Control Laws for Continuous TSK Fuzzy Models Using a Simultaneous Stabilization of a Collection of SIMO Models

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ABSTRACT: Most of the works dealing with stability of Takagi-Sugeno-Kang models (TSK) (excepted Feng (1997),Leung (1996))use control laws based on a PDC (Parallel Distributed Compensation), in fact it sums up in local linear feedback. The goal of this paper is to propose some control laws that are not directly based on a PDC in the case of continuous SIMO TSK fuzzy models.

KEYWORDS: Fuzzy models, Lyapunov stability, Fuzzy observer, Linear Matrix Inequalities.

INTRODUCTION

This paper deals with the stabilization of continuous fuzzy systems. In precedent works, (Kim 1995, Ma 1998, Tanaka 1998) with Takagi-Sugeno-Kang fuzzy models (TSK) (Takagi 1985, Sugeno 1988), the derived control laws are based on local linear feedback called PDC (Parallel Distributed Compensation) (Tanaka 1996). Stability analysis is done with a Lyapunov approach (Tanaka 1992, 1996, 1998).

In the paper we propose some control laws not directly based on a PDC approach. In the first part a new approach is discussed. The propositions are following the ideas found in petersen (1987), of simultaneous quadratic stabilization of SIMO linear systems. In the second part we give a separation principle with a fuzzy observer for the derived control law. An example illustrating the approach is also provided. We are interested in the following TSK fuzzy model :

$$\dot{x}(t) = \sum_{i=1}^{r} h_i\left(z(t)\right)\left[A_i x(t) + B_i u(t)\right], \quad y(t) = \sum_{i=1}^{r} h_i\left(z(t)\right) C_i x(t) \tag{1}$$

APPROACH USING SIMULTANEOUSLY STABILIZING A COLLECTION OF SIMO MODELS

Theorem 1 : Let (Σ) a fuzzy model described in equation (1). Suppose there exists a matrix $P > 0$ and a real $a > 0$ such that the following equations are satisfied :

$$\forall i \in \{1,\ldots,r\},\ \exists Q_i > 0, F_i, \qquad A_i^T P + PA_i - PB_i F_i - F_i^T B_i^T P + Q_i \le 0 \tag{2}$$

$$\forall j \ \ Q_i + PB_i F_j + F_j^T B_i^T P > a\, I \tag{3}$$

then the control law : $u(x) = g_1(x) + g_2(x)$ with :

$$g_1(x) = \min_i\left(0, \frac{x^T Q_i x - a\, \|x\|^2}{2B_i^T Px} - F_i x\right) \qquad \text{for :} \quad i \ \text{ such that : } B_i^T Px > 0 \tag{4}$$

$$g_2(x) = \max_i\left(0, \frac{x^T Q_i x - a\, \|x\|^2}{2B_i^T Px} - F_i x\right) \qquad \text{for :} \quad i \ \text{ such that : } B_i^T Px < 0 \tag{5}$$

stabilizes globally asymptotically the closed-loop system.

This approach is called SSF (Simultaneous Stabilization for Fuzzy models) (Vermeiren 1998). To establish the proof, the following lemma is necessary :

Lemma 1 : If the condition (3) is satisfied then : $\forall x \in R^n$, $g_1(x) = 0$ or $g_2(x) = 0$.

Proof :

By reductio ad absurdum . Let us consider $\exists\, x \in R^n$ such that : $g_1(x) < 0$ et $g_2(x) > 0$, thus :

$\exists\, i \in \{1,\dots,r\}$ such that : $B_i^T Px > 0$ and $x^T Q_i x - a\left\|x\right\|^2 - 2x^T PB_i F_i x < 0$; then : $2x^T PB_i F_i x > x^T Q_i x - a\left\|x\right\|^2$ (6)

and

$\exists\, j \in \{1,\dots,r\}$ such that : $B_j^T Px < 0$ and $x^T Q_j x - a\left\|x\right\|^2 - 2x^T PB_j F_j x < 0$; then $x^T Q_j x < a\left\|x\right\|^2 + 2x^T PB_j F_j x$ (7)

Let us assume for the pair (i, j) (resp. (j,i)), the condition (3) is satisfied : $x^T Q_i x + 2x^T PB_i F_j x > a\left\|x\right\|^2$, then

$$2x^T PB_i F_j x > a\left\|x\right\|^2 - x^T Q_i x \qquad (8)$$

(6)+(8) gives : $2x^T PB_i\left(F_i + F_j\right)x > 0$ and as $B_i^T Px > 0$ thus $\left(F_i + F_j\right)x > 0$.

For the pair (j,i) , according to (7) we can write : $x^T Q_j x + 2x^T PB_j F_i x < a\left\|x\right\|^2 + 2x^T PB_j\left(F_i + F_j\right)x$

or $B_j^T Px < 0$ and $\left(F_i + F_j\right)x > 0$ therefore : $x^T Q_j x + 2x^T PB_j F_i x < a\left\|x\right\|^2$, which contradicts condition (3).

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Proof of theorem 1 :

Let us consider the Lyapunov function : $V(x) = x^T Px$, the derivative is given by :

$\dot{V}(x) = \sum_{i=1}^{r} h_i(z)\left(x^T\left(A_i^T P + PA_i\right)x + 2\left(B_i^T Px\right)u\right)$ and according to equation (2) :

$$\dot{V}(x) \le \sum_{i=1}^{r} h_i(z)\left\{-x^T Q_i x + 2x^T PB_i F_i x + 2\left(B_i^T Px\right)\left(g_1(x) + g_2(x)\right)\right\} = \sum_{i=1}^{r} h_i(z)\,\dot{V}_i(x)$$

case 1 : $g_1(x) \ne 0$

According to lemma 1, $g_2(x) = 0$, from which we deduce :

$\dot{V}_i(x) = -x^T Q_i x + 2x^T PB_i F_i x + 2\left(B_i^T Px\right)g_1(x)$, and as $g_1(x) \ne 0$:

$\exists\, j \in \{1,\dots,r\}$ such that $B_j^T Px > 0$ and $g_1(x) = \dfrac{x^T Q_j x - a\left\|x\right\|^2}{2B_j^T Px} - F_j x < 0$

case 1a : i is such that $B_i^T Px > 0$, as j corresponds to minimum of $g_1(x)$ then :

$g_1(x) \le \dfrac{x^T Q_i x - a\left\|x\right\|^2}{2B_i^T Px} - F_i x$, and directly : $\dot{V}_i(x) \le -a\left\|x\right\|^2$.

case 1b : i is such that $B_i^T Px = 0$, et : $\dot{V}_i(x) = -x^T Q_i x$.

case 1c : i is such that $B_i^T Px < 0$, we have then :

$$\dot{V}_i(x) = -x^T Q_i x + 2x^T PB_i F_i x + \frac{B_i^T Px}{B_j^T Px}\left(x^T Q_j x - a\left\|x\right\|^2\right) - 2x^T PB_i F_j x$$

$$= -\left(x^T Q_i x + 2x^T PB_i F_j x\right) + \frac{B_i^T Px}{B_j^T Px}\left(x^T Q_j x + 2x^T PB_j F_i x - a\left\|x\right\|^2\right)$$

as $\dfrac{B_i^T Px}{B_j^T Px} < 0$, and according to condition (3) for the pairs (i, j) and (j,i) : $\dot{V}_i(x) \le -a\left\|x\right\|^2$.

case 2 : $g_2(x) \neq 0$

According to lemma 1, $g_1(x) = 0$, from which we deduce : $\dot{V}_i(x) = -x^T Q_i x + 2x^T PB_i F_i x + 2\left(B_i^T Px\right) g_2(x)$, and as $g_2(x) \neq 0$: $\exists\, j \in \{1,\ldots,r\}$ such that $B_j^T Px < 0$ and $g_2(x) = \dfrac{x^T Q_j x - a\,\|x\|^2}{2B_j^T Px} - F_j x > 0$

cas 2a : i is such that $B_i^T Px > 0$, we have then :

$$\dot{V}_i(x) = -x^T Q_i x + 2x^T PB_i F_i x + \frac{B_i^T Px}{B_j^T Px}\left(x^T Q_j x - a\,\|x\|^2\right) - 2x^T PB_i F_j x$$

$$= -\left(x^T Q_i x + 2x^T PB_i F_j x\right) + \frac{B_i^T Px}{B_j^T Px}\left(x^T Q_j x + 2x^T PB_j F_i x - a\,\|x\|^2\right)$$

as $\dfrac{B_i^T Px}{B_j^T Px} < 0$, and according to condition (3) for the pairs (i, j) and (j, i) : $\dot{V}_i(x) \leq -a\,\|x\|^2$.

cas 2b : i is such that $B_i^T Px = 0$, and : $\dot{V}_i(x) = -x^T Q_i x$.

cas 2c : i is such that $B_i^T Px < 0$, as j correspond to maximum of $g_2(x)$ then :

$g_2(x) \geq \dfrac{x^T Q_i x - a\,\|x\|^2}{2B_i^T Px} - F_i x$, and directly : $\dot{V}_i(x) \leq -a\,\|x\|^2$.

case 3 : $g_1(x) = g_2(x) = 0$, from which we deduce : $\dot{V}_i(x) = -x^T Q_i x + 2x^T PB_i F_i x$

cas 3a : i is such that $B_i^T Px > 0$, as $g_1(x) = 0$: $x^T Q_i x - a\,\|x\|^2 - 2x^T PB_i F_i x \geq 0$, and : $\dot{V}_i(x) \leq -a\,\|x\|^2$.

cas 3b : i is such that $B_i^T Px = 0$, and : $\dot{V}_i(x) = -x^T Q_i x$.

cas 3c : i is such that $B_i^T Px < 0$, as $g_2(x) = 0$: $x^T Q_i x - a\,\|x\|^2 - 2x^T PB_i F_i x \geq 0$, and : $\dot{V}_i(x) \leq -a\,\|x\|^2$.

In any case, with : $I = \left\{i,\, B_i^T Px = 0\right\}$, we have :

$\dot{V}(x) \leq -\sum_{i \in I} h_i(z) x^T Q_i x - a\,\|x\|^2 \sum_{i \notin I} h_i(z) < 0$, that completes the proof.

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Equations (2) and (3) can be written in LMIs' form, (El Ghaoui 1997). Using standard manipulations, Schur complement is used for condition (10), the stability conditions of theorem 1 are equivalent to the following problem :

find $a > 0$, $X > 0$, $Y_i > 0$ and $M_i > 0$ satisfying

$$XA_i^T + A_i X - B_i M_i - M_i^T B_i^T + Y_i \leq 0 \tag{9}$$

$$\begin{bmatrix} Y_i + B_i M_j + M_j^T B_i^T & X \\ X & \frac{1}{a} I \end{bmatrix} > 0 \tag{10}$$

where $P = X^{-1}$, $F_i = M_i P$ et $Q_i = PY_i P$.

This approach is interesting insofar as there is no restriction on the used TSK SIMO models. Nevertheless, the proposed control law allows to simultaneously stabilize all the sub-models. As a result, the obtained stability conditions are certainly more conservative than those found in the litterature (Tanaka 1998), but we can expect a better robustness with regards to parametric variations. This point is illustrated, in the last section, with an application to the problem of balancing and swinging-up an inverted pendulum on a cart for the real case application see (Vermeiren 1999).

Some interesting results in stability, were found in the special case where $\forall i \in \{1,\ldots,r\},\ \exists k_i > 0,\ B_i = k_i B_1$ (Guerra 1998, Vermeiren 1998) allowing to reduce the conservatism of classical conditions of (Tanaka 1998). In this special case, it is also possible to simplify the proposed control laws.

Theorem 2 : Let (Σ) a SIMO fuzzy model described equation (1) verifying : $\forall i \in \{1,\ldots,r\},\ \exists k_i > 0,\ \ B_i = k_i B_1$. Suppose there exists a matrix $P > 0$ such that :

$$\forall i \in \{1,\ldots,r\},\ \exists F_i, \qquad A_i^T P + PA_i - PB_i F_i - F_i^T B_i^T P < 0 \tag{11}$$

then the control law : $$u(x) = -\begin{cases} \max\limits_{i\in\{1,\ldots,r\}} (F_i x) & \text{when } B_1^T Px \geq 0 \\ \min\limits_{i\in\{1,\ldots,r\}} (F_i x) & \text{when } B_1^T Px < 0 \end{cases} \tag{12}$$

stabilizes globally asymptotically the closed-loop system.

Proof : Let us consider the Lyapunov fonction $V(x) = x^T Px$, the derivative becomes :

$\dot{V}(x) = \sum\limits_{i=1}^{r} h_i(z)\left(x^T\left(A_i^T P + PA_i\right)x + 2\left(B_i^T Px\right)u\right)$ and according to equation (11) :

$$\dot{V}(x) < \sum_{i=1}^{r} h_i(z)\left\{2x^T PB_i F_i x + 2\left(B_i^T Px\right)u\right\} = \sum_{i=1}^{r} h_i(z) 2\left(B_i^T Px\right)\left(F_i x + u\right) \ = 2\sum_{i=1}^{r} h_i(z) k_i \left(B_1^T Px\right)\left(F_i x + u\right)$$

case 1 : $B_1^T Px \geq 0$ then : $\forall i \in \{1,\ldots,r\}$ $u = -\max\limits_{i\in\{1,\ldots,r\}} (F_i x) \leq -F_i x$ et $F_i x + u \leq 0$

case 2 : $B_1^T Px < 0$ then : $\forall i \in \{1,\ldots,r\}$ $u = -\min\limits_{i\in\{1,\ldots,r\}} \geq -F_i x$ et $F_i x + u \geq 0$

and : $\forall i \in \{1,\ldots,r\}$ $\left(B_1^T Px\right)\left(F_i x + u\right) \leq 0$ and : $\dot{V}(x) < 0$

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Notice that the obtained control law is not necessarily continuous.

Finally let us quote that in the case of quadratic stabilisation of this special case, i.e. using Riccati equations, we can propose a theorem for linear control laws (Vermeiren 1998).

Theorem 3 : Let (Σ) be a TSK SIMO fuzzy model described equation (1) verifying the condition : $\forall i \in \{1,\ldots,r\},\ \exists k_i > 0,\ B_i = k_i B_1$. Suppose there exists a matrix $P > 0$ such that :

$$\forall i \in \{1,\ldots,r\},\ \exists Q_i > 0, r_i > 0,\ A_i^T P + PA_i - r_i^{-1} PB_i B_i^T P + Q_i = 0 \tag{13}$$

the following linear feedback law : $$u(t) = -a\ B_1^T Px(t) \qquad \text{with : } a = \max_{i\in\{1,\ldots,r\}} \left(r_i^{-1} k_i\right) \tag{14}$$

stabilizes globally asymptotically the closed-loop system.

Proof is straightforward.

GLOBAL CLOSED-LOOP

Two observers are available (Tanaka 1998). In the following part, as for the work of Ma (1998) the premises variables are supposed to be measurable ones. In this case the observer has the following form :

Rule i , $i = 1, 2, \ldots, r$: If $z_1(t)$ is F_i^1 and … and $z_n(t)$ is F_i^n Then $$\begin{cases} \dot{\hat{x}}(t) = A_i \hat{x}(t) + B_i u(t) + K_i\left(y(t) - \hat{y}(t)\right) \\ \hat{y}_i(t) = C_i \hat{x}(t) \end{cases} \tag{15}$$

The control law is given, theorem 1, replacing $x(t)$ by $\hat{x}(t)$, i.e. : $u(\hat{x}) = g_1(\hat{x}) + g_2(\hat{x})$. With (1) and (15), the global model can be described, with $\tilde{x}(t) = x(t) - \hat{x}(t)$, as : $$\begin{cases} \dot{x} = \sum\limits_{i=1}^{r} h_i(z)\left(A_i x + B_i u(\hat{x})\right) \\ \dot{\tilde{x}} = \sum\limits_{i=1}^{r}\sum\limits_{j=1}^{r} h_i(z) h_j(z)\left(A_i - K_j C_i\right)\tilde{x} \end{cases} \tag{16}$$

Then the following separation theorem can be derived.

Theorem 4 : Suppose there exists two matrices $P_c > 0$ and $P_o > 0$ with the following property :

i) $\forall x \neq 0, V(x) = x^T P_c x$ is strictly decreasing ;

ii) $\forall \tilde{x} \neq 0, \tilde{V}(\tilde{x}) = \tilde{x}^T P_o \tilde{x}$ is strictly decreasing ;

then the global closed-loop system (16) is globally asymptotically stable.

The proof is too long and omitted here. Following the idea of Ma (1998), it is based on the comparison principle (Vermeiren 1998).

APPLICATION TO THE INVERTED PENDULUM SYSTEM

To illustrate the performance of the whole approach, consider the problem of balancing an inverted pendulum on a cart. In most works concerned with the fuzzy control of an inverted pendulum based on a TSK model, either they are concerned with the SIMO case with restricted area for the angle $q(t) \in \left]-\frac{p}{2}, \frac{p}{2}\right[$ (Ma 1998), or with the SISO case, i.e. without considerations of the cart position, $q(t) \in [-p, p]$ (Wang 1996, Leung 1996). In order to take into account the whole pendulum domain, it is possible to use a swinging sequence when $|q(t)| > \frac{p}{2}$ (Wei 1995).

The motion equations of the inverted pendulum device are :

$$\ddot{X}(t) = \frac{F(t) - f\,\dot{X}(t) + m\left(\frac{g}{2}\sin(2q(t)) - L\dot{q}^2(t)\sin(q(t))\right)}{M + m\sin^2(q(t))} \tag{17}$$

$$\ddot{q}(t) = \frac{g(M+m)\sin(q(t)) + F(t)\cos(q(t)) - f\,\dot{X}(t)\cos(q(t)) - mL\dot{q}^2(t)\sin(q(t))\cos(q(t))}{L\left(M + m\sin^2(q(t))\right)} \tag{18}$$

with :

m :	mass of pendulum (0.025 kg)	L :	half a arm length (0.1 m)
M :	mass of cart (20 kg)	f :	dry friction (150 Nms/rad)
G :	power gain (67)	$u(t)$:	command (V)
$q(t)$:	angle of pendulum (rad)	$X(t)$:	cart position (m)

First of all a TSK fuzzy model, which represents the dynamics of the nonlinear system can be obtained. When $q(t) = \pm p/2$, this model remains uncontrollable. With $q(t) \in [-q_0, q_0]$, (17) and (18), the following matrices of a simplified TSK model, with two rules, are obtained, with the state vector : $x(t) = \begin{bmatrix} q(t) & X(t) & \dot{X}(t) & \dot{q}(t) \end{bmatrix}^T$:

$$A_1 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ \dfrac{mg}{M} & 0 & \dfrac{-f}{M} & 0 \\ \dfrac{g}{L}\left(1+\dfrac{m}{M}\right) & 0 & \dfrac{-f}{ML} & 0 \end{bmatrix} \quad B_1 = \begin{bmatrix} 0 \\ 0 \\ \dfrac{G}{M} \\ \dfrac{G}{ML} \end{bmatrix} \quad A_2 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ \dfrac{mg\sin(q_0)\cos(q_0)}{Mq_0} & 0 & \dfrac{-f}{M} & 0 \\ \dfrac{g}{L}\left(1+\dfrac{m}{M}\right)\dfrac{\sin(q_0)}{q_0} & 0 & \dfrac{-f\cos(q_0)}{ML} & 0 \end{bmatrix}$$

$$B_2 = \begin{bmatrix} 0 \\ 0 \\ \dfrac{G}{M} \\ \dfrac{G\cos(q_0)}{ML} \end{bmatrix}$$

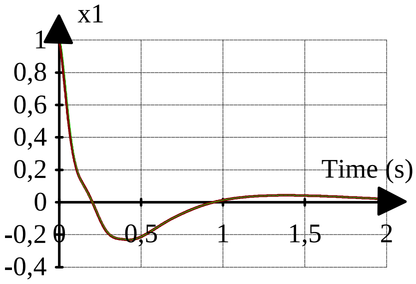
$$C_1 = C_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

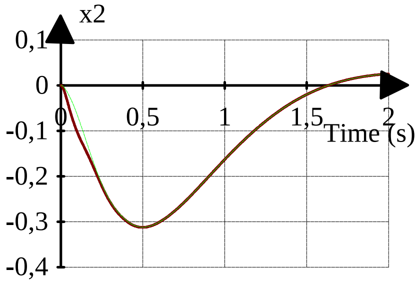
For this application, we use the control law of theorem 1. The fuzzy model uses measurable premises variable $q(t)$, so, the separation principle is available. The observer is obtained according to LMIs found in Tanaka (1998). Results are :

$$K = \begin{bmatrix} -9{,}9475 & 49{,}7292 \\ 15{,}1310 & -9{,}9475 \\ 113{,}9497 & 157{,}5444 \\ -487{,}6541 & 1235{,}976 \end{bmatrix} \qquad K_2 = \begin{bmatrix} -9{,}0670 & 38{,}2757 \\ 17{,}0550 & -9{,}0670 \\ 136{,}5423 & -120{,}9060 \\ -380{,}7782 & 723{,}6224 \end{bmatrix}$$

A pole placement is used for the control part. Results are : $F_1 = [12{,}4929 \;\; -4{,}5643 \;\; -6{,}0424 \;\; 1{,}2610]$ $F_2 = [22{,}4201 \;\; -5{,}4194 \;\; -6{,}7550 \;\; 2{,}4657]$. These gains satisfy the stability conditions of theorem 1. An example of result is shown, figures 1 and 2, with initial conditions : $x(0) = [1 \;\; 0 \;\; 0 \;\; -5]^T$ and $\hat{x}(0) = [1 \;\; 0 \;\; 0 \;\; 0]^T$.

A complete study of the inverted pendulum with comparison with other control laws (linear feedback, PDC approach, fuzzy approach with output feedback) in simulation or in real time is done in (Vermeiren 1998, Vermeiren 1999).





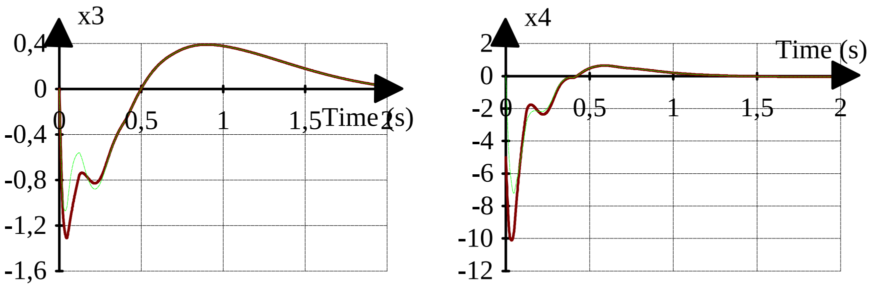


Figure 1 : State variables for the global system (non linear model + fuzzy SSF control law + fuzzy observer) with : $x(0) = \begin{bmatrix} 1 & 0 & 0 & -5 \end{bmatrix}^T$ and $\hat{x}(0) = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}^T$

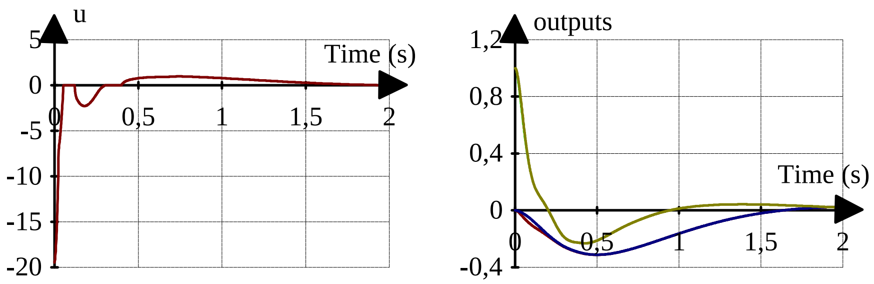


Figure 2 : Control law and Outputs for global system with : $x(0) = \begin{bmatrix} 1 & 0 & 0 & -5 \end{bmatrix}^T$ and $\hat{x}(0) = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}^T$

CONCLUSION

Some control laws that are not directly based on a PDC were derived, following the ideas of using a division in the control law (Guerra 1998, Vermeiren 1998). Due to a separation principle, the global stability results are maintained when the observer uses measured premises variables. It is obvious that we cannot obtain necessary and sufficient conditions using such an approach, as long as antecedent part of the rules are not taken into account. Nevertheless, a way for introducing such knowledge can be found in Marin (1995).

Application in simulation and in real time of the well-known inverted pendulum as shown the efficiency of the approach (Vermeiren 1998).

At last let us say that the use of LMIs allows also to take into account performances such as decay rate, constraints on the output and on the command (Boyd 1994, Tanaka 1998).

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