t\}$$

Example 9

Values of the function q and all admissible situations in our example are presented in Table IX:.

situations	a	b	k	l	m	q(S)
S1	1	0	0	0	0	0
S2	1	0	1	0	0	0
S3	1	0	1	1	0	0
S4	1	0	2	0	0	0
S5	1	0	2	1	0	0
S6	1	0	2	2	0	0
S7	1	1	0	0	0	0
S8	1	1	1	0	0	0
S9	1	1	1	1	0	0
S10	1	1	2	0	0	0
S11	1	1	2	1	0	0
S12	1	1	2	2	0	0
S13	1	0	0	0	1	0
S14	1	0	1	0	1	0
S15	1	0	1	1	1	0
S16	1	0	2	0	1	0
S17	1	0	2	1	1	0
S18	1	0	2	2	1	0
S19	1	1	0	0	1	0
S20	1	1	1	0	1	0
S21	1	1	1	1	1	1/3
S22	1	1	2	0	1	0
S23	1	1	2	1	1	1/3
S24	1	1	2	2	1	2/3
S25	1	0	0	0	2	0
S26	1	0	1	0	2	0
S27	1	0	1	1	2	1/3
S28	1	0	2	0	2	0
S29	1	0	2	1	2	1/3
S30	1	0	2	2	2	1/3
S31	1	1	0	0	2	0
S32	1	1	1	0	2	0
S33	1	1	1	1	2	1/3
S34	1	1	2	0	2	1/3
S35	1	1	2	1	2	1/3
S36	1	1	2	2	2	2/3
S37	1	2	0	0	2	0
S38	1	2	1	0	2	0
S39	1	2	1	1	2	1/3
S40	1	2	2	0	2	0
S41	1	2	2	1	2	2/3
S42	1	2	0	0	1	0
S43	1	2	1	0	1	0
S44	1	2	1	1	1	1/3
S45	1	2	2	0	1	0
S46	1	2	2	1	1	2/3
S47	1	2	2	2	1	1
S48	2	0	1	0	0	0
S49	2	0	2	0	0	0
S50	2	0	2	1	0	0
S51	2	0	1	0	1	0
S52	2	0	2	0	1	0
S53	2	0	2	1	1	0
S54	2	1	1	0	1	0

situations	a	b	k	l	m	q(S)
S55 (current)	2	1	2	0	1	0
S56	2	1	2	1	1	1/3
S57	2	0	1	0	2	0
S58	2	0	2	0	2	0
S59	2	0	2	1	2	0
S60	2	1	1	0	2	0
S61	2	1	2	0	2	0
S62	2	1	2	1	2	2/3
S63	2	2	1	0	2	0
S64	2	2	2	0	2	0
S65	2	2	2	1	2	2/3

Table IX:. Admissible situations with the quality score

Let us find the rules for admissible situations with the quality score not lower than 2/3 – these rules are going to be used in the calculations in the next section.

$a1\wedge b1\wedge k2\wedge l2\wedge m1\vee a1\wedge b1\wedge k2\wedge l2\wedge m2 \vee a1\wedge b2\wedge k2\wedge l1\wedge m2 \vee a1\wedge b2\wedge k2\wedge l1\wedge m1\vee a1\wedge b2\wedge k2\wedge l2\wedge m1\vee$
$a2\wedge b1\wedge k2\wedge l1\wedge m2 \vee a2\wedge b2\wedge k2\wedge l1\wedge m2 \rightarrow q(S) \geq 2/3$

3.5 SYSTEM WITH CONSTRAINTS

The multi-agent system, with defined the local states for each agent and the global situations satisfying constraints, will be called the *system with constraints*. We denote our system with constraints by M_{Ag}.

4 CONFLICT DEFINITION

In Section 3.1-3.4 the system with constraints has been defined. In such systems, conflict can be defined on several different levels.

4.1 LOCAL CONFLICT

The agent ag is in the *local conflict* on state s iff s does not belong to the ε-neighbourhood of s', for any s' from the set of ag-targets where ε is a given threshold.

Local conflicts for agent ag arise from the low level of subjective evaluation of the current state by ag. The value $Cl_{ag}(s)$, which can be treated as a degree of the local conflict for ag at $s\in U_{ag}$ is defined by.

$$Cl_{ag}(s) = \begin{cases} f_{ag}(s) - e, \text{when } f_{ag}(s) > e \\ 0, \text{ otherwise} \end{cases}$$

where ε is a given threshold. The function f_{ag} evaluates the distance from the state s to the set of targets of ag, i.e. $f_{ag}(s)=\min\{|e_{ag}(s) - e_{ag}(s')|: s'\in T(ag)\}$, where $e_{ag}(s)$ is the subjective evaluation of ag at the local state s.

Example 10

We choose the threshold ε in our example to be equal 0, i.e., we want to obtain the state without local conflict. For the state s2 of agent ag_1, $Cl_{ag1}(s2)=0$ – the current state belongs to the set of targets. However, $Cl_{ag2}(s5)=2/3-0=2/3$ – i.e. the agent ag_2 is in a local conflict at s5, the current state s5 is not satisfactory for agent ag_2.

4.2 GLOBAL CONFLICT

A situation S is called t-*conflicting* for Ag where t is a given threshold iff S does not belong to the set $Score_{Ag}(t)$. When the current situation is conflicting for Ag then agents from Ag are in the *global conflict*. The difference between the situation score and the given threshold can be treated as a global conflict degree, i.e.,

$$Cg_{Ag}(S) = \begin{cases} t - q(S), \text{when } t > q(S) \\ 0, \text{ otherwise} \end{cases}$$

where t is the given threshold and q is the quality function.

In our example, let us take t=2/3. Thus the current situation S55 is t-conflicting for $\{ag_1, ag_2\}$ and the global conflict factor is equal to $Cg_{Ag}(S55)=2/3-0=2/3$.

5 PROBLEMS

The introduced above conflict model gives us possibility, first to understand and, then, to analyse conflicts of everyday life. Particularly, the fundamental problems described below can be analysed deeply. Firstly the most important problem is investigated, that is, the possibility to achieve *consensus*. As in everyday life consensus can be found on several levels and under some conditions – what is discussed in Section 5.1. The next sections encounter problems concerning another aspects of conflicts.

5.1 CONSENSUS

Input
The system with constraints M_{Ag} defined in Section 3.
t – an acceptable threshold of the global conflict for Ag.
Output
The set of all situations with eliminated global conflict i.e., $Cg_{Ag}(S')=0$, where S' is any new, reconstructed situation. That means that quality score of the new, reconstructed situation cannot be lower then the given threshold t.

Algorithm

The algorithm must analyse all admissible situations and find these with the quality score not lower than the given threshold t.

Finding the solution consists in retrieving the formulas which describe the set $Score_{Ag}(t)$. To do this, the information table with admissible situations is converted into a decision table. We are looking for a formula (rule) describing the decision that the situation is not conflicting. How to create such a formula has been shown in Example 10.

One can find that changing the global situation does not solve all the problems. The quality of the local states of the agents is not considered - the local conflict can be even stronger then before. In the next sections, we are going to analyse the problem more deeply and we will try to eliminate local conflicts as well.

Example 11

For t=2/3 the solution of compromise problem, for our conflict, is presented by the formula f_φ or, equivalently, in the form of a table (Table X:).

$f_\varphi = a1 \wedge b1 \wedge k2 \wedge l2 \wedge m1 \vee a1 \wedge b1 \wedge k2 \wedge l2 \wedge m2 \vee a1 \wedge b2 \wedge k2 \wedge l1 \wedge m2 \vee a1 \wedge b2 \wedge k2 \wedge l1 \wedge m1 \vee a1 \wedge b2 \wedge k2 \wedge l2 \wedge m1 \vee a2 \wedge b1 \wedge k2 \wedge l1 \wedge m2 \vee a2 \wedge b2 \wedge k2 \wedge l1 \wedge m2$.

Situations	a	b	k	l	m
S1	1	1	2	2	1
S2	1	1	2	2	2
S3	1	2	2	1	2
S4	1	2	2	1	1

Situations	a	b	k	l	m
S5	1	2	2	2	1
S6	2	1	2	1	2
S7	2	2	2	1	2

Table X:. Global situations without the conflict (considering conflict threshold t=2/3)

5.2 CONSENSUS ON THE LOCAL PREFERENCES

In this section a conflict analysis is proposed where the local information tables and the set of local goals are taken into consideration.

Input
The system with constraints M_{Ag} defined in Section 3.
t – an acceptable threshold of the global conflict for Ag.
Output
All situations with the global conflict reduced to degree at most t, without local conflict for any agent.

The problem in this section consists in looking for a more optimal compromise: additionally it is required that the new situation is constructed in the way that all local states in the situation are favourable for the agents.

Algorithm

Algorithm is based on verification of the global situations from $Score_{Ag}(t)$ with the local set of goals of the agents. The solution is described by the formula f:

$$f = \bigwedge_{ag\in Ag} t(ag) \wedge f_j$$

where t(ag) is the disjunction of targets of the agent ag, and f_φ is the formula (shown in Example 9) representing all admissible situations without the global conflict regarding the threshold t.

The situations, which can be found using this algorithm, are better then the previous one – the local preferences are considered.

Example 12

The way f_φ is created is presented in Example 9.

$$f = \bigwedge_{ag\in Ag} t(ag) \wedge f_j$$

$t(ag_1)$ and $t(ag_2)$ are based on the set of goals of the agents ag_1 and ag_2 respectively. Example 5 shows the way the formula $t(ag_1)= a2\wedge b2 \vee a2\wedge b1$ can be found. The formula $t(ag_2)$ is found in the same way: $t(ag_2)= l2$.

Thus, the formula f is the following conjunction:
$f = (a2\wedge b2 \vee a2\wedge b1) \wedge l2 \wedge (a1\wedge b1\wedge k2\wedge l2\wedge m1\vee a1\wedge b1\wedge k2\wedge l2\wedge m2 \vee a1\wedge b2\wedge k2\wedge l1\wedge m2 \vee a1\wedge b2\wedge k2\wedge l1\wedge m1\vee a1\wedge b2\wedge k2\wedge l2\wedge m1\vee a2\wedge b1\wedge k2\wedge l1\wedge m2 \vee a2\wedge b2\wedge k2\wedge l1\wedge m2)$.

Within the given data there is no solution for this problem – the goals of the agents cannot coexist, are rejected by the constraints. We will look for the solution in close neighbourhood of the local targets (in the local closeness).

5.3 CONSENSUS ON THE LOCAL CLOSENESS

Input
The system with constraints M_{Ag} defined in Section 3.
t – an acceptable threshold of the global conflict for Ag.
The closeness threshold χ

Output
All situations with the global conflict degree at most t, and without local conflict for any agent. The new situation can be constructed from the local states closeness, i.e., from the states having the distance from those in the information table less than χ.

Algorithm

The algorithm is similar to the previous one, but each state from the set of goals of any agent is enlarged on the closeness. Precisely, the Boolean formula f' defines the solution.

$$f' = \bigwedge_{ag \in Ag} t'(ag) \wedge f_j$$

where t'(ag) is the formula which describes the agent ag's set of targets with closeness, that is each state from the set of targets and the state closeness is considered. The formula f_φ describes the global situations from the set $Score_{Ag}(t)$.

Example 13

The formula f_φ has been defined in Example 9.
Let us consider the closeness of the goals with the threshold χ=1/2, i.e. if d(s, s')<1/2, then a local state s' is close to s.
Let the distance function be defined by

$$d(s, s') = \frac{1}{card(A(ag))} \sum_{a \in A(ag)} |s(a) - s(a')|.$$

For agent ag_1 the closeness neighbourhood with the threshold 1/2 is not giving new states, thus the formula for the local goals of this agent remains the same as found in Example 5. The local set of goals for the agent ag_2 is shown in Table XI:.

Local states	k	l	m	decision	order e_{x2}
s1	2	2	2	1	1
$s1_1$	2	1	2	1	
$s1_2$	2	2	1	1	
s2	1	2	2	1	2/3
$s2_1$	0	2	2	1	
s3	1	1	2	0	1/3
$s3_1$	0	1	2	0	
$s3_2$	1	0	2	0	
s4	1	1	1	0	1/3
$s4_1$	0	1	1	0	
$s4_2$	1	0	1	0	
$s4_3$	1	1	0	0	
s5	2	0	1	0	0
$s5_1$	2	1	1	0	
$s5_2$	2	0	2	0	
$s5_3$	2	0	0	0	

Table XI:. Local states of agent ag_2 with closeness

One can notice that the states from the closeness neighborhood of s1 can be the same as those from the closeness neighborhood of state s5 while the states s1 and s5 have completely different evaluation values. We will take the upper boundary of the set of target states (as specified in Table XI: by the states with decision 1).

To find out the minimal rules for decision 1 the discernibility between the set of local goals and the other states has to be found. The discernibility matrix is presented in Table XII:.

	s1	$s1_1$	$s1_2$	s2	$s2_1$
s3	k, l	k	k, l	l	k, l
$s3_1$	k, l	k	k, l, m	k, l	l
$s3_2$	k, l	k, l	k, l, m	l	k, l
s4	k, l, m	k, m	k, l	l, m	k, l, m
$s4_1$	k, l, m	k, m	k, l	k, l, m	l, m

	s1	$s1_1$	$s1_2$	s2	$s2_1$
$s4_2$	k, l, m	k, l, m	k, l	l, m	k, l, m
$s4_3$	k, l, m	k, m	k, l, m	l, m	k, l, m
s5	l, m	l, m	l	k, l, m	k, l, m
$s5_1$	l, m	m	l	k, l, m	k, l, m
$s5_2$	l	l	l, m	k, l	k, l
$s5_3$	l, m	l, m	l, m	k, l, m	k, l, m

Table XII:. Discernibility matrix

The prime implicants for each considered state are as follows: s1: l, $s1_1$: $k\wedge l\wedge m$, $s1_2$: l, s2: l and $s2_1$: l. These attributes are considered while generating the decision rules and consequently $t'(ag_2)$ is the formula as follows. We are always looking for the minimal rules to simplify the formula and hasten the computation.

$t'(ag_2)= l2 \vee k2\wedge l1\wedge m2 \vee l2 \vee l2 \vee l2 = l2 \vee k2\wedge l1\wedge m2$
$t'(ag_1)$ has been found in Example 5, i.e., $t'(ag_1)= a2\wedge b2 \vee a2\wedge b1$.

Formula f' is as follows:
$f'= (a2\wedge b2 \vee a2\wedge b1) \wedge (l2 \vee k2\wedge l1\wedge m2) \wedge (a1\wedge b1\wedge k2\wedge l2\wedge m1\vee a1\wedge b1\wedge k2\wedge l2\wedge m2 \vee a1\wedge b2\wedge k2\wedge l1\wedge m2 \vee a1\wedge b2\wedge k2\wedge l1\wedge m1\vee a1\wedge b2\wedge k2\wedge l2\wedge m1\vee a2\wedge b1\wedge k2\wedge l1\wedge m2 \vee a2\wedge b2\wedge k2\wedge l1\wedge m2)$
$f'= a2\wedge b1\wedge k2\wedge l1\wedge m2 \vee a2\wedge b2\wedge k2\wedge l1\wedge m2$

Thus, the situations presented in Table XIII: are proposed as the solution in the conflict – the consensus.

Situations	a	b	k	l	m
S1	2	1	2	1	2
S2	2	2	2	1	2

Table XIII:. Not conflicting situations

5.4 FINDING THE SMALLEST GROUP OF AGENTS INVOLVED IN CONFLICT

We are looking for new situations with the global conflict level lower than a given threshold, and the minimal number of agents, who have to change their actual states to access the new situations.

Input
The system with constraints M_{Ag} defined in Section 3.
t – an acceptable threshold of the global conflict for Ag.
Output
All situations with acceptable conflict level and a minimal set of agents involved in the conflict.

Algorithm

One of the ways to resolve the problem is to resolve the consensus problem first, i.e. use the algorithm described in Section 5.1-5.3 according to this problem detailed specification. This first step gives the disjunction of acceptable situations – formula f1. Then, the formula representing the current global state has to be specified – formula f2. It comes from the compounding of the local current states. The last steps are the conjunction of the formulas $f= f1\wedge f2$ and the decomposition of formula f components into each agent parts. The component (components) composed from the biggest number of agents parts (e.g. m agents) is the problem solution. The number of agents, which has to change its view, is the difference n-m where n is the total number of agents.

6 CALCULATION STRATEGIES

The Boolean calculations of formulas described in the previous section can be exhausted or time consuming. In the consensus problems we have to verify the local goals f1,...,fn against the formula of admissible situations f. This usually yields long formulas looked like this: $g = f1\wedge f2\wedge ...\wedge fn\wedge f$. Calculating prime implicants of such formulas is usually

NP-hard problem. Therefore depending of the formula, the simple strategy or eventually quite complex heuristics must be used to resolve the problem in the real time. The important notice can be used especially in the advanced strategies is that the result (if any) is the disjunction of selected components of formula f.

The discussed problems, especially consensus problem, can be treated as the numerical CSP problems as well see e.g. Beringer (1998), Botelho (1998), Puget (1998). The entry points are the constraints, which in this case will not be transformed into the propositional form. The quality score of the global situation must be set to the already computed situation. If the score does not satisfy the threshold, a next solution has to be searched.

7 CONCLUSIONS

We have presented and discussed the extension of the Pawlak conflict model. The understanding of the underlying local states as well as constraints in the given situation is the basis for any analysis of our world. The local goals and the evaluation of the global situation are observed as factors defining the strength of the conflict and can suggest the way to reach the consensus.

Some fundamental problems and the way of solving them have been presented with attempt of deeper insight into structure of conflicts.

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