Local Model Networks for Nonlinear Predictive Control

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ABSTRACT: This paper uses a Local Model Network (LMN) to identify a highly nonlinear chemical process. The LMN is constructed from local Auto Regressive with eXternal input (ARX) models, and is trained using a hybrid learning approach. This LMN structure is exploited for long-range nonlinear predictive control, within a Dynamic Matrix Control (DMC) framework. The internal linear model representation of the nonlinear plant for conventional DMC is replaced by the LMN, which introduces transparency while offering distinct advantages for nonlinear model-based control. Simulation results for a pH neutralisation process are used to illustrate the performance benefits of LMN's for two novel nonlinear Dynamic Matrix Control schemes. The problem of applying both input and output constraints to the nonlinear controllers is discussed.

KEYWORDS: Process control; Predictive control; Nonlinear identification; Nonlinear control; Neural Networks.

INTRODUCTION

Since the early 1980s [Cutler (1980), Richalet (1978)], a number of alternative model based predictive control (MBPC) strategies have emerged, including Dynamic Matrix Control which utilises a step response model in the control law and Generalized Predictive Control (GPC) [Clarke (1987)] which incorporates a linear Controlled Auto Regressive with Integrated Moving Average (CARIMA) plant model. The strength of research interest within MBPC has significantly increased in the past decade with a theoretical basis beginning to emerge [Morari (1999)].

If the process is linear, with no constraints and the desired process output is constant for the foreseeable horizon, then all of the controllers generally yield approximately the same result. That is to say, all of these linear control laws have the same structure with a sufficient number of degrees of freedom (after some manipulation) [Soeterboek (1990)]. However, since control relies on a linear model of the process generated at a particular operating point, the controller's internal model will be less representative of the dynamics of a nonlinear process as the process moves away from this point. This in turn reduces the robustness of the closed-loop system.

Recently, there has been much interest in studying the capabilities of neural networks, such as Multi-layer Perceptrons (MLPs), Radial Basis Functions (RBFs) and B-Splines, for non-linear modelling and identification of dynamical systems. Progress on understanding the principles, relating to issues including training, generalisation and validation, have been accompanied by industrial applications. Work has also been done on replacing the linear models of model-based control systems by neural models in an attempt to accrue control benefits from improved plant representations [Irwin (1995)].

While of undoubted value for non-linear identification and control of dynamical systems, neural networks have a number of limitations for practical applications. Thus, in online training, due consideration must be given to the necessity for regularisation with noisy data and to the choice of network architecture. More fundamentally, the non-transparent, black-box nature of neural models make it difficult to include 'a priori' system information, and to interpret the final structure meaningfully in terms of physical process characteristics. Neural approaches also fail to exploit the significant body of theoretical results available for conventional model-based control, making it difficult to analyse the closed-loop behaviour in terms of stability and robustness.

The aim of this paper is to describe a non-linear modelling architecture, called the local model network (LMN), which introduces transparency while offering distinct advantages for non-linear model-based control. Simulation results for a pH neutralisation process are used to illustrate the performance benefits of LMNs for two novel non-linear Dynamic Matrix Control (DMC) schemes. The problem of applying both input and output constraints to the nonlinear controllers is discussed.

LOCAL MODEL NETWORKS (LMNs)

In contrast to the neural networks such as the Multi-layer Perceptron (MLP) or Radial Basis Function (RBF), the Local Model Network forms a global plant model from a set of locally valid sub-models [Johansen (1993)]. The general feedforward structure of the Local Model Network contains sub-models that could be neural networks or linear plant models. Linear models and *a priori* information from a physical modelling exercise can easily be incorporated within this structure. The outputs of each sub-model are passed through a local processing function that effectively acts to generate a window of validity for the model in question. These nonlinear weighting functions utilise only a subset of the available modelling data to generate the desired partitioning of the model space. The resultant localised outputs are then combined as a weighted sum at the model output node.

The LMN itself is a generalised form of RBF neural network. The basis functions in this instance are used to weight general functions of the inputs as opposed to simply the weights. The network output can be described as:

$$\hat{y}(k+1) = F(\underline{\psi}(k), \underline{\phi}(k)) = \sum_{i=1}^{M} f_i(\underline{\psi}(k))\rho_i(\underline{\phi}(k)) \qquad (1)$$

Here the M local models $f_i(\psi)$ are linear or non-linear functions of the measurement vector $\underline{\psi}$, and are multiplied by a basis function $\rho_i(\underline{\phi})$ that is a function of the current operating region vector, $\underline{\phi}$. The latter does not necessarily need to be the full model input vector, $\underline{\psi}$, but can be a subset of the measurement data available. For comparison note that in the RBF neural network, the functions $f_i(\underline{\psi})$ are constants and the basis functions $\rho_i(\underline{\phi})$ are radial. The basis functions $\rho_i(\underline{\phi})$ in equation (1) are commonly chosen to be normalised Gaussian functions where c_i and σ_i are the widths and centres respectively, i.e.

$$\rho_i(\underline{\phi}) = \frac{exp(-\|\underline{\phi} - c_i\|^2 / 2\sigma_i^2)}{\sum_{i=1}^{M} exp(-\|\underline{\phi} - c_i\|^2 / 2\sigma_i^2)} \qquad i = 1,2,...M \qquad (2)$$

This function gives a value close to 1 in parts of $\underline{\phi}$ where the local f_i is a good approximation to the unknown F, and a value of 0 elsewhere. If, for example, the local models are of the ARX form,

$$f_i(\underline{\psi}) = b_{oi}u(k) + b_{1i}u(k-1) + ... + b_{si}u(k-s) + ... + a_{1i}y(k) + a_{2i}y(k-1) + ... + a_{(r+1)i}y(k-r) \qquad (3)$$

where *r*, *s* are the orders of *y(k)* and *u(k)* respectively, then the local model network constitutes a nonlinear ARX plant model as follows:

$$\hat{y}(k+1) = \hat{F}(\underline{\psi}(k), \underline{\phi}(k)); \quad \underline{\psi}(k) = [y(k), y(k-1), ..., y(k-r), ...u(k), u(k-1), ..., u(k-s)]^T; \quad \underline{\phi}(k) \subset \underline{\psi}(k) \qquad (4)$$

Combining equations (1) and (4) yields the following expansion:

$$\hat{y}(k+1) = B_0u(k) + B_1u(k-1) + ... + B_su(k-s) + ... + A_1y(k) + A_2y(k-1) + ... + A_{(r+1)}y(k-r) \qquad (5)$$

The resulting model is still ARX in structure, with the parameters A_i and B_i defined at each operating point, $\underline{\phi}(k)$, by equation (6).

$$B_i = \sum_{j=1}^{M} \rho_j(\underline{\phi}(k))b_{ij} \quad A_i = \sum_{j=1}^{M} \rho_j(\underline{\phi}(k))a_{ij} \qquad (6)$$

The underlying assumption in the local modelling strategy is that the plant to be modelled undergoes significant changes in operating conditions. For most batch and continuous processes in the chemical, biotechnological and power industries, definite regimes can be identified during procedures such as start-up, shut-down and product shifts. Incorporating simpler models in each operating region can reduce the complexity of the overall model. For example, local state-space and ARMAX models can be formed using localised perturbation signals and then interpolated to give global non-linear state-space and NARMAX (non-linear ARMAX) models [Johansen (1993)].

The identification of local operating regimes for an unknown plant is difficult. Any such identification strategy has to take into account the complexity of the target mapping, the representational ability of the local models associated with the basis functions and the availability of the data. The problem is therefore to identify those variables, which describe the system operating behaviour. *A priori* knowledge of the plant can be used at this stage. When little knowledge of the actual regimes exists, however, it may be beneficial to use unsupervised learning methods, such as k-means clustering and nearest neighbours, to give an initial estimate of the normalised Gaussian interpolation regions. These clustering methods are valid in this case since many plants tend to operate in distinct regions. Despite these difficulties, successful applications of the Local Model technique have been reported in the biotechnology and chemical engineering industries[Johansen & Foss (1995)].

As with conventional neural networks, training is a crucial issue for LM networks, since there is the added complexity of identifying the local models as well as the parameters of the interpolation functions. This paper employs a hybrid learning approach for LM networks constructed from ARX local models and normalised Gaussian basis functions [McLoone (1998)]. Singular Value Decomposition is used to identify the local linear models in conjunction with Quasi-Newton optimisation for determining the centres and widths of the interpolation functions. To avoid overtraining problems in nonlinear dynamic modelling with noisy data, the SVD minimises a one-step-ahead prediction error, while the nonlinear optimisation is performed on a model-based error.

THE pH NEUTRALISATION PLANT

The neutralisation of pH represents a highly non-linear process, and offers a suitable case study for the demonstration and evaluation of LMN techniques. Figure 1 shows a schematic of the plant. The process consists of weak concentration acid, base and buffer streams being continuously mixed within a reaction vessel whose effluent pH is measured at a distance from the plant, introducing a time delay. The aim is control the pH value of the outlet stream by varying the inlet base flow rate (Q_2). The acid and buffer flow rates of the process, Q_1 and Q_3 respectively, are controlled using peristaltic pumps. The outlet flow rate (Q_4) is dependent upon the fluid height (h) within the vessel and the position of the outlet valve, which is set manually. Ramping the base flow rate while recording the outlet stream pH value produced an estimate of the process titration curve, as shown in figure 2.

For this plant, the static gain between the base flow rate and the pH (the slope of the titration curve) varies considerably as these variables change. It can be seen that there are five regions in which the gain is nearly constant. Also, within each region, linear identification experiments suggested that a second-order model was sufficient to describe the dynamic process behaviour. A LMN with five, local, second-order, linear ARX models was constructed to give

$$pH(k+1) = \sum f_i(\underline{\Psi}) r_i(\underline{\Phi}) \tag{7}$$

$$\underline{\Psi}(k) = [pH(k), pH(k-1), q_2(k-d), q_2(k-1-d)]^T, \underline{\Phi}(k) = [pH(k), q_2(k-1-d)]^T \tag{8}$$

The titration curve was used to initialise the non-linear activation regions with the operating point defined using both the pH and the base flow rates. Brown *et al.* (1997) gives further details about the pH plant, together with results from the identified LMN model trained on actual data from an experimental pilot plant. Results are now presented from two non-linear controllers based upon this identified LMN representation of the pH plant.

DYNAMIC MATRIX CONTROL

DMC [Luyben (1989)] uses a discrete-time, step-response model of the process to predict the behaviour of the controlled outputs over a finite receding horizon. The overall objective is to calculate the control moves for the plant, without violating any pre-specified constraints. DMC uses only a single step-response model, generated at one particular operating point. Thus, as plant conditions vary due to disturbances or operating point changes, this linear model becomes less representative of the current plant conditions, resulting in degraded closed-loop performance. However, if it were possible to improve the representation of the plant by the internal model, it can be reasonably expected that this would result in better closed-loop performance.

LOCAL MODEL DYNAMIC MATRIX CONTROL (LM-DMC)

The LMN, described previously, forms a global plant model from a set of locally valid sub-models. Since a global plant model is now available a step-response model can be extracted from each of the local linear models. These are passed through the interpolation functions to provide a step-response model for the DMC controller that is linear and locally valid at any particular operating point. The control sequence can then be solved analytically at each sample instant. The new local model structure [Townsend (1998)] is illustrated in figure 3.

This DMC strategy was tested in simulation on the pH neutralisation process described earlier. The performance was compared with conventional linear DMC based on a linear internal model generated at the neutral pH value of 7.0 Figure 4 shows the results obtained from a series of step changes in pH from 4.0 to 10.0, a range over which the process gain varies significantly as was shown in figure 2. The deterioration in the linear DMC is clear, particularly for the pH outlet value of 8.0 which corresponds to a high-gain region. The LM-DMC gave more consistent tracking across the range, albeit with a slightly slower response time in some regions.

System disturbances were introduced by reducing the buffer flow rate (Q3) from its nominal value of 0.55 ml/s at different operating points. Figure 5 shows that the LM-DMC outperformed the conventional DMC since it was able to tolerate a larger disturbance in the base flow rate (0.55 ml/s compared to 0.3ml/s) before large oscillations appeared.

LOCAL CONTROLLER DYNAMIC MATRIX CONTROL (LC-DMC)

This technique is very similar to gain scheduling, which is probably the most commonly used approach to control highly nonlinear systems [Astr m (1989)]. A gain scheduling controller is constructed by interpolating between the members of a family of linear controllers. Simple design, tuning and relatively low computational burden means that this remains a very favourable control strategy amongst practising control engineers.

The LM network consists of a set of locally-valid, sub-models together with an appropriate interpolation function. A controller is now designed about each of the local models. The output of each controller is then passed through the interpolation function which effectively generates a window of validity for each of the individual controllers. The interpolated outputs are then summed and used to supply the control commands to the process. The resultant LC-DMC structure is as shown in figure 6 for the pH application.

This control strategy was again tested in simulation on the pH neutralisation process. These studies again showed an improvement over the conventional DMC controller. Figure 7 shows the tracking performance of the local controller DMC compared with the local model DMC. In this case the local controller shows a faster rise time for each of the steps. Figure 8 shows the disturbance performance of the local controller DMC when the outlet pH value is 9.0. The controller was tested for disturbances of various magnitude. In this case the local controller DMC shows slightly more oscillatory action than the local model DMC.

CONSTRAINTS WITH LM-DMC AND LC-DMC CONTROLLERS

One of the strengths of model based predictive control has to be its ability to include constraints in a systematic fashion within the control algorithm. Constraints come in two forms; *hard constraints* which are usually associated with control increments or control absolute values and *soft constraints* which are usually associated with controlled variables. It is not possible to violate a hard constraint due to the physical mechanics of say an actuator. However it does provide valuable information which may be used by the control algorithm. By comparison soft constraints may violate the limits imposed upon them, since the physical performance of say a controlled variable may not always be accuratly predicted by the internal model used by the control algorithm.

In general, if the cost function is quadratic and subjected to constraints, there exists no analytical solution for the control problem. The most commonly recommended technique to solve this dilemma is quadratic programming (QP) which is a highly computationally intensive technique. Infeasibility may also be a problem as it may not be possible to find a solution which satisfies all of the constraints [Clarke (1994)].

In the previous section, two novel nonlinear controllers were introduced. The LM-DMC controller consists of a single controller whose internal model is regularly updated. It is possible therefore to apply both input and output constraints to this scheme in a manner similar to the normal linear DMC controller.

By comparison the LC-DMC scheme consists of a family of controllers. Input constraints may be applied to each controller and, when the outputs of the controllers are passed through the interpolation function, no constraint will be broken (assuming that a valid solution was found for each). The model uses a 'partition of unity' so this assumption is not invalid. It is however believed that output constraints may not be applied to the LC-DMC controller. Further, each controller of the network has to predict how the controlled variables will react across a future prediction horizon independently from the rest of the controllers in the network. However, there is no way of knowing that when the individual controller outputs are passed through the interpolation function and summed that the resultant input to the plant will not violate an output constraint.

CONCLUSIONS

LMNs introduce transparency into non-linear identification of dynamical systems while offering possibilities for non-linear model-based control, which in this case is Dynamic Matrix Control.

Two novel predictive control schemes were presented and tested in simulation on a pH neutralisation process. The results have shown a significant improvement over the conventional DMC controller.

The LM-DMC strategy requires only one set of tuning parameters. It is therefore necessary to tune this nonlinear controller so that the highest frequency dynamics of the process always take precedence. This is seen to be a limiting factor since, when the pH process moves to an operating point, with a lower gain, the tuning parameters are such that the best control performance may not be achieved.

For this application, the LC-DMC controller consisted of five local DMC controllers, each with a set of tuning parameters. Since the controllers may be tuned independently, this avoids the effect of high frequency dynamics taking precedence. However, tuning these controllers was not easy due to both their number and the effect of the normalised basis functions of the LMN. This may explain why we receive faster rise times but more oscillatory performance when disturbances occur.

The issue of applying constraints to both of the nonlinear controllers is discussed, however this is an area for further work.

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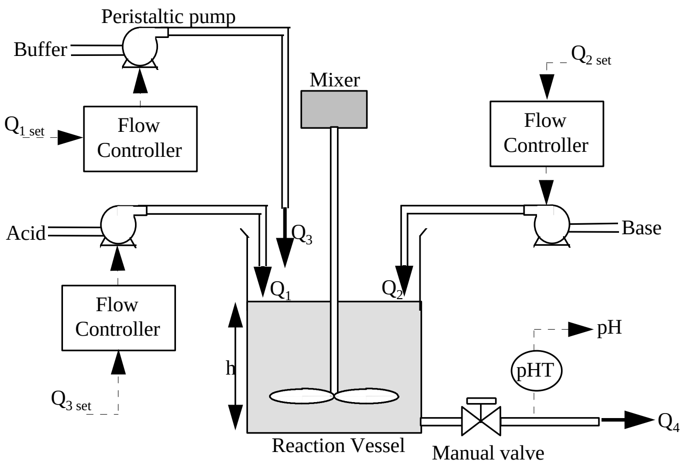

Fig. 1. Schematic of pH neutralisation plant

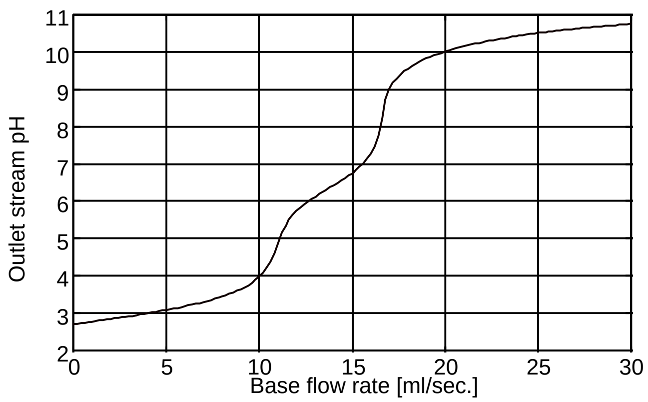

Fig. 2. Titration curve for pH process

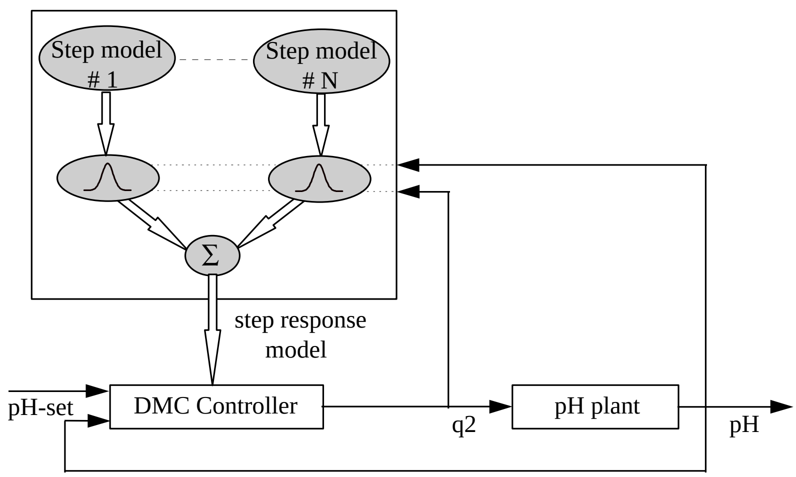

Fig. 3. Local model network based Dynamic Matrix Control (LM-DMC)

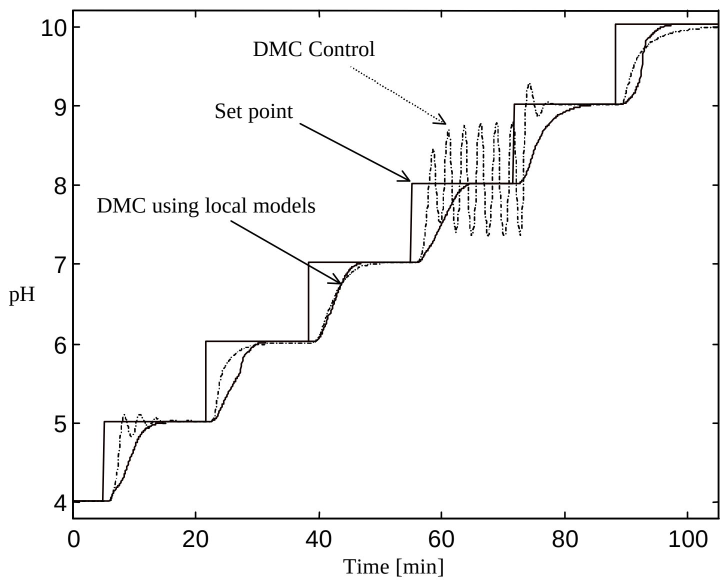

Fig. 4. Set-point tracking for linear and local model based DMC

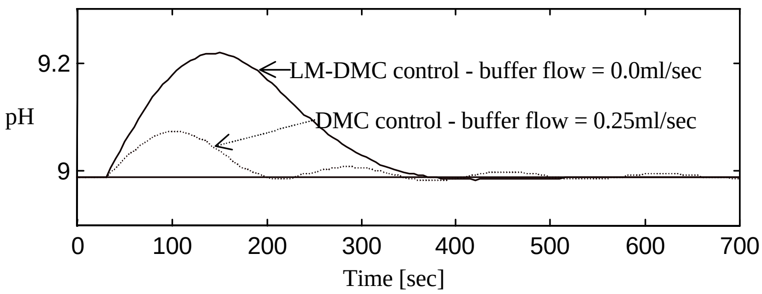

Fig. 5. Disturbance rejection for linear and local model based DMC

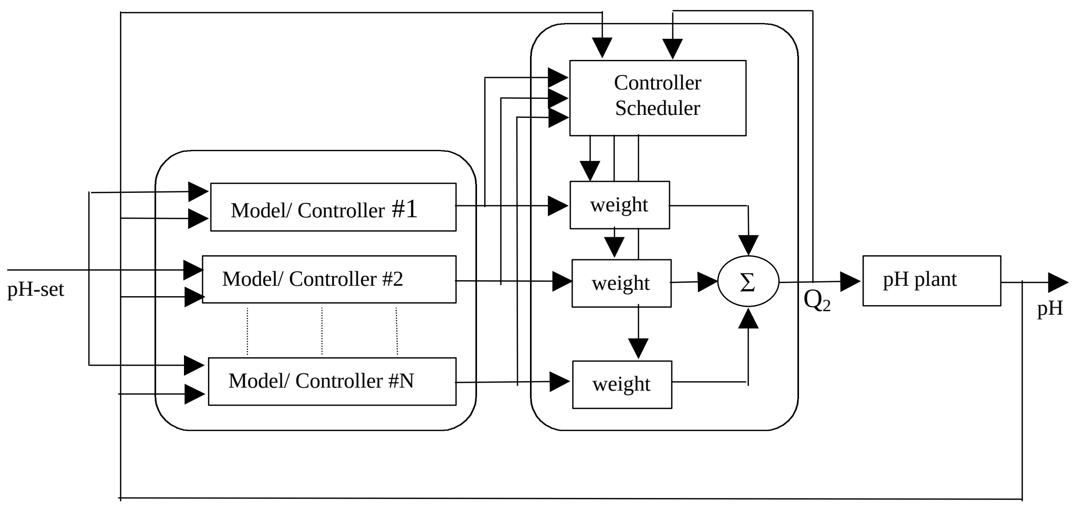

Fig. 6. Local controller network based Dynamic Matrix Control (LC-DMC)

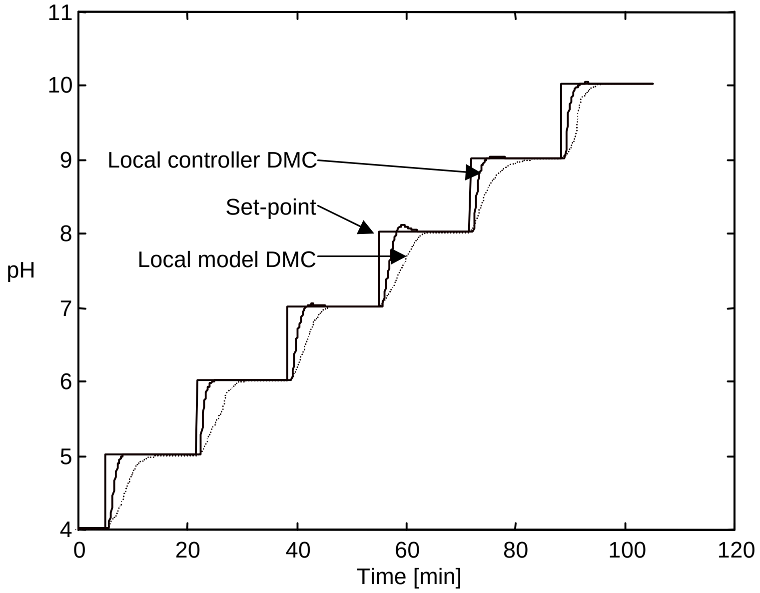

Fig. 7. Set-point tracking for local model based DMC and local controller DMC

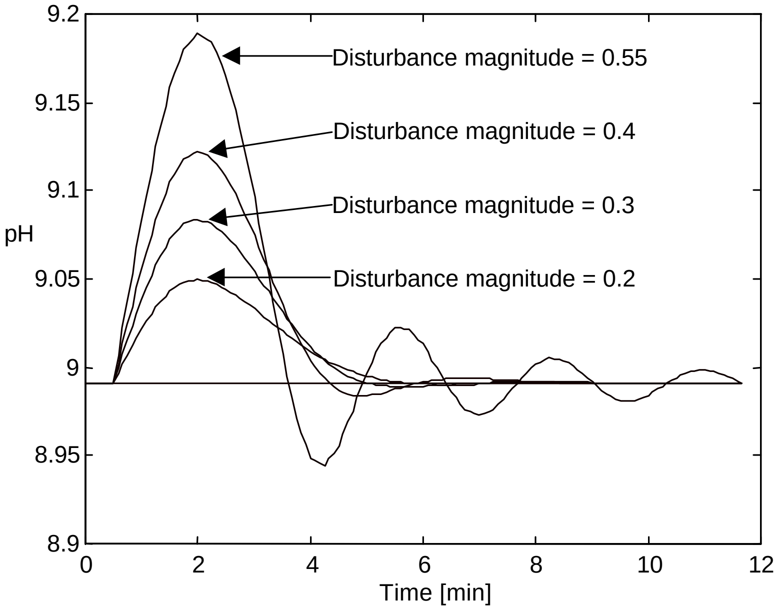

Fig. 8. Disturbance rejection for local controller DMC (LC-DMC)