Stability Analysis of a Sliding-Mode Fuzzy Logic Controller Using Petri Net Modeling and Learning Automata Theory

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ABSTRACT: A hybrid intelligent control system that combines fuzzy logic with variable structure system (VSS) control is presented. This control architecture is viewed as a Discrete Event System and its properties are studied using the corresponding Petri Net Diagram. The equivalence between this controller and learning finite state automata is revealed. The validity of the method is tested through simulation examples.

KEYWORDS : adaptive fuzzy control, sliding mode control, Lyapunov stability theory, finite state automata, Petri Nets, supervisory control, Discrete Event Systems, hybrid intelligent control.

INTRODUCTION

This paper aims to connect the theory of automata and supervisory control of DES with systems theory and fuzzy logic. Previous attempts to bridge the fields of automata and systems theory were done by Kalman et al. (1969) and Ramadge and Wohnam (1989).

A fuzzy logic-based approach to Sliding-Mode Controllers (SMC), which also exploits basic elements of finite state automata, is presented. The control principle is to maintain or change the control action according to the sign of the product $e\dot{e}$ where $e = x - x^d$ is the system error, and the control action can be either an increase or a decrease of the control signal which is realized through the use of fuzzy rules. The proposed controller , called *Reduced Complexity - Sliding Mode Fuzzy Logic Controller* (RC-SMFLC) is characterized by its simplicity, and can facilitate significantly the solution of control problems in the presence of parametric uncertainties and external disturbances (see Tzafestas and Rigatos (1998)) .

It is shown that that the closed-loop system formed by the adaptive fuzzy controller and the plant can be viewed as a *discrete event system* (DES). The control actions are rather event driven than time driven , since a change of the control action is determined by the sign of $e\dot{e}$. The Petri Net model of this discrete event system is derived and its equivalence to a learning finite state automaton is proved. The adaptive fuzzy controller is analyzed from the point of view of formal languages theory.

The mathematical formulation of the Petri Net model is used in the study of the behavior of the adaptive fuzzy controller and helps to ensure that the closed-loop system possesses the following desirable properties : (i) reachability , i.e. no useless information is included in the rule base and the fuzzy controller is able to infer appropriate conclusions from certain plant conditions. (ii) cyclic behavior , i.e. to guarantee that the closed-loop system is not going to get trapped in oscillations between states that are far from the desirable final state , (iii) stability , i.e. to assure the boundedness of the parameters of the plant, and (iv) asymptotic stability , i.e. to guarantee the convergence of the closed loop system to the desirable setpoint.

The conventional controllers (e.g. PID, robust or adaptive control schemes) or even the majority of intelligent controllers (e.g. fuzzy and neural controllers) require knowledge of the error e and the error's first or higher order derivatives, in order to produce the appropriate control signal. On the contrary RC-SMFLC uses only the sign of e and $\dot{e}$ and does not take into account the magnitude of these parameters (see Tzafestas and Rigatos (1998)) . This facilitates the hardware implementation of the controller with the use of flip-flop circuits or Programmable Logic Arrays .

The structure of the paper is as follows : First the concept and the properties of the RC-SMFLC architecture are given. The RC-SMFLC is viewed as a discrete event system and the basic principles of DES analysis are introduced. Then the Petri Net diagram of the closed-loop system is derived and is used to study the properties of reachability , cyclic behavior and asymptotic stability. The equivalence between RC-SMFLC and a learning finite state automaton is proved and an insight of RC-SMFLC is given using formal languages theory. Some evaluation tests of RC-SMFLC are presented. The efficiency of the proposed hybrid intelligent controller is verified in the case of : i) regulation of the geometrical and thermal characteristics of the arc-welding process , and ii) motion control of a mobile robot that moves in a partially unknown environment . Finally the conclusions of the RC-SMFLC analysis are given.

DESIGN OF AN RC-SMFLC

THE CONCEPT

For the design of a *Reduced-Complexity SMFLC* the basic conditions for the convergence of a system to the desirable set-point are taken into account. Consider an n-order nonlinear non-autonomous system

$$x^{(n)}(t) = f(x,t) + b(x,t)u + \tilde{d} \qquad (1)$$

with output $x(t)$ and desirable set-point $x_d(t)$.

The tracking error is $e(t) = x(t) - x_d(t)$ and the rate of error change is $\dot{e}(t) = \dot{x}(t) - \dot{x}_d(t)$.
The convergence conditions are :

$$\text{If} \quad e(t)\dot{e}(t) < 0 \quad \text{then} \quad x(t) \to x_d(t) \Rightarrow e(t) \to 0 \qquad (2)$$

$$\text{If} \quad e(t)\dot{e}(t) > 0 \quad \text{then} \quad x(t) \text{ deviates from } x_d(t) \qquad (3)$$

The same conditions could have been derived from the Lyapunov function $V = \frac{1}{2}e^2 \Rightarrow \dot{V} = e\dot{e}$.

From (2) and (3) it is obvious that the greater part of the information, needed to achieve convergence to the desirable setpoint, is contained in $e(t)$ and $\dot{e}(t)$.
Define now the sliding surface $s(x,t)$

$$s(x,t) = e(t)\dot{e}(t) < 0 \qquad (4)$$

Then the control law can be expressed as follows :

- If $\text{sgn}(e(t)\dot{e}(t)) < 0$, then the control action leads to convergence and should be maintained
- If $\text{sgn}(e(t)\dot{e}(t)) > 0$, then the control action leads to divergence and should be altered.

Once the state vector $[e(t), \dot{e}(t)]^T$ is found in the semi-plane $s(x,t) = e(t)\dot{e}(t) < 0$, it gradually approaches the null vector $[0,0]^T$. Thus, the goal is to find a control law u that will be able to keep the state vector in the semi-plane $s(x,t) = e(t)\dot{e}(t) < 0$.

There two possible control actions : increase or decrease the control signal u . To ensure that the control signal is increased with the use of the FLC the following rules are employed :

IF u_k is U_1 THEN u_{k+1} is U_2
IF u_k is U_2 THEN u_{k+1} is U_3
.
.
IF u_k is U_{n-1} THEN u_{k+1} is U_n

where $U_{1,} U_2$, ……, U_{n-1}, U_n are the fuzzy subsets in which the fuzzy phase plane U of the control input is divided.
The decrease of the control signal is performed via a similar superposition of n-1 fuzzy rules .

If the fuzzy phase plane U is partitioned by n equal membership functions the widths of which remain unchanged , then it can be verified that the above rule base can lead the system to oscillations round the desirable set-point. Consequently, *in order to achieve convergence the nonlinear transfer characteristic of the fuzzy controller should be such that the smaller the distance from the setpoint is the smaller the change of the control signal becomes*.

THE ENHANCED RULE BASE OF THE RC-SMFLC

There are two requirements for the control law in RC-SMFLC :

a) To keep the error state vector $[e(t),\dot{e}(t)]^T$ inside the sliding surface $s(x,t) = e(t)\dot{e}(t) < 0$

b) Its magnitude to be proportional to the distance from the point $\{e = 0, \dot{e} = 0\}$.

The rule-base presented above has to be modified in order to satisfy requirement (b) , i.e. the magnitude of the control signal |u| must decrease when the distance from $\{e = 0, \dot{e} = 0\}$ decreases. If membership functions with widths that remain constant are used, then the distance from $\{e = 0, \dot{e} = 0\}$ is not taken into account and a control law of the form :

$u_{fuzz} = -K \operatorname{sgn}(s)$, where K is constant, is produced.

To overcome this problem, the width of the membership functions should be modified in every crossing of $e = 0$, i.e. at every change of the error sign ($\operatorname{sgn}(e_k e_{k-1}) = -1$). The last two control signals u_{k-1} and u_k are utilized :

u_{k-1} is the last control signal below (above) $e = 0$
u_k is the last control signal above (below) $e = 0$

Recalling the bisection method , the control signal u^* that will produce zero error should be searched in the range $[u_{k-1}, u_k]$
The new fuzzy subsets $U_1, U_2, \ldots\ldots, U_{n-1}, U_n$ correspond to the division of the interval between these two control signals $[u_{k-1}, u_k]$ in n equal segments.

As $e = 0$ is approached, the width of the membership functions is reduced, and consequently the control gain K is reduced too. In this way a control law of the form

$$u_{fuzz} = -K_{fuzz}(\operatorname{sgn}(e_k e_{k-1}))\operatorname{sgn}(s) \qquad (s = e\dot{e}), \qquad (5)$$

where, e_k is the error at the k-th step of the algorithm, and e_{k-1} is the error at the (k-1)-th step of the algorithm, is derived. This control law bears resemblance to the diagonal-type FLC (see Palm et. al (1996))

$$u_{fuzz} = -K_{fuzz}(e, \dot{e}, l)\operatorname{sgn}(s) \qquad (6)$$

and to the conventional SMFLC (see Slotine and Li (1991)) .

$$u_{fuzz} = -K(|s|\operatorname{sgn}(s)) \qquad (7)$$

where |s| is the distance from the diagonal. The above can be summarized in the following enhanced rule base , which consists of 6 rules $R_1^/ - R_6^/$:

$R_1^/$: IF $e_k > 0$ AND $\dot{e}_k > 0$ AND $\operatorname{sgn}(e_k e_{k-1}) = -1$ THEN change control action
AND reduce $||\Delta u_k||$

$R_2^/$: IF $e > 0$ AND $\dot{e} > 0$ AND $\operatorname{sgn}(e_k e_{k-1}) = 1$ THEN change control action
AND maintain $||\Delta u_k||$

$R_3^/$: IF $e > 0$ AND $\dot{e} < 0$ AND $\operatorname{sgn}(e_k e_{k-1}) = 1$ THEN maintain control action
AND maintain $||\Delta u_k||$

$R_4^/$: IF $e < 0$ AND $\dot{e} > 0$ AND $\operatorname{sgn}(e_k e_{k-1}) = 1$ THEN maintain control action
AND maintain $||\Delta u_k||$

$R_5^/$: IF $e_k < 0$ AND $\dot{e}_k < 0$ AND $\operatorname{sgn}(e_k e_{k-1}) = -1$ THEN change control action

AND reduce $||\Delta u_k||$

$R_6^{/}$: IF $e<0$ AND $\dot{e}<0$ AND $\text{sgn}(e_k e_{k-1})=1$ THEN change control action
AND maintain $||\Delta u_k||$

While the algorithm evolves, each change of $\text{sgn}(e_k e_{k-1})$ clips the fluctuation ranges of the control signal. This causes a reduction of the parameter K_{fuzz}. However, between two successive changes of $\text{sgn}(e_k e_{k-1})$, K_{fuzz} remains unchanged .

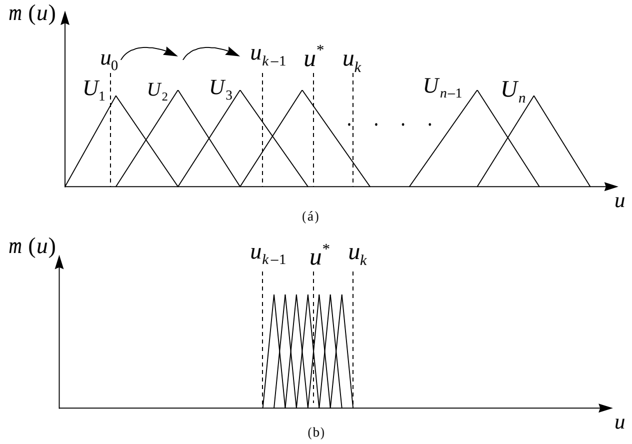


Figure 1 : Adaptation of the membership functions in RC-SMFLC

THE RC-SMFLC CONTROL LAW

The control law in conventional SMC is

$$u=\hat{b}^{-1}(\tilde{u}-\hat{f})$$
$$\tilde{u}=G(\hat{u}-K(x,t)sat(s/\Phi)) \qquad (8)$$
$$\hat{u}=x_d^{(n)}-\sum_{k=1}^{n-1}\binom{n-1}{k}l^{\,k}e^{(n-k)}$$

The control law in RC-SMFLC will be the same with only one change concerning the control term $K(x,t)sat(s/\Phi)$ which now becomes $K(\text{sgn}(e_k e_{k-1}))\,\text{sgn}(e_k\,\dot{e}_k)$, i.e. :

$$u=\hat{b}^{-1}(\tilde{u}-\hat{f})$$
$$\tilde{u}=G(\hat{u}-K(\text{sgn}(e_k e_{k+1}))\,\text{sgn}(e_k\,\dot{e}_k)) \qquad (9)$$
$$\hat{u}=x_d^{(n)}-\sum_{k=1}^{n-1}\binom{n-1}{k}l^{\,k}e^{(n-k)}$$

A previous estimation of the system’s parameters $f(x,t)$ and $b(x,t)$ is not always necessary . Assume that the system’s poles are strictly stable (e.g. the system n Strictly Positive Real) and $\hat{b}^{-1}=1$ and $\hat{f}=0$. Assume also that G and $\hat{u}$ are selected as : $G=1$ and $\hat{u}=u_0$, where u_0 is a randomly selected value in the interval of the permitted input values. Introducing the previous assumptions in (9) yields :

$$u=\tilde{u}$$
$$\tilde{u}=\hat{u}-K(\text{sgn}(e_k e_{k-1}))\,\text{sgn}(e_k\,\dot{e}_k) \qquad (10)$$

$\hat{u} = u_0$

LINKING FUZZY CONTROL WITH DES AND PETRI NET MODELING

THE MODEL OF AN EXPERT CONTROLLER

The RC-SMFLC presented in the previous section is a fuzzy expert controller that operates in a DES logic. Therefore the main principles of DES analysis and modeling can also be applied on RC-SMFLC. The model of the expert control system is illustrated in Fig. 2 (Tzafestas (1989)). The inputs of the expert controller are the current state of the plant x_k and the reference input x_k^d .

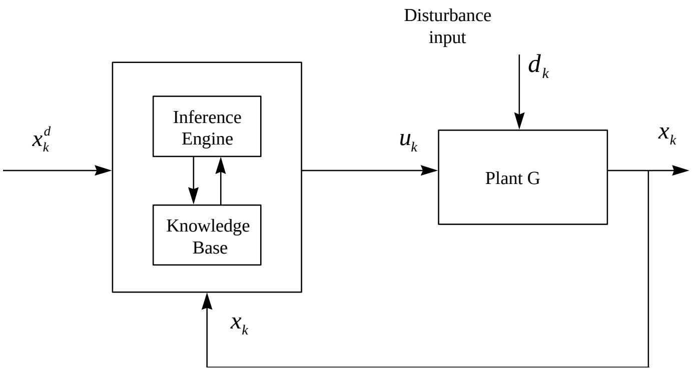


Figure 2 : Expert Control System

The closed loop of the expert controlled system can be described by the 6-tuple (Passino and Burgess (1998))

$$C = (P, T, A, W, M_0, E_v) \qquad (11)$$

where

1) P is the set of the places (states) of the closed-loop system, with $P = P_p \times P_c$ and P_p is the set of states of the plant , while P_c is the set of states of the controller.

2) T is the set of transitions and represents the events that take place in the expert controller . Two kinds of events can be related to a transition i.e. an increase or a decrease of the control signal.

3) $A \subseteq (P \times T) \cup (T \times P)$ is the set of arcs that emanate from a place p_i and end at a transition t_j or start from a transition t_j and end at a place p_i .

4) $W : A \to R$ is the set of arc weights. The arc weights represent the magnitude of the changes of the control signal. These changes are produced by a transition t_i (increase or decrease) . In contrast to many DES (e.g. DES modeled by ordinary Petri Nets), where the arc weights are predefined and remain unchanged , in the case of the RC-SMFLC , the arc weights are adapted and their values depend on the distance of the current state of the plant from the state { $e = 0$ and $\dot{e} = 0$ } . The updated values of the arc weights are produced via the fuzzy inference mechanism .

5) $M_0 : P \to \{0,1\}$ is the initial marking of the PN diagram of the closed-loop system . It is assumed that the system starts from a random state. The state of the system at the beginning of the control algorithm corresponds to a place which is assigned one token while all the other places are given no token. The existence of a token at one place indicates the current state of the closed-loop system.

6) E_v is the set of the controller event trajectories that are physically possible . In the case of the fuzzy expert controller E_v consists of the transitions that connect the live places of the PN diagram.

From the above it becomes clear that the C model , $C = (P, T, A, W, M_0, E_v)$, that was used for the description of an expert control system can be considered as an extension of a Petri Net. Therefore the basic principles of the PN design and analysis can also be applied on an expert control system.

ANALYSIS TECHNIQUES FOR EXPERT CONTROL SYSTEMS

The following properties are of interest in the analysis of a DES expert controller

Definition 1 : *Lyapunov Asymptotic Stability for Discrete Event Systems*

A closed invariant set $X_m \subset X$ of the states of a plant, with an initial state $x_o \in X_v$ and a set of events E_a , is stable in the sense of Lyapunov , if for every $e > 0$ it is possible to find a $d > 0$ such that

$$\text{when} \quad r\,(x_o, X_m) < d \quad \text{to hold} \quad r\,(X(x_0, E_k, k), X_m) < e \tag{12}$$

for all the events $E_k \in E_a(x_0)$ and $k \geq 0$.

Definition 2 : Asymptotic stability for Discrete Event Systems

A closed invariant set $X_m \subset X$ of the states of a plant, with an initial state $x_o \in X_v$ and a set of events E_a , is asymptotically stable if it possible to find a $d > 0$ such that

$$\text{when} \quad r\,(x_o, X_m) < d \quad \text{to hold} \quad \lim_{k \to \infty} r\,(X(x_0, E_k, k), X_m) = 0 \tag{13}$$

for all the events $E_k \in E_a(x_0)$ and $k \geq 0$.

STUDY OF THE RC-SMFLC PROPERTIES USING THE CORRESPONDING PETRI NET

In the sequel it will be proved that if the plant under control is *Strictly Positive Real* (SPR) then RC-SMFLC leads to an asymptotically stable closed-loop system.

Theorem : A transfer function $H(s) = \dfrac{b_m s^m + b_{m-1} s^{m-1} + \ldots + b_0}{s^n + a_{n-1} s^{n-1} + \ldots + a_0}$ is SPR if and only if (see Slotine and Li (1991))

i) $H(s)$ is a strictly stable transfer function

ii) the real part of $H(s)$ is strictly positive along the jw axis , i.e. $\forall\, w \geq 0 \quad \mathrm{Re}[H(jw)] > 0$

It should be reminded that in adaptive control of nonlinear systems the first aim is to transform the system under control via linearization to a SPR one and then apply the adaptive control law to the SPR equivalent of the initial model (Slotine and Li (1991)).

We prove the following lemma :

Lemma 1 : Assume that $H(s)$ is the transfer function that describes the dynamic error equation of a system P i.e. $e(s) = H(s)u(s)$. Consider also function $e_k = f(u_{k-T})$ which is the corresponding discrete function that relates the error to the control input (k indicates the k -th repetition of the control action with a period T >settling time) . Then if $H(s)$ is SPR, it holds that $e_k = f(u_{k-T})$ is monotone.

Proof : The transfer function $H(s)$ is

$$H(s) = \frac{b_m s^m + b_{m-1}s^{m-1} + \ldots + b_0}{s^n + a_{n-1}s^{n-1} + \ldots + a_0} \quad \text{with } a_0, b_0 \neq 0 \tag{14}$$

Then by multiplying both sides of $e(s) = H(s)u(s)$ with s and applying the final value theorem one gets :

$\lim_{s\to 0}[s\,e(s)] = \lim_{s\to 0} H(s) \bullet \lim_{s\to 0}[su(s)]$ i.e. $\lim_{t\to\infty} e(t) = \lim_{s\to 0} H(s) \bullet \lim_{t\to\infty} u(t)$ i.e.

$\lim_{t\to\infty} e(t) = \frac{b_0}{a_0} \bullet \lim_{t\to\infty} u(t)$. Naming $e_k = \lim_{t\to\infty} e(t)$ and $u_{k-T} = \lim_{t\to\infty} u(t)$, the initial relation becomes $e_k = \frac{b_0}{a_0} u_{k-T}$. It has to be noted that $t \to \infty$ stands for a large time interval , normally greater than the settling time of the closed-loop system .

Since $H(s)$ is SPR the system described by $H(s)$ is stable (i.e. bounded), $u(t)$ produces bounded $e(t)$. Thus the ratio $\frac{b_0}{a_0}$ is bounded i.e. $\exists M \in R^+$ such that $|\frac{b_0}{a_0}| < M$ .Equation $e_k = \frac{b_0}{a_0} u_{k-T}$ shows that in the steady state $e_k = f(u_{k-T})$ is indeed monotonous.

THE PN DIAGRAM OF RC-SMFLC USING THE ENHANCED RULE BASE

To avoid the cyclic behavior, the system's representation has to be enhanced by exploiting the information coming from the change of the error's sign . Thus the rule-base $R_1^/ - R_6^/$ is used .The places of the enhanced Petri Net diagram will be :

p_1 : $e > 0$ and $\dot{e} > 0$ and last control action = increase and $\text{sgn}(e_k e_{k-1}) = 1$

p_2 : $e > 0$ and $\dot{e} > 0$ and last control action = increase and $\text{sgn}(e_k e_{k-1}) = -1$

p_3 : $e > 0$ and $\dot{e} > 0$ and last control action = decrease and $\text{sgn}(e_k e_{k-1}) = 1$

p_4 : $e > 0$ and $\dot{e} > 0$ and last control action = decrease and $\text{sgn}(e_k e_{k-1}) = -1$

p_5 : $e > 0$ and $\dot{e} < 0$ and last control action = increase and $\text{sgn}(e_k e_{k-1}) = 1$

p_6 : $e > 0$ and $\dot{e} < 0$ and last control action = increase and $\text{sgn}(e_k e_{k-1}) = -1$

p_7 : $e > 0$ and $\dot{e} < 0$ and last control action = decrease and $\text{sgn}(e_k e_{k-1}) = 1$

p_8 : $e > 0$ and $\dot{e} < 0$ and last control action = decrease and $\text{sgn}(e_k e_{k-1}) = -1$

p_9 : $e < 0$ and $\dot{e} > 0$ and last control action = increase and $\text{sgn}(e_k e_{k-1}) = 1$

p_{10} : $e < 0$ and $\dot{e} > 0$ and last control action = increase and $\text{sgn}(e_k e_{k-1}) = -1$

p_{11} : $e < 0$ and $\dot{e} > 0$ and last control action = decrease and $\text{sgn}(e_k e_{k-1}) = 1$

p_{12} : $e < 0$ and $\dot{e} > 0$ and last control action = decrease and $\text{sgn}(e_k e_{k-1}) = -1$

p_{13} : $e < 0$ and $\dot{e} < 0$ and last control action = increase and $\text{sgn}(e_k e_{k-1}) = 1$

p_{14} : $e < 0$ and $\dot{e} < 0$ and last control action = increase and $\text{sgn}(e_k e_{k-1}) = -1$

p_{15} : $e < 0$ and $\dot{e} < 0$ and last control action = decrease and $\text{sgn}(e_k e_{k-1}) = 1$

p_{16} : $e < 0$ and $\dot{e} < 0$ and last control action = decrease and $\text{sgn}(e_k e_{k-1}) = -1$

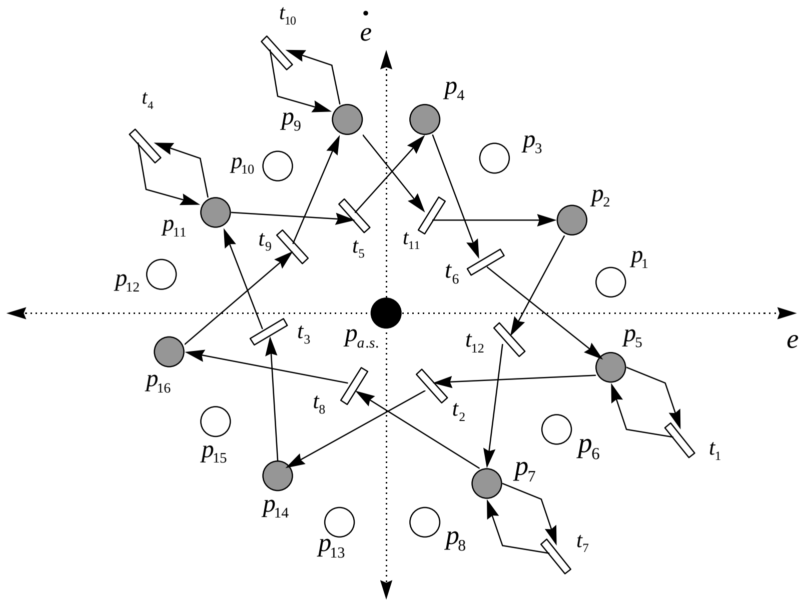


Figure 3: Enhanced Petri Net diagram of RC-SMFLC using rule base $R_1^{/} - R_6^{/}$

Lemma 2 :

If the control law, described by the rule base $R_1^{/} - R_6^{/}$, is applied on a SPR plant then

i) place p_1 is reachable only if the closed-loop system is randomly initialized at place p_5 and $e_k = f(u_{k-T})$ is an increasing function.

ii) place p_{15} is reachable only if the closed-loop system is randomly initialized at place p_{11} and $e_k = f(u_{k-T})$ is an increasing function.

iii) place p_3 is reachable only if the closed-loop system is randomly initialized at place p_7 and $e_k = f(u_{k-T})$ is a decreasing function.

iv) place p_{13} is reachable only if the closed-loop system is randomly initialized at place p_9 and $e_k = f(u_{k-T})$ is a decreasing function.

From { p_1, p_3, p_{13}, p_{15} } the state sequence of the closed-loop system returns to the set of states { $p_2, p_5, p_7, p_9, p_{11}, p_{14}, p_{16}$ }

Proof :

If $e_k = f(u_{k-T})$ is an increasing function and the system is randomly initialized at states p_5 or p_{11} then the following state sequences may occur (Fig.4):

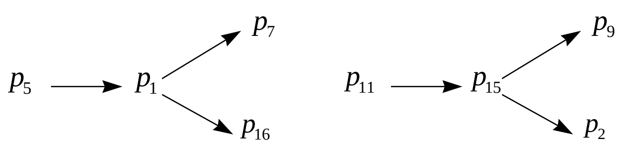


Figure 4 : $e_k = f(u_{k-T})$ is an increasing function.

If $e_k = f(u_{k-T})$ is a decreasing function and the system is randomly initialized at states p_7 or p_9 then the following state sequences may occur (Fig. 5):

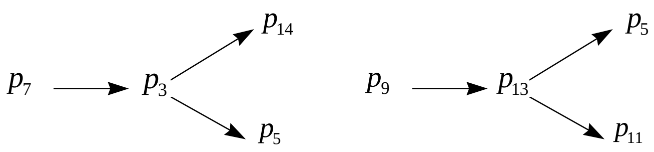


Figure 5 : $e_k = f(u_{k-T})$ is a decreasing function.

If the system enters in one of the states { $p_2, p_5, p_7, p_9, p_{11}, p_{14}, p_{16}$ } then it can not move to { p_1, p_3, p_{13}, p_{15} } because the error e_k and the first derivative of error $\dot{e}_k$ get the same sign only when a change of $\operatorname{sgn}(e_k)$ occurs. This happens when the system goes before through the places p_2, p_4, p_{14} and p_{16} respectively .Then according to the PN diagram the following transitions are expected to take place :

$$p_2 \xrightarrow{t_{12}} p_7\,,\quad p_4 \xrightarrow{t_6} p_5\,,\quad p_{14} \xrightarrow{t_3} p_{11} \;\text{ or }\; p_{16} \xrightarrow{t_9} p_9\,.$$

Therefore there is no sequence of events that can bring the system back to places p_1, p_3, p_{13} and p_{15} and these can be removed from the PN diagram .

Lemma 3 :

If the control law, described by the rule base $R_1^/ - R_6^/$, is applied on a SPR plant then places p_6, p_8, p_{10} and p_{12} are not reachable.

Proof :

The proof is given for place p_6 and is similar for the other three places p_8, p_{10} and p_{12}. For p_6 the following relations hold :

$$e_k > 0 \tag{15a}$$

$$\dot{e}_k < 0 \tag{15b}$$

$$\operatorname{sgn}(e_k e_{k-1}) = -1 \tag{15c}$$

Form (15a) and (15c) one gets $e_{k-1} < 0$ .Combining with (15a) follows that $e_k - e_{k-1} > 0$ i.e. $\dot{e}_k > 0$ which conflicts (15b). Since places p_6, p_8, p_{10} and p_{12} are not reachable they can be removed from the PN diagram.

Lemma 4 : If the control law, described by the rule base $R_1^/ - R_6^/$, is applied on a SPR plant then the only permitted set of states in the resulting closed-loop system is $\{p_2, p_4, p_5, p_7, p_9, p_{11}, p_{14}, p_{16}\}$

Proof :

Since the plant is SPR the dynamic error equation of the closed-loop system $e_k = f(u_{k-T})$ will have a monotone behavior. Without loss of generality assume that $e_k = f(u_{k-T})$ is monotonous decreasing and the current state of the system is p_5. Then taking into account Lemma 2 and Lemma 3 and the PN diagram of Fig. 3, the control signal will keep on increasing and the state of the system will continue to be p_5 until a change of the error sign occurs and then system moves to place p_{14}. At p_{14} the weights of arcs are reduced thus a decrease in the control signal will not cause change of the error sign and the system will move to place p_{11} (instead of p_4). The control signal will keep on decreasing and the system will remain at place p_{11} until an a change of the error sign occurs and then the system moves to place p_4. At p_4 the weights of arcs are reduced once more thus an increase in the control signal will not cause change of the error sign and the system will move back to place p_5 (instead of p_{16}). Consequently the system will remain on the closed path

$$p_5 \to p_{14} \to p_{11} \to p_4 \to p_5$$

Similarly it can be shown that the alternative closed path on which the system can move is

$$p_7 \to p_{16} \to p_{19} \to p_2 \to p_7$$

Examining the liveness of the PN it can be deduced that the set of the live places consists of p_2, p_4, p_5, p_7, p_9, p_{11}, p_{14} and p_{16}. The transitions that link these places are :

t_1 : increase	t_5 : decrease	t_9 : increase
t_2 : increase	t_6 : increase	t_{10} : increase
t_3 : decrease	t_7 : decrease	t_{11} : increase
t_4 : decrease	t_8 : decrease	t_{12} : decrease

We prove the following theorem :

Theorem :

If the control law, described by the rule base $R_1^/ - R_6^/$, is applied on a SPR plant then the resulting closed-loop system is asymptotically stable .

Proof :

The asymptotically stable state $x_{a.s.} = \{e = 0, \dot{e} = 0\}$ is defined . It has to be shown that

$$r\ (X(x_0, E_k, k), x_{a.s.}) \to 0 \qquad \text{for all } E_k \text{ such that } E_k \in E_a(x_0) \text{ and } k \to \infty .$$

From Lemma 2, 3 and 4 it is deduced that the state of the closed-loop system keeps on moving on the cyclic paths

$$p_5 \to p_{14} \to p_{11} \to p_4 \to p_5 \quad \text{or} \quad p_7 \to p_{16} \to p_{19} \to p_2 \to p_7$$

and that the weights of the arcs $W: A \to R$ that connect the above places diminish at every change of the error sign $(e_k e_{k-1} < 0)$. These weights declare the magnitude of the change of the control signal Δu_k which enable the transition from one place to the other . Δu_k is proportional to the distance from $x_{a.s.}$

Therefore as $k \to \infty$ the states of the system states will approach each thus the system will end to a state for which simultaneously hold :

$$X(x_0, E_\infty, k = \infty) : e\dot{e} > 0 \text{ with last control action } \Delta u = 0 \tag{16}$$

$$\text{and} \quad X(x_0, E_\infty, k = \infty) : e\dot{e} < 0 \text{ with last control action } \Delta u = 0 \tag{17}$$

$$\text{i.e.} \quad X(x_0, E_\infty, k = \infty) : e = 0 \ \& \ \dot{e} = 0 \qquad \text{i.e.} \quad X(x_0, E_\infty, k = \infty) = x_{a.s.}$$

$$\text{i.e.} \quad r\ (X(x_0, E_\infty, k = \infty),\ x_{a.s.}) = 0$$

VIEWING RC-SMFLC AS A LEARNING AUTOMATON.

OVERVIEW OF THE THEORY OF FORMAL LANGUAGES AND FINITE STATE AUTOMATA

The following definition reveals that RC-SMFLC is indeed a finite state automaton :

A finite automaton is defined by the six-tuple $G = (P, T, A, W, M_0, E_v)$ (see Jafari (1995)) , where

P is the set of states q of the automaton

T is the input alphabet or output symbols s of a state (i.e. the set of transitions between states)

A is the set of arcs that connect the states to transitions and vice-versa

W is the transition function (equivalent to the weights of the arcs)
M_0 is the initial state of the automaton (equivalent to the .the initial marking)
E_v is the set of all physically possible transitions i.e. the language L of the automaton

The automaton described above is a learning automaton. It has the ability to observe the response of the environment, update its internal structure and select the next action to be applied on the environment . This adaptation is performed by the change of the arc weights W .

Comparing the six-tuple that describes RC-SMFLC, to the six-tuple that describes the learning automaton it can be deduced that RC-SMFLC is a learning finite state automaton. The language generated by RC-SMFLC is

$L(G)$ = {increase, decrease, increase increase, increase decrease, decrease increase, decrease decrease, increase increase increase, increase increase decrease , etc .}

$L(G)$ should contain only those sequences of events that lead the system to a reachable state.

SIMULATION RESULTS

The proposed adaptive fuzzy controller has been tested in many different cases among which are :
i) the regulation of the thermal and geometrical characteristics of the arc-welding process and (Tzafestas et al. (1997))
ii) the motion control of a mobile robot that in a partially unknown environment (Tzafestas et al. (1996)).
The simulation code was written in C++.

THE LINEARIZED MODEL OF ARC-WELDING

As far as its thermal characteristics are concerned, the arc-welding thermal model considers as outputs the *weld nugget cross section* NS, the *heat affected zone* HZ and the *centerline cooling rate* CR, and as inputs the *thermal power of the torch*.

$$\frac{NS(s)}{U(s)} = \frac{K_a}{t_a s+1} \;,\; \frac{HZ(s)}{U(s)} = \frac{K_b(t_b s+1)}{(t_1 s+1)(t_2 s+1)} \;\text{ and }\; \frac{CR(s)}{U(s)} = \frac{K_c}{(t_a s+1)(t_b s+1)} \qquad (18)$$

The following 2nd order linear system was tested (input : thermal power of the torch Q -output : CR)

$$G_m(z) = \frac{0.11(z+1)^2}{(z-0.96)(z-0.63)}$$

Further more the tracking capability of the proposed fuzzy expert controller was evaluated in the case of a ramp setpoint with either positive or negative slope.

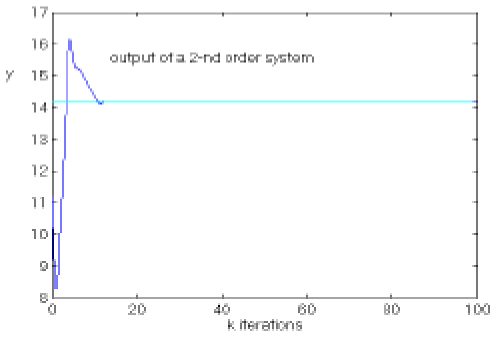


Figure 6 : Control of the cooling rate of the arc-welding process

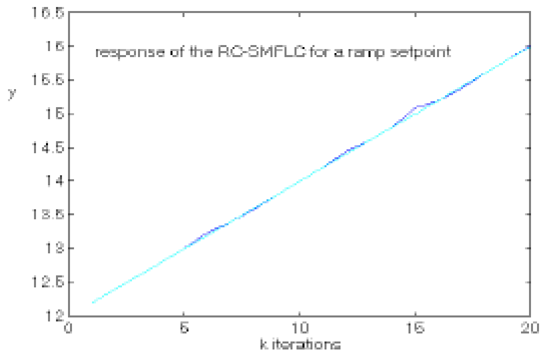


Figure 7: Tracking of a ramp set-point with positive slope

VELOCITY CONTROL OF A MOBILE ROBOT IN A PARTIALLY UNKNOWN ENVIRONMENT

The performance of the proposed fuzzy expert controller was tested for various cases of robot motion in a partially unknown workspace.

THE MODEL OF THE MOBILE ROBOT

The model of the mobile robot is described by the following differential equation (see Rigatos et al. (1998)) :

$$(m+I_R(s))\ \ddot{s}(t)\ +\ I_R^{/}(s)\,\dot{s}^2(t)+mg\,z^{/}(s)\ =\ u(t) \tag{19}$$

where $(\bullet)^{/} \overset{\Delta}{=} \dfrac{d(\bullet)}{ds}$. A useful equivalent formulation can be obtained by using as an independent variable the parameter s instead of the time t. This is accomplished by introducing the definition $v(s) \overset{\Delta}{=} \dfrac{ds}{dt}(s)$. Therefore

$$(m+I_R(s))\ v(s)v^{/}(s)+\ I_R^{/}(s)v^2(s)+mg\,z^{/}(s)\ =\ u(s) \tag{20}$$

i.e.

$$(m+I_R(s))\ \dot{v}(s)+\ I_R^{/}(s)v^2(s)+mg\,z^{/}(s)\ =\ u(s) \tag{21}$$

HYBRID CONTROL ARCHITECTURE

The first step is to transform the nonlinear model into an SPR one. This is achieved by applying a hybrid control scheme that includes a nonlinear control element and a PD controller . The appropriate control signal is

$$u(t) = \ddot{s}_d(t)+\frac{I_R^{/}(s)}{(m+I_R(s))}s^2(t)+K_p e(t)+K_d\,\dot{e}(t)+u_{RC-SMFLC}(t) \tag{22}$$

which results in :

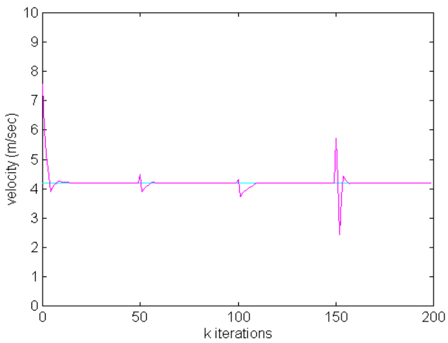
$$\ddot{s}(t)-\ddot{s}_d(t)+K_p e(t)+K_d\,\dot{e}(t)=u_{RC-SMFLC}(t)-\frac{mgz^{/}(s)}{(m+I_R(s))} \tag{23}$$

where $e(t)=s_d(t)-s(t)$. Thus

$$\ddot{e}(t)+K_p e(t)+K_d\,\dot{e}(t)=\frac{mgz^{/}(s)}{(m+I_R(s))}-u_{RC-SMFLC}(t) \tag{24}$$

The poles of the system described by (24) are determined by the gains K_p and K_d. Thus if K_p and K_d are appropriately selected, then the model of the position of the mobile robot comes in an SPR form. RC-SMFLC is then going to compensate the term $\dfrac{mgz^{/}(s)}{(m+I_R(s))}$ and assures that $\lim_{t\to\infty} e(t)=0$.

It was assumed that the mass m, the moment of inertia I and the ″reflected″ inertia I_R of the robot were known, and the uncertainty of the robotic model concerned only the existence of slopes $\dfrac{¶\, z}{¶\, s}$ in the robot trajectory. The robot could mount an uphill slope $(¶\, z\,/\,¶\, s>0)$, or could go down a downhill slope $(¶\, z\,/\,¶\, s<0)$. Both the magnitude and the sign of the slope were unknown and time varying.



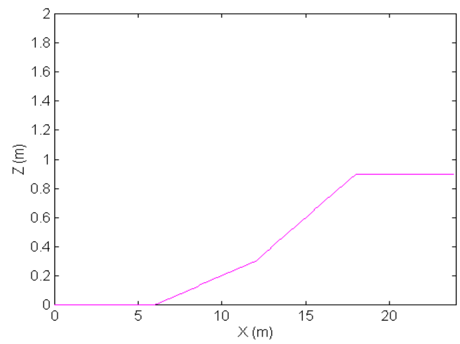


Figure 8 : Velocity fluctuation when the robot mounts an uphill slope.

Figure 9 : The corresponding robot trajectory on an uphill slope.

CONCLUSIONS

In this paper the RC-SMFLC hybrid intelligent control system was presented. The proposed controller combines the basic elements of fuzzy logic control and sliding mode control theory. The rule base of RC-SMFLC consists of six rules which decide a change of the control action according to the sign of the error e and the first derivative of error $\dot{e}$. The possible control actions are either an increase or a decrease of the control signal. This change of the control signal value is implement via fuzzy inference . The closer the system approaches the desired setpoint , the smaller the changes of the contol signal become. It must be noted that in contrast to conventional controllers or other intelligent controllers RC-SMFLC does not make use of the value of error and its derivative but only of their sign.

The Petri Net diagram of the control system was derived and the properties of *reachability*, *cyclic behavior* and *asymptotic stability* were studied. The non reachable states of the control system were identified and removed from the Petri Net diagram thus reducing the complexity of the control architecture. It was also proved that if the plant under control is or has been transformed to a SPR form ,then RC-SMFLC assures convergence to the desired setpoint.

Furthermore the equivalence between the control scheme and a learning finite state automaton was revealed. This enabled to study RC-SMFLC from the point of view of formal languages theory. The representation of RC-SMFLC as a finite state automaton facilitates the hardware implementation of this controller with the use of simple flip-flop circuits.

Finally the credibility of RC-SMFLC was tested in the case of two plants : i) the arc-welding process, and ii) a mobile robot that moves in a partially unknown environment. In both cases the results were quite satisfactory.

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