Identification of State-Space Models with RGO

S. Garrido, L.Moreno and M.A. Salichs
Universidad Carlos III de Madrid
Leganés 28911, Madrid (Spain)

ABSTRACT: This paper presents an improved identification method of non-linear dynamic systems with State-Space models. This method is based on Genetic algorithms with a new technique to simulate the behaviour of the gradient method without using the concept of derivatives. This method uses an identification algorithm that begins by calculating what ARX model adapts better to the system. Then the algorithm uses Restricted Genetic Optimization (RGO) which improves the previous identification.

1 LINEAR MODELS IN THE STATE-SPACE

A general linear invariant system can be described in discrete time as

$$y(t) = G_0(q)u(t) + H_0(q)e_0(t), \tag{1}$$

where y(t) is the output vector of dimension p, $u(t)$ is the input vector of dimension m and they are supposed to be known. The unknown output perturbations that appear in the output are supposedly generated by the second term, where $e_0(t)$ is the noise vector of dimension p. $e_0(t)$ is supposed to be a sequence of independent sthochastic variables that verify

$$E[e_0(t)] = 0, \quad \text{y} \quad E[e_0(t)e_0(t)^T] = \Lambda_0 \tag{2}$$

where E is the mean operator.

As the system is of finite order, it can be described in the State-Space by the introduction of an auxiliar vector of states $x(t)$ of dimension n, where n is the dimension of the system. The input-output relationship can be described as

$$\begin{aligned} x(t+1) &= Ax(t) + Bu(t) + Ke_0(t) \\ y(t) &= Cx(t) + e_0(t) \end{aligned} \tag{3}$$

where $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $K \in \mathbb{R}^{n\times p}$ and $C \in \mathbb{R}^{p\times n}$.

The way of representing the noise in 3 is called innovations. State-space model 3 is equal to system 1 if matrices A, B, C and K verify

$$G_0(q) = C(qI-A)^{-1}B, \text{ y } H_0(q) = C(qI-A)^{-1}K + I. \tag{4}$$

None of the matrices A, B, C and K is unique. Changing the base for the states is possible the following way:

$$\tilde{x}(t) = T^{-1}x(t) \tag{5}$$

where $T \in \mathbb{R}^{n\times n}$ is a non singular matrix and the system in the new state variables is

$$\begin{aligned} \tilde{x}(t+1) &= \tilde{A}\tilde{x}(t) + \tilde{B}u(t) + \tilde{K}e_0(t) \\ y(t) &= \tilde{C}\tilde{x}(t) + e_0(t) \end{aligned} \tag{6}$$

where $\tilde{A} = T^{-1}AT$, $\tilde{B} = T^{-1}B$, $\tilde{K} = T^{-1}K$ and $\tilde{C} = CT$.

It is easy to see that

$$\begin{aligned} C(qI-A)^{-1}B &= \tilde{C}(qI-\tilde{A})^{-1}\tilde{B} \\ C(qI-A)^{-1}K + I &= \tilde{C}(qI-\tilde{A})^{-1}\tilde{K} + I \end{aligned} \tag{7}$$

2 PARAMETRIZATION AND IDENTIFIABILITY

A model structure $\mathcal{M}$ is globally identifiable if $\mathcal{M}(\theta) = \mathcal{M}(\theta^*) \Rightarrow \theta = \theta^*$.

The identifiable model structures have been often used to identify systems because of the bijective correspondence between the models and the values of the parameters. But it is difficult to find a well conditioned parametrization for some systems.

The identifiable model structures $\mathcal{M}_\mathcal{I}$ were introduced in [10] and can be defined as:

Definition 1 *Let $A(\theta)$ a matrix filled with zeros and with ones in the upper row of the diagonal. Let the rows numbered $r_1, r_2, \ldots, r_p$, where $r_p = n$ filled with parameters. Let $B(\theta)$ and $K(\theta)$ filled with parameters and $C(\theta)$ with zeros and ones in row r_i, column $r_i + 1$. (It is considered that $r_0 = 0$ and that p is the dimension of $y(t)$).*

Ljung [8] demonstrates the next theorem, 4A.2.

Theorem 1 *The state-space model structure $\mathcal{M}_\mathcal{I}$ defined is globally identifiable in θ^* if and only if $\{A(\theta^*), [B(\theta^*)K(\theta^*)]\}$ is controllable.*

3 COMPLETELY PARAMETRIZED MODELS IN THE STATE-SPACE

If we consider model 3, with matrices A, B, C and K filled with parameters, the model is over-parametrized and part of its identifiability is lost. To be more precise, for the identifiable model structure, $2np + nm$ parameters are needed.However, the number of parameters for completely parametrized model structure is

$$d = n^2 + 2np + nm. \tag{8}$$

That is, for the completely parametrized model structure, n^2 parameters more than for the identifiable model structure are needed.

Nevertheless, with this state-space model structure, it is not necessary to find out what multi-index structure is the best for the problem because the completely parametrized model structure contains all the others.

Moreover, it has the advantage that the structure is more flexible and it is possible to do transformations in order to get a description that is numerically better conditioned.

4 BALANCED REALIZATIONS

The matrix

$$W_0 = \sum_{k=0}^{\infty} (A^T)^k C^T C A^k \tag{9}$$

is called *Observability Gramian* for the system in the State Space. The eigenvalues of this matrix describe how the initial state *x(0)* influence the output signal *y(t)* when $u(t) \equiv 0$. This matrix satisfies the next equation of Lyapounov:

$$W_0 = A^T W_0 A + C^T C. \tag{10}$$

The dual matrix

$$W_c = \sum_{k=0}^{\infty} A^k B B^T (A^T)^k \tag{11}$$

is called Controllability Gramian. This matrix describe how the input signal u influences the state vector x. W_c satisfies

$$W_c = A W_c A^T + B B^T. \tag{12}$$

The Gramian matrices are symmetric by construction.

Definition. A realization in the State Space is balanced if

$$W_o = W_c = \Sigma = \text{diag}\ (\sigma_1\ \ldots, \sigma_n) \tag{13}$$

with $\sigma_1 \geq \ldots \geq \sigma_n > 0$.

5 INTRODUCTION TO THE RESTRICTED GENETIC OPTIMIZATION.

In the literature, the use of genetic algorithms as an optimization method is off-line and the time used is quite long. New mechanisms were used the same way Nature does: on-line. We have demonstrated [4, 5], that genetic algorithms can be a fast optimization method of time-varying functions.

Usually GAs are used as a parallel, global search technique. It evaluates many points simultaneously, improving the probability of finding the global optimum. In time-varying optimization, this characteristic is useful in the first generations, to find the correct basin of attraction, however, it consumes a large computation time later. For this reason, a fast semi-local optimization method, such as RGO is better.

The proposed method is called after the mechanism of restricted search. It consists of carrying out the search in the neighbourhood of a point, and of taking the best adapted point as the center of the search of the new generation.

We get new generations oriented in the direction of the steepest slope of the cost function, and with a distance to the center as close as possible to the correct one. This distance corresponds to the velocity at which the system is changing.

This behaviour simulates the gradient method without using derivatives and can be used when signals are very noisy. If these signals are not very noisy, a gradient-based method can be used.

We can carry out a search in a big neighbourhood at the beginning and then reduce its radius (The radius is taken as proportional to uncertainty) . In fact, the method does a global search at first and a local search at the end. This method reduces the probability of finding a local minimum.

In the algorithm, the following operators are implemented: reproduction, cross, mutation, elitism, immigration, ranking and restricted search.

A ranking mechanism was used to regulate the number of offspring that an individual can have because if an individual is very good, it can have too many descendants and the genetic diversity of the population can decrease considerably. This mechanism is implemented by using the fitness function $F_n(i) = \frac{1}{cte+V_n(i)}$, where $V_n(i) = \Sigma_{k=1}^{buff}(y_{n-k} - \hat{y}_{n-k})^2$ is the loss function and cte is a constant with a high value (10^8).

5.1 RGO IDENTIFICATION.

The excitation input u(t) was chosen as a pseudo random binary sequence (PRBS), with coloured noise.

At first, a least squares estimation using an ARX model is done. Then, this model is transformed in a State-Space model. The estimation is transformed in a balanced realization in order to have a numerically better conditioned model.

To improve this estimation, a search of models is carried out. In order to get this minimization, the algorithm searches for new models in a neighbourhood of the last estimate. These models must have coefficients close to the ones previously calculated and their loss function must be as small as possible. The radius of this neighbourhood is taken as proportional to the uncertainty.

The search is carried out using completely parametrized matrices and the estimation is transformed in a balanced realization at the end of each generation.

At each generation, the last model is substituted by the best of the new generation as center of the neighbourhood where the search is realized. In this way our model tracks the exact model. In this neighborhood the individuals of the new generations will be oriented mostly in the direction of the steepest slope of the loss function and at the distance which corresponds to the velocity of the real system changing with time.

The algorithm used is:

1. A first estimation using least squares and an ARX model is done. Then, this model is transformed in a State-Space model.
2. The model is transformed in a balanced realization.
3. k is assigned the value 1.
4. The minimization problem
$$\hat{\theta}^k = \arg\min_{\theta} V_N(\theta) \tag{14}$$
is solved using RGO (Restricted Genetic Optimization) with completely parametrized matrices.
5. The estimation is transformed in a balanced realization.

6. The value of k is incremented by one and steps 4-6 are repeated until the condition

$$V_N(\hat{\theta}_b^{k-2}) - V_N(\hat{\theta}_b^{k-1}) < \varepsilon \tag{15}$$

is satisfied, where ε is a priori known constant.

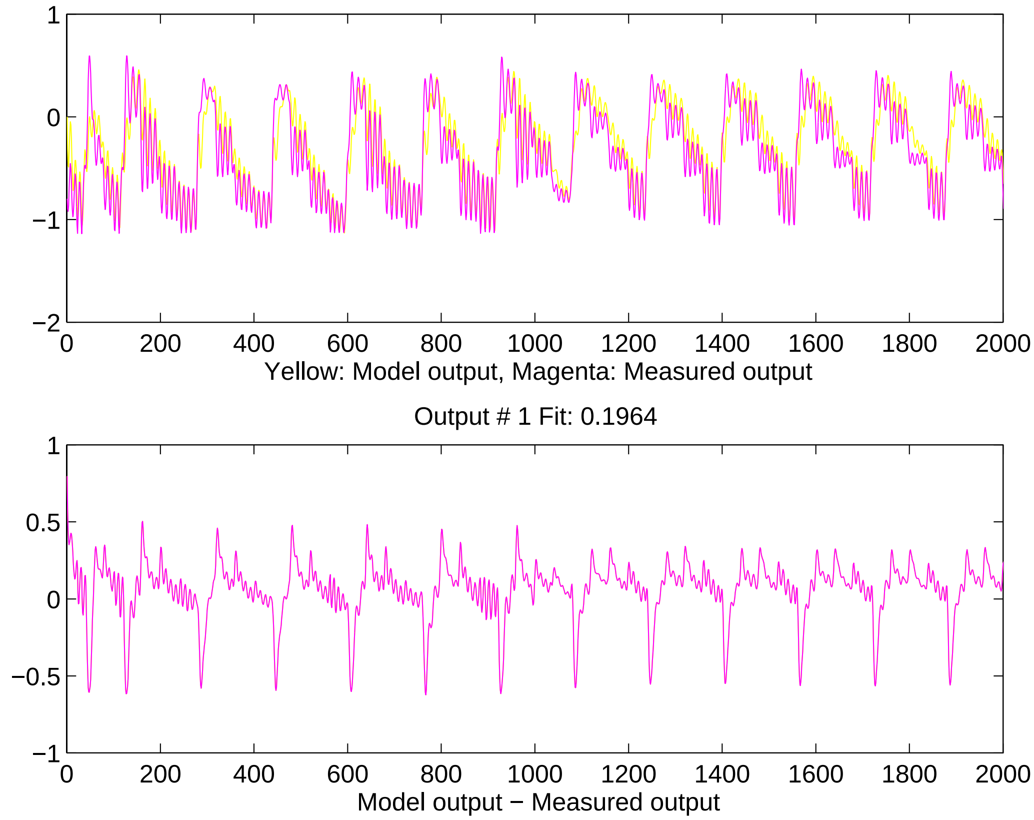

Figure 1: Results of identification of the Plant with the ARX method. Top: Model output vs. system output. Bottom: Error

6 RESULTS

In order to contrast the results, the Relative Root Mean Square (RMS) error is used as a performance index:

$$\text{RMS error} = \sqrt{\frac{1}{N}\Sigma_{i=1}^{N}(y_i - \hat{y}_i)^2} \tag{16}$$

$$\text{Relative RMS error} = \frac{\sqrt{\frac{\Sigma_{i=1}^{n}(y_i-\hat{y}_i)^2}{N}}}{\sqrt{\frac{\Sigma_{i=1}^{n}(y_i)^2}{N}}} \times 100\% \tag{17}$$

To compare the various identification methods we consider the application to a real system: the Twin Rotor Mimo System (TRMS) which is a laboratory set-up designed for control experiments.

In certain aspects its behaviour resembles that of a helicopter. From the control point of view it exemplifies a high order non-linear system with different dynamics in each position.

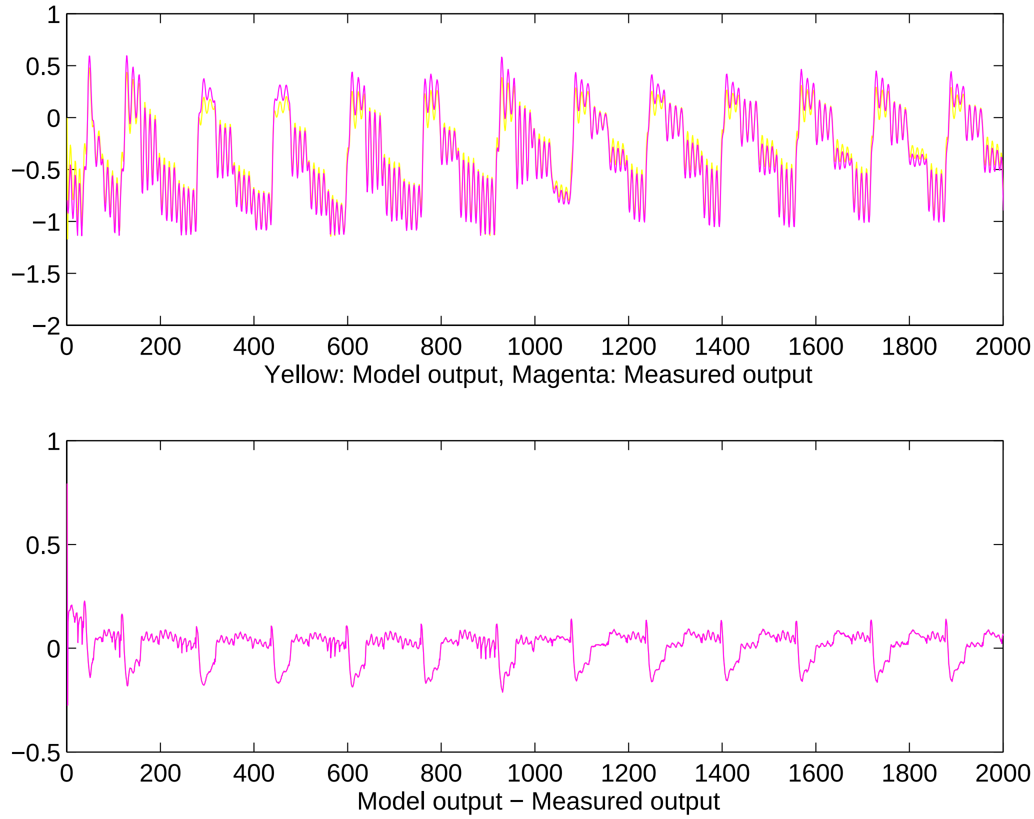

Figure 2: Results of identification of the Plant with the Gauss-Newton method. Top: Model output vs. system output. Bottom: Error

The obtained results of relative RMS errors can be summarized in the next table :

	Rel. RMS error
ARX	0.1964
Gauss-Newton	0.06425
RGO	0.03378

References

[1] Armingol, J.M.,Moreno,L.E., Garrido,S., de la Escalera, A.Salichs,M.A.: Mobile Robot Localization Using a Non-Linear Evolutionary Filter.(1998).*3rd IFAC Symp. on Intelligent Autonomous Vehicles(IAV'98).*

[2] Chen, S. and S. A. Billings. (1989). Representations of non-linear systems: the NARMAX model. *Int. J. Control*, *49*, 1013-1032.

[3] Flockton, S.J. and M.J. White. (1993). Pole-zero system identification using genetic algorithms. *Proceedings 5th International Conference on Genetic Algorithms.* 17-21 July, University of Illinois at Urbana Champaign, USA. pp. 531-535.

[4] Garrido,S.,Moreno,L.E.,Balaguer,C.(1998). State Estimation for Nonlinear Systems using Restricted Genetic Optimization. *11th Int. Conference on Industrial and Engineering Applications of Artificial Intelligence and Expert Systems.(IEA-AIE'98)*

[5] Garrido,S.,Moreno,L.E.,Salichs,M.A.(1998). Nonlinear On-line Identification of Dynamic Systems with Restricted Genetic Optimization. *6th European Congress on Intelligent Techniques & Soft Computing. (EUFIT'98)*

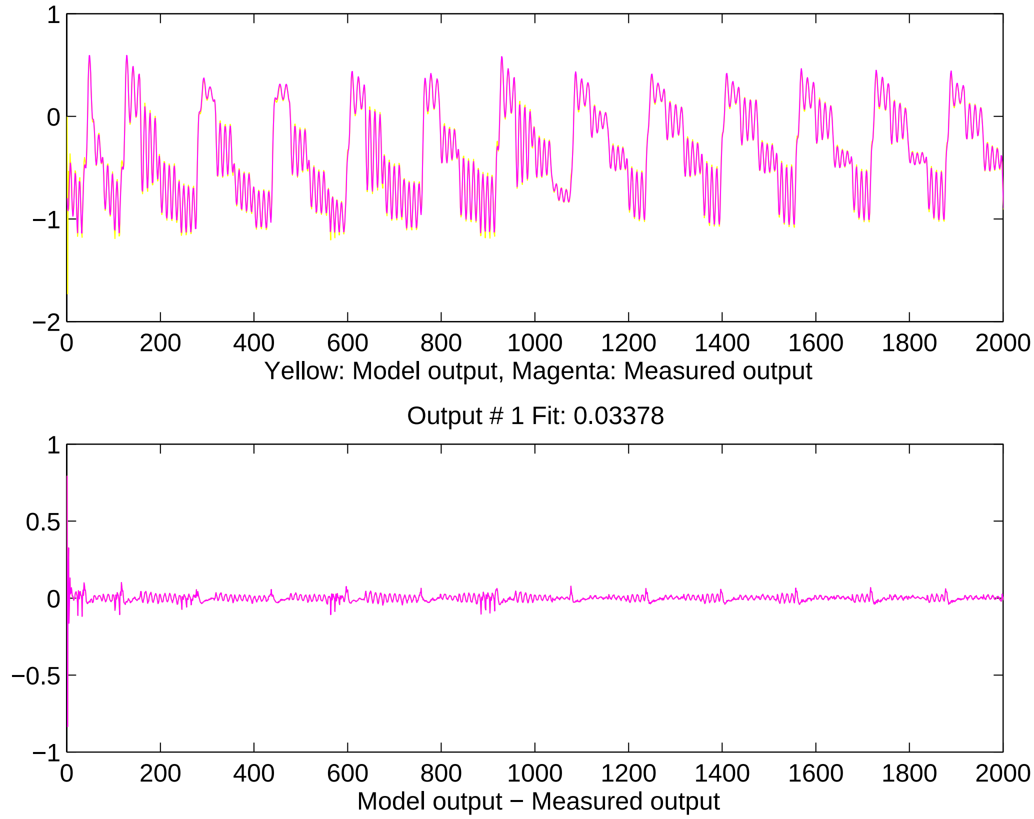

Figure 3: Results of identification of the Plant with the RGO method. Top: Model output vs. system output. Bottom: Error

[6] Iba, H., H. deGaris and T. Sato. (1995). A numerical approach to Genetic Programming for System Identification. *Evolutionary Computation*, *3*, 4, 417-452.

[7] Kristinson, K. and Dumont G. (1992). System identification and control using genetic algorithms. *IEEE Transactions on Systems, Man, and Cybernetics*, *22*(5), 1033-1046.

[8] Ljung, L.(1987) *System Identification: Theory for the User*. Prentice-Hall, Englewood Cliffs, NJ.

[9] Luo,W. and S.A. Billings. (1995). Adaptive model selection and estimation for nonlinear systems using a sliding data window. *Signal Processing*, *46*, 179-202.

[10] van Overbeek,A.J.M. and Ljung,L.(1982). *On-line structure selection for multivariable state space models.* Automatica, **18(5)** 529-543.

[11] Tan, K.C., Y. Li, D. J. Murray-Smith and K.C. Sharman (1995). System identification and linearization using genetic algorithms with simulated annealing. *Proc. 1st IEE/IEEE International Conference on Genetic Algorithms in Engineering Systems: Innovations and Applications*. Sheffield, UK.