THE APPLICATION OF FUZZY METHODS TO SUSTAINABLE DECISION MAKING

M.A. Hersh,
Centre of Systems and Control and Department of Electronics and Electrical Engineering,
University of Glasgow, Glasgow G12 8LT, Scotland.
Tel: +44 141 330 4906. Fax: +44 141 330 6004. Email: m.hersh@elec.gla.ac.uk

ABSTRACT: The concept of an augmented fuzzy set was recently developed by the author to allow representation of the degree of certainty of imprecise and uncertain information, in addition to the information itself. In this work this concept is further extended to additionally allow representation of the degree of importance or priority of the information and the representation of data with different signs. This is particularly relevant in sustainable decision making where both positive and negative factors occur. Rules for calculation of the associated set operations are presented. The methodology is illustrated by application to the evaluation of the case for and against a proposed urban motorway, the M74 northern extension in Glasgow, Scotland.

KEYWORD: Fuzzy sets, set operations, uncertainty, priority, information, decision making, transport.

INTRODUCTION

Sustainable decision making (Hersh, 1998) supports moves towards a sustainable society and is becoming of ever greater importance due to the increasing burden human activity is placing on the natural environment and the rapid pace of technological change. This presents challenges and opportunities, but also the possibility of disaster. Thus sustainable decision making should lead to processes, projects and products which contribute to the developmental needs of humanity, while protecting and conserving the natural environment and maintaining the ability of future generations to meet their own needs. It should also consider possible long term and indirect consequences as well as short term and direct ones and attempt to harmonise the sometimes conflicting goals of environmental integrity, societal development, economic efficiency and equity.

Decision making is highly reliant on the availability of information and the ability to process and represent it. Much of the data required to support the environmental, developmental and equity aspects of sustainable decision making is of a qualitative nature, imprecise, uncertain and incomplete. This makes the use of fuzzy methods particularly appropriate for the analysis and representation of this data.

AUGMENTED FUZZY SETS

Fuzzy sets (Zadeh, 1974, 1975; Zadeh et al, 1975; Klir et al, 1992; Novak et al, 1995) are generally defined in terms of a membership function taking values in the closed interval [0 1]. Although useful for representing imprecise information, a traditional fuzzy set cannot be used to give an indication of its reliability. To allow representation of the degree of certainty with which information holds in addition to the information itself, the author (Hersh et al, 1998ab) introduced and defined the concept of an augmented fuzzy set. This is represented by an ordered pair of numbers in [0, 1] and defined as

$$f: X \to I \times I$$
$$f(x) = (\mu, \nu), \quad \forall x \in X, \quad \mu, \nu \in I$$

The first element in the ordered pair (μ, ν) is a fuzzy set membership function, whereas the second element can be treated either as a fuzzy set membership function or a probability. The normal rules for combination of fuzzy sets have to be modified slightly to take into account the degree of certainty with which the information is known. Full details are given in (Hersh et al 1998ab) and illustrated by applications from life cycle analysis and design in the computer industry (Hersh et al, 1998ab, 1999).

THIRD ORDER AUGMENTED FUZZY SETS

Decision making frequently involves factors of varying degrees of importance. To allow the relative importance or priorities of different variables to be taken into account in addition to the values of the variables and the degree of certainty with which they are known, the author (Hersh, 1999) introduced and defined the concept of a third order augmented fuzzy set. These sets can also be used to investigate the effect on decision making of attaching different degrees of relative importance to the different factors through the use of different weights on the variables. A third order augmented fuzzy set is defined by an ordered triple as follows:

$$f: X \rightarrow I \times I \times I$$
$$f(x) = (\mu, \nu, \pi), \quad \forall x \in X, \ \mu,\nu,\pi \in I$$

The first and third elements μ and π of the ordered triple (μ, ν, π) are fuzzy set membership functions, which represent the extent to which a particular (imprecise) property holds and its degree of priority or importance. The second element ν can be treated as either a fuzzy set membership function or a probability. In the first case it represents the degree of certainty with which the information is known and in the second case the probability of the membership function having the value μ. Only the membership function case will be considered here.

AGGREGATION: 'WEIGHTING APPROACH'

Augmented fuzzy sets can be used to represent, for instance environmental or social impacts at a given product or process life cycle stage. Calculation of the total environmental or social impact requires aggregation of the impact at each stage and/or aggregation of the different categories of impacts at each life cycle stage. There are two different approaches to calculating the set operations. In the first case μ is weighted by π and in the second case all three elements of the triple (μ,ν,π) are treated symmetrically. Only the first case will be considered here, as the use of weighting to define the importance or priority of different factors is particularly appropriate in decision making.

The union of n fuzzy sets with membership functions $f_i = (\mu_i, \nu_i, \pi_i)$ (i=1...n) has membership function

$$f_{sup} = \bigvee_{i=1}^{n} f_i : X \rightarrow I \times I \times I$$

$$f_{sup} = (\mu_{sup}, \nu_{sup}, \pi_{sup}), \quad \forall x \in X, \ \mu_{sup}, \nu_{sup}, \pi_{sup} \in I$$

with

(a) $\mu_{sup} = \mu'_u / \pi'_u, \quad \nu_{sup} = \nu_u, \quad \pi_{sup} = \pi_u$

if (ai) $\mu'_u + \delta \geq \mu'_h$ (aii) $\nu_u - \varepsilon \geq \nu_h$

(b) $\mu_{sup} = \mu'_m / \pi'_m, \ \nu_{sup} = \nu_m, \ \pi_{sup} = \pi_m$,

if (ai) but not (aii) is satisfied, (bi) $\mu'_m + \delta \geq \mu'_h$, (bii) $\nu_m - \varepsilon \geq \nu_h$

(c) $\mu_{sup} = \mu'_h / \pi'_h, \ \nu_{sup} = \nu_h, \ \pi_{sup} = \pi_h$ otherwise

where δ and ε are given and

$$h = \min j \in J, \quad J = \{j: \nu_j = \sup_{k \in K} \nu_k\}, \quad K = \{k: \mu'_k = \sup_{i=1...n} \mu'_i\}$$

$$m = \min p \in P, \quad P = \{p: \mu'_p = \sup_{q \in Q} \mu'_q\}, \quad Q = \{q: \nu_q = \sup_{i=1...n} \nu_i\}$$

$$u = \min w \in W, \quad W = \{w: \mu'_w = \sup_{z \in Z} \mu'_z\}, \quad Z = \{z: \mu'_z \nu_z = \sup_{i=1...n} (\mu'_i \nu_i)\}$$

$$\mu'_i = \mu_i \pi'_i, \quad \pi'_i = \frac{\pi_i}{\sum_{i=1}^{n} \pi_i}, \quad i = 1, \ldots, n$$

This approach to the definition of the union operator is similar for the first two elements of the triple to that for the second order augmented fuzzy set (Hersh et al, 1998ab) with each value of μ weighted according to its importance and is analogous to maximisation of a weighted function of two variables.

The constants δ and ε can be chosen by the user to determine the relative importance of maximising values, the certainty with which they are known or both values and certainty. A number of special cases are likely to be of interest. Taking $\delta = 0$, $0 \le \varepsilon \le 1$ gives the normal fuzzy set operator with value $\mu'_{sup} = \sup_{i=1\ldots n} \mu'_i$. The associated certainty ν_{sup} and importance π_{sup} can then be obtained as $\nu_{sup} = \sup_{j\in J} \nu_j$, $\pi_{sup} = \sup_{j\in J} \pi_j$ for $J = \{j: \mu'_j = \sup_{i=1\ldots n} \mu'_i\}$, so that $\mu_{sup} = \mu'_{sup}/\pi_{sup}$. Conversely choice of $\delta = 1$ means that the μ' condition is always satisfied and only maximisation of the certainty is required. Although the third element π_j of the ordered triple which defines a third order augmented fuzzy set is not used directly in the maximisation from which the union operator is obtained, the value of π_{sup} still has to be calculated so μ_{sup} can be obtained from μ'_{sup}.

AUGMENTED FUZZY SETS WITH NEGATIVE COEFFICIENTS

In decision making both positive and negative impacts, generally referred to as benefits and costs, have to be considered. However, since (augmented) fuzzy sets only take values between 0 and 1, they can only be used to measure either costs or benefits, but not both together. This problem can be resolved by an alternative definition in which the first element of the augmented fuzzy set is allowed to take values between -1 and 1. The definition of the aggregation will then also require modification. Thus a third order augmented fuzzy set with its first element allowed to take negative values can be defined as follows:

$$f: X \to I_\pm \times I \times I$$
$$f(x) = (\mu, \nu, \pi), \quad \forall x \in X, \ \mu \in I_\pm, \nu, \pi \in I, \ \text{where } I_\pm = [-1, 1]$$

The union of n fuzzy sets with membership functions $f_i = (\mu_i, \nu_i, \pi_i)$ (i=1...n) is obtained by combining the fuzzy sets with positive first and negative first elements separately to give the fuzzy sets $f_+ = (\mu_+, \nu_+, \pi'_+)$ and $f_- = (\mu_-, \nu_-, \pi'_-)$ respectively and then combining these two fuzzy sets, taking into account the signs of μ_+ and μ_-. Thus

$$f_{sup} = f_+ \vee f_-, \ X \to I_\pm \times I \times I$$
$$f_{sup} = (\mu_{sup}, \nu_{sup}, \pi_{sup}), \quad \forall x \in X, \ \mu_{sup} \in I_\pm, \ \nu_{sup}, \pi_{sup} \in I$$

$$\mu_{sup} = \frac{\mu_+\pi_+ + \mu_-\pi_-}{\pi_+ + \pi_-}, \qquad \nu_{sup} = \frac{\nu_+\pi_+ + \nu_-\pi_-}{\pi_+ + \pi_-}$$

$$f_+ = \bigvee_{j\in J_+} f_j: \ X \to I \times I \times I \ \text{ for } J_+ = \{j: \mu_j \ge 0\}$$

$$f_+ = (\mu_+, \nu_+, \pi'_+) = \left(\mu_{\sup_{j\in J_+}}, \ \nu_{\sup_{j\in J_+}}, \ \pi_{\sup_{j\in J_+}}\right), \quad \forall x \in X, \ \mu_+, \nu_+, \pi'_+ \in I$$

$$f_- = \bigvee_{j\in J_-}^{n} f_i: \ X \to I_- \times I \times I, \quad \text{for } J_- = \{j: \mu_j \le 0\}$$

$$f_- = (\mu_-, \nu_-, \pi'_-) = \left(-\mu^*_{\sup_{j\in J_-}}, \ \nu_{\sup_{j\in J_-}}, \ \pi_{\sup_{j\in J_-}}\right), \quad \forall x \in X, \ \mu_- \in I, \ \nu_-, \pi_- \in I_- \ \text{ for } I_- = [-1, 0], \ \mu^*_j = -\mu_j$$

where $\left(\mu_{\substack{\sup \\ j\in J_+}}, \ \nu_{\substack{\sup \\ j\in J_+}}, \ \pi_{\substack{\sup \\ j\in J_+}}\right)$ and $\left(-\mu^*_{\substack{\sup \\ j\in J_-}}, \ \nu_{\substack{\sup \\ j\in J_-}}, \ \pi_{\substack{\sup \\ j\in J_-}}\right)$ are as defined in the previous section for the fuzzy sets with the first element μ positive and negative respectively.

The priorities π_+ and π_- used in combining the fuzzy sets of positive and negative values or costs and benefits are chosen by the user and depend on the perceived relative importance of costs and benefits. They are not necessarily equal to π'_+ and π'_-, as indicated by the notation, though they may be. The value of π_{sup} has deliberately not been defined, as its value is only relevant if the fuzzy set is further aggregated and in that case it is most appropriately chosen by the user according to its priority relative to the variables it is aggregated with.

ILLUSTRATION: THE CASE FOR AND AGAINST THE M74 NORTHERN EXTENSION, A GLASGOW URBAN MOTORWAY

To illustrate the methodology the case for and against building the M74 northern extension, an urban motorway on the south side of Glasgow (Scotland) which is intended to link the city centre into the existing motorway network, will be considered. This motorway extension is supported by Glasgow City Council and some local businesses, particularly those involved in road freight or electronics, but opposed by residents throughout the city, particularly in the area the road will go through, environmental groups and businesses and the local council in the area affected by the road. The planning process and limited consultation exercises concentrated on a particular route and did not investigate alternatives to building the road.

The methodology developed in the previous section will be applied to this decision problem. Since this example is intended to be illustrative rather than an in depth analysis, only three options will be considered. These are: construction of the M74, maintaining the status quo and an integrated package of measures, including the closure of urban motorways in the Glasgow conurbation and improvements in the cycling network and public transport facilities, intended to give a model shift away from private car use to public transport, cycling and walking and from road freight to rail or mixed road rail freight. It should be noted that each of these options would be in addition to the 'route action plans', that the cycle network already agreed on by Glasgow City Council has still to be fully implemented and that there may be some interaction between the two sets of measures. Although there are studies of the effects of building the M74 northern extension, available data is limited, as well as uncertain, imprecise, qualitative and incomplete, and numerical data is highly dependent on the assumptions made, making the use of augmented fuzzy sets particularly appropriate.

The main social and environmental variables of interest are travel times, accidents, stress on the Kingston Bridge (a motorway bridge over the river Clyde), noise, community separation, energy consumption employment, energy consumption, economic development, equity, costs, traffic generation and emissions. These variables give the space I = $\{x_1 - x_{12}\}$ of environmental and social impacts to be considered. Many of the relevant impacts are the changes in these variables, as in the case of travel time savings (or losses), rather than their absolute values. This will result in a mixture of positive and negative changes, frequently called benefits and costs and can be represented by the modification to (augmented) fuzzy set techniques discussed in the previous section to allow the variables to take values between -1 and 1, rather than between 0 and 1. The two life cycle stages of construction and use will be considered for each option, with the total impact in each category for each option equal to the sum of the impacts for the two life cycle stages.

When appropriate studies have been carried out, which is not the case here, it is generally possible to obtain reasonably accurate measures of the relative impacts of the different options for each category of interest. These relative measures can then be converted to fuzzy set values using either local or global scaling. Global scaling has the advantage of giving a measure of the overall severity of the impact, though there may be insufficient data to do this. If global scaling is not used then scaling of the severity of the different categories of impacts relative to each other is required to allow them to be combined. To give good discrimination and make full use of the range [0 1] it may be necessary to use a multiplying factor when global scaling is used, but this will not be done here.

The fuzzy sets for M74 construction, maintaining the status quo and the integrated package of measures are given below. Only values of the first two coefficients μ and ν are given to allow the effects of changing the value of priority π on the decision reached to be investigated and different values of π to be used when aggregating the two life cycle stages or the different categories of impact for each alternative. Since the data is presented from the point of view of the negative environmental and social impacts of the different options, increasing positive values correspond to increasing degrees of impact and amelioration of the impact is indicated by negative values.

M74 Northern Extension: Construction:

x	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}
μ	0.2	0.05	0.3	0.8	0.5	0.3	-0.1	-0.2	0.8	1	0.1	0.3
ν	0.3	0.2	0.2	0.7	0.5	0.3	0.3	0.5	0.7	1	0.3	0.4

M74 Northern Extension: Use

x	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}
μ	-0.2	-0.3	0.7	0.7	0.7	0.7	0.1	0.1	0.8	0.3	0.8	0.9
ν	0.2	0.3	0.3	0.6	0.6	0.5	0.2	0.2	0.4	0.5	0.5	0.7

Status Quo: Construction

x	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}
μ	0	0	0	0	0	0	0	0	0	0	0	0
ν	1	1	1	1	1	1	1	1	1	1	1	1

Status Quo: Use

x	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}
μ	0.2	0.1	0.3	0.3	0	0.3	0	0	0.2	0.1	0.2	0.5
ν	0.2	0.2	0.3	0.5	0.8	0.3	0.7	0.7	0.5	0.5	0.5	0.5

Integrated Package of Measures: Construction

x	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}
μ	0.1	0	0	0.1	0	0.1	-0.1	-0.1	-0.6	0.1	0	0
ν	0.3	0.3	0.4	0.2	1	0.7	0.4	0.5	0.6	0.4	0.8	0.7

Integrated Package of Measures: Use

x	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}
μ	-0.4	-0.6	-0.8	-0.8	-0.6	-0.7	-0.4	-0.4	-0.8	0.05	-0.8	-0.7
ν	0.2	0.4	0.5	0.5	0.5	0.5	0.3	0.3	0.5	0.6	0.5	0.5

The fuzzy sets for construction and use can be combined to give an overall fuzzy set for each option. Taking $\pi = 0.3$ for construction and $\pi = 0.7$ for use in all cases and $\delta = \varepsilon = 0.1$ gives the following fuzzy sets:

M74 Northern Extension

x	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}
μ	-0.08	-0.195	0.7	0.8	0.7	0.7	0.04	0.01	0.8	1	0.8	0.9
ν	0.23	0.27	0.3	0.7	0.6	0.5	0.23	0.29	0.7	1	0.5	0.7

Status Quo

x	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}
μ	0.2	0.1	0.3	0.3	0	0.3	0	0	0.2	0.1	0.2	0.5
ν	0.2	0.2	0.3	0.5	0.8	0.3	0.7	0.7	0.5	0.5	0.5	0.5

Integrated Package of Measures

x	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}
μ	-0.25	-0.6	-0.8	-0.53	-0.6	-0.46	-0.4	-0.4	-0.8	0.1	-0.8	-0.7
ν	0.23	0.4	0.5	0.41	0.5	0.56	0.3	0.3	0.5	0.4	0.5	0.5

The data for the three options can be compared in a number of different ways. The fuzzy sets as presented could form the basis of comparison or the data could be aggregated, for instance to give a smaller number of categories of impacts, aggregate values of 'benefits' and 'costs' or an overall estimate of the impact, including the associated degree of certainty of the information, for each option. However it should be noted that there are tradeoffs between quality of information and simplification. Thus, although use of one overall index can greatly simplify comparison of different options, it considerably reduces the quality of the information and does not allow differentiation of different types of data. The overall impact will first be calculated for the first option. The fuzzy set for this option, including relative priorities for the different impacts, is presented below.

M74 Northern Extension

x	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}
μ	-0.08	-0.195	0.7	0.8	0.7	0.7	0.04	0.01	0.8	1	0.8	0.9
ν	0.23	0.27	0.3	0.7	0.6	0.5	0.23	0.29	0.7	1	0.5	0.7
π	0.3	0.9	0.6	0.9	0.7	0.9	0.9	0.7	0.8	0.6	0.9	0.9

For ease of evaluation a table of values of $\mu'=\mu_i\pi_i$ (where the factor $\sum_{i=1}^{n}\pi_i$ can be omitted as it is common to all terms), ν_i, and π_I is presented below:

M74 Northern Extension

x	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}
μ'	-0.8	-0.18	0.42	0.72	0.49	0.63	0.036	0.007	0.64	0.6	0.72	0.81
ν	0.23	0.27	0.3	0.7	0.6	0.5	0.23	0.29	0.7	1	0.5	0.7
π	0.3	0.9	0.6	0.9	0.7	0.9	0.9	0.7	0.8	0.6	0.9	0.9

The first two variables x_1 and x_2 have negative values and can therefore be combined to give (μ_-, ν_-). For all values of δ and ε, combination of these impacts gives $\mu'_- = -0.18$, $\nu_- = 0.27$, $\pi'_- = 0.9$, so that $\mu_- = -0.2$

The ten impacts x_3 - x_{12} with positive values can now be combined in a similar way. For x_3 - x_{12}

$\mu'_h = \mu'_{12} = 0.81$, $\nu_h = \nu_{12} = 0.7$, $\pi_h = \pi_{12} = 0.9$
$\mu'_m = \mu'_u = \mu'_{10} = 0.6$, $\nu_m = \nu_u = \nu_{10} = 1$, $\pi_m = \pi_u = \pi_{10} = 0.6$

Therefore for $\delta = \varepsilon = 0.1$ conditions (ai), (aii), (bi) and (bii) are not satisfied and so
$\mu'_+ = 0.81$, $\nu_+ = 0.7$, $\pi'_+ = 0.9$, so that $\mu_+ = 0.9$

When the positive and negative impacts are equally weighted i.e. $\pi_+ = \pi_- = 0.5$, combination of (μ_+,ν_+) and (μ_-,ν_-) then gives $\mu_{sup} = 0.35$, $\nu_{sup} = 0.49$. Since ten of the impacts have positive values, more appropriate weights could be $\pi_+ = 0.9$, $\pi_- = 0.1$, giving $\mu_{sup} = 0.79$, $\nu_{sup} = 0.66$ or $\pi_+ = 0.7$, $\pi_- = 0.3$, giving $\mu_{sup} = 0.57$, $\nu_{sup} = 0.57$

The impacts for the other two options can be combined in a similar way, using the same weights or priorities for each impact to give a fair comparison. Tables of μ', ν_i and π_i for the status quo and integrated package of measures options are given below:

Status Quo

x	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}
μ'	0.06	0.09	0.18	0.27	0	0.27	0	0	0.16	0.06	0.18	0.45
ν	0.2	0.2	0.3	0.5	0.8	0.3	0.7	0.7	0.5	0.5	0.5	0.5
π	0.3	0.9	0.6	0.9	0.7	0.9	0.9	0.7	0.8	0.6	0.9	0.9

Integrated Package of Measures

x	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}
μ'	-0.08	-0.54	-0.48	-0.48	-0.42	-0.41	-0.36	-0.28	-0.64	0.06	-0.72	-0.63
ν	0.23	0.4	0.5	0.41	0.5	0.56	0.3	0.3	0.5	0.4	0.5	0.5
π	0.3	0.9	0.6	0.9	0.7	0.9	0.9	0.7	0.8	0.6	0.9	0.9

For the status quo option, all values are positive and therefore $\mu_- = 0$, $\nu_- = 1$
$\mu'_h = \mu'_u = \mu'_{12} = 0.45$, $\nu_h = \nu_u = \nu_{12} = 0.5$, $\pi_h = \pi_u = \pi_{12} = 0.9$
$\mu'_m = \mu'_5 = 0$, $\nu_m = \nu_5 = 0.8$, $\pi_u = \pi_5 = 0.7$

Therefore for all values of δ and ε conditions (ai) and (aii) are satisfied and

$\mu'_+ = 0.45$, $\nu_+ = 0.5$, $\pi'_+ = 0.9$, so that $\mu_+ = 0.5$

For the integrated package of measures option, only x_{10} has a positive value, so that
$\mu'_+ = \mu'_{10} = 0.06$, $\nu_+ = \nu_{10} = 0.4$, $\pi'_+ = \pi_{10} = 0.6$, so that $\mu_+ = 0.1$

The remaining eleven variables can now be aggregated, using $\mu^*_i = -\mu^*_i$, $\mu'^*_i = -\mu_i\pi_i$

$\mu'^*_h = \mu'^*_u = \mu'^*_{11} = 0.72$, $\nu_h = \nu_u = \nu_{11} = 0.5$, $\pi_h = \pi_u = \pi_{11} = 0.9$
$\mu'^*_m = \mu'^*_6 = 0.41$, $\nu_m = \nu_6 = 0.56$, $\pi_m = \pi_6 = 0.9$

Therefore for all values of δ and ε conditions (ai) and (aii) are satisfied and

$\mu'_- = -\mu'^*_{11} = -0.72$, $\nu_- = \nu_u = \nu_{11} = 0.5$, $\pi'_- = \pi_{11} = 0.9$, so that $\mu_- = -0.8$

When the positive and negative impacts are equally weighted i.e. $\pi_+ = \pi_- = 0.5$, combination of (μ_+,ν_+) and (μ_-,ν_-) then gives $\mu_{sup} = -0.35$, $\nu_{sup} = 0.45$. However, since the majority of the impacts are negative a more appropriate choice of weights would be $\pi_+ = 0.1$, $\pi_- = 0.9$, giving $\mu_{sup} = -0.71$, $\nu_{sup} = 0.49$

Values of μ_{sup} and ν_{sup} are tabulated below for the three options, denoted a, b and c, for six different sets of values of π_+ and π_-

Values of π_+ and π_-	$(\mu_{a.sup}, \nu_{a.sup})$	$(\mu_{b.sup}, \nu_{b.sup})$	$(\mu_{c.sup}, \nu_{c.sup})$
$\pi_+=0.9$, $\pi_-=0.1$	(0.79, 0.66)	(0.45, 0.55)	(0.01, 0.41)
$\pi_+=0.7$, $\pi_-=0.3$	(0.57, 0.57)	(0.35, 0.65)	(-0.17, 0.43)
$\pi_+=0.5$, $\pi_-=0.5$	(0.35, 0.49)	(0.25, 0.75)	(-0.35, 0.45)
$\pi_+=0.3$, $\pi_-=0.7$	(0.13, 0.40)	(0.15, 0.85)	(-0.53, 0.47)
$\pi_+=0.1$, $\pi_-=0.9$	(-0.09, 0.31)	(0.05, 0.95)	(-0.71, 0.49)
$\pi_+=0$, $\pi_-=1$	(-0.2, 0.27)	(0, 1)	(-0.8, 0.5)

Thus this example illustrates the way the use of priority values or weights effects the decision making process. In this case the integrated package of measures is clearly better than the other options for all values of π_+ and π_- or relative weightings of 'costs' and 'benefits'. The option of maintaining the status quo is better than that of building the M74 northern extension, except for the case of $\pi_+ = 0$ and $\pi_- = 1$ i.e. when the disadvantages, such as the the environmental costs, are totally ignored.

CONCLUSIONS

Ordinary fuzzy sets allow the representation of uncertain information, but give no indication of its reliability or the relative importance of different items of data. Second order augmented fuzzy sets allow the representation of the degree of certainty of the information and third order augmented fuzzy sets in addition allow representation of the degree of priority of the data. This is particularly useful in sustainable decision making. The definition of third order augmented fuzzy sets has been further extended to allow the first element to take values between -1 and 1, permitting the representation of both positive and negative data.. This is useful in decision making, where it is frequently required to represent both advantages and disadvantages or costs and benefits. Set combination rules have been derived and presented for this case. The use of third order augmented fuzzy sets in decision making and the effects of changing the relative importance of different factors on the decision have been illustrated by the example of the M74 northern extension, a proposed urban motorway in Glasgow, Scotland.

REFERENCES

Hersh, M.A., 1998, "Sustainable Decision Making and Decision Support Systems", Computing and Control, vol. 9, pp. 289-295.

Hersh, M.A. and Hamburg, I., 1998a, "Fuzzy approaches to 'green design' with examples from the computer industry" University of Glasgow Control Systems Centre Report.

Hersh, M.A., Hamburg, I. and P. Hamburg, 1998b, "The Application of Fuzzy Methods to Environmentally Compatible Production in Manufacturing Industry", Analele Univ. Craiova.

Hersh, M.A. and Hamburg, I., 1999, "Fuzzy Logic and Green Design: A New Approach", CIMCA ’99, Vienna, Austria.

Hersh, M.A., 1999, "Augmented fuzzy sets: applications to sustainable design and design making", University of Glasgow Control Systems Centre Report.

Klir, G.J. and Yuan, B., 1995, 2Fuzzy sets and fuzzy logic, theory and applications", Prentice Hall, 1995

Novák, V. et al (eds.), 1992, "Fuzzy approach to reasoning and decision-making", Kluwer academic publishers.

Zadeh, L.A. 1974, "The concept of a linguistic variable and its application to approximation reasoning" I and II, Information sciences, vol. 8 pp. 199-249, 301-375.

Zadeh, L.A. 1975, "The concept of a linguistic variable and its application to approximation reasoning" III, Information sciences, vol. 9 pp. 43-79

Zadeh, L.A. K.-S. Fu, K. Tanaka and M. Shimura (eds.), 1975, "Fuzzy sets and their applications to cognitive and decision processes", Academic Press Inc.