# The Use Of Fuzzy Sets For Brain Description

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ABSTRACT: A major problem in the description of the human brain is the high variability of its anatomy. To cope with this variability, a methodology for a precise and yet comprehensive description of the brain is presented, that uses natural language words. The classification is obtained using minimum and maximum values derived from a population and a mathematical model according to fuzzy rules. Following this strategy the usual sharp boundaries between classes are avoided.

KEYWORDS: brain anatomy, brain atlas, classification, intersubject standardization, modelling

## INTRODUCTION

While surgery considers only the individual anatomy of a single patient, population studies make use of comparable groups of people. An interesting problem in the interindividual analysis of brain anatomy in general and especially of brain imaging is the morphological variability of the brain, Keyserlingk (1988).
When searching for an approach to this variability, the use of crisp numerical values in the form of tables or of ratios, derived from these numerical values often leads to an overwhelming amount of data and does not help in the comprehension of the functional differences.

Our aim was therefore to develop a method for a description of the brain - and also of other organs - in natural language words with the same precision as with digits but with the advantage of better comprehension and an easier integration into medical documents.

## MATERIAL AND METHODS

### ATLAS REGISTRATION

Standard models like a 3-dimensional brain atlas can be used to obtain such a description. Hence a spatial normalization is necessary. This normalization corresponds to a geometrical transformation of images, or of the standard model, to reduce the variability as described earlier, Berks (1998). In addition, it increases the accuracy of the analysis of images derived from different people. When dealing with a standard model on sets of images of different persons matrix transformations are therefore required, to assign every single point in the atlas to a corresponding point in the image data set.
Based on a three-dimensional superpositioning of a digital brain atlas onto several individual image data sets a procedure was established to map corresponding cortical regions in different image modalities. In this way a map was obtained which could than be used for elastic transformations. The transformation brings the image data sets into register. The respective transformation matrix is stored and can be used to perform the analysis of the corresponding brain parameters.

### DETERMINATION OF VARIABLES

After the necessary transformation matrices were obtained, the MR images of 30 healthy volunteers could be evaluated. The brains were categorized with respect to the variables 'length', 'width' and 'height' according to the following rules. Let the linguistic variables, Zadeh (1973), of the whole 3D set be named 'human brain'. Within this 3D-set three one-dimensional subsets exist, named x-, y-, and z-axis, i.e., the midsagittal axis, the longitudinal and the midtransversal axis (Figure 1).

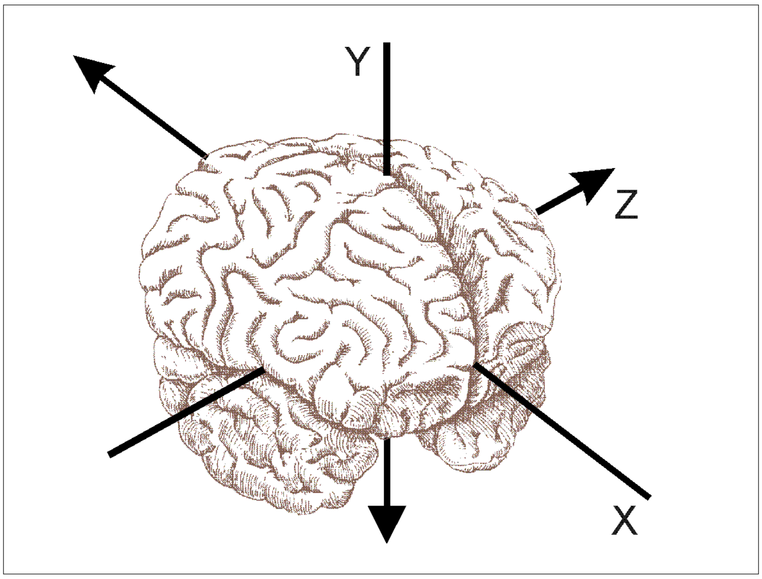


Figure 1: The x-, y-, and z-axis as subsets of the data set 'human brain'. The orientation of the axes is determined by the digital brain atlas.

These subsets present further subdivisions described with the variables 'long' and 'short' for the midsagittal axis, with 'high' and 'flat' for the longitudinal axis, and with 'broad' and 'small' for the midtransversal axis. The middle part, usually referred to as 'normal' is omitted.
In order to perform computations in the fuzzy domain, the minimum and maximum value of the three axes have to be determined. The images of the 30 volunteers were used to find these values in all three directions of space. The absolute measure ranges between 130 and 169 mm for the x-axis, between 93 and 119 mm for the y-axis, and between 64 and 75 mm for the z-axis. These values were then used to calculate membership functions, Zimmermann (1994), for the described variables. The adjectives: long - short, high - flat, broad - narrow are considered as extraordinary conditions. In the quantitative fuzzy approach a normal or central part has to be taken into consideration. We assigned the membership values between 0.33 and 0.66 to be inconspicuous or normal. That means the definition of the thresholds are $a$=0.66 and $a'$ = 0.33. A sigmoid membership function seems to be more suitable than a linear one (Figure 2).

This sigmoid membership function is achieved by the following conditions.
Let $E$ be a set of attributes $E = \{long, high, broad\}$ and let $V$ be the corresponding set of basic values $v$. For every $u \in E$ and every $v \in V$, the membership function $m_u(v)$ is defined as

$$m_u(v) = \begin{cases} 0, & 0 \le v \le 1 \\ 2 * \left(\frac{v-a}{1-a}\right)^2, & a \le v \le \frac{a+1}{2} \\ 1 - 2 * \left(\frac{v-1}{1-a}\right)^2, & \frac{a+1}{2} \le v \le 1 \end{cases}$$

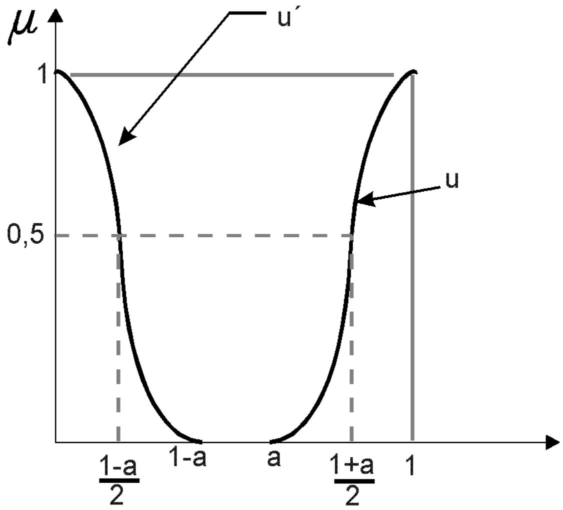


Figure 2: Function for the determination of the membership values to the requested attributes.

For every $u'$ of a second set $E' = \{short,\ flat,\ narrow\}$ containing the complimentary variables, the respective membership function appears as a mirror:

$$m_{u'}(v) = m_u(1-v), \quad 0 \le v \le 1$$

The linguistic description of an individual brain may become even more precise when specifying adjectives are used like very long, very very long etc. In fuzzy mode, this means a further subdivision of the descriptive sets into additional subsets. It has become usage to account this subsets mathematically by taking the exponential form, like

$$m_{very\ u}(v) = \left(m_u(v)\right)^2$$ or

$$m_{very\ u'}(v) = \left(m_u(v)\right)^{\frac{1}{2}}$$, respectively.

An example concerning the attribute ‘long’ is written

$$m_{very\ long}(v) = \left(m_{long}(v)\right)^2 .$$

The description of the brain may be further specified by designing agglomerations of subsets to new compound subsets. A compound 3D set titled with the linguistic variable ‘size’ may be derived by combination of the three orthogonal one-dimensional subsets into one 3D set in space. The set ‘size’ should like the other subsets be divided into three parts ‘large’, ‘normal’, and ‘small’ with the threshold values 0.33 and 0.66. For joining the three one-dimensional subsets into the one three-dimensional set all t-norms operations would be suitable. It seems to reasonable, however, to take the simplest operator, the intersection.

Let $C = \{large, small\}$ be a compound set of attributes composed as an intersections large = long ∩ high ∩ broad and small = short ∩ flat ∩ narrow with the values $w \in W$ .
Then the following formulae

$$m_{large}(w) = \begin{cases} 0, & w \le 0.66 \\ \min\{m_1(v), m_2(v), m_3(v)\} & \end{cases}$$ and

$$m_{small}(w) = \begin{cases} 0, & w \ge 0.33 \\ \max\{m_1(v), m_2(v), m_3(v)\} & \end{cases}$$

are given to evaluate the membership function of the ‘size’ of the brain. A graphic interpretation of the different subsets is given in Figure 3.

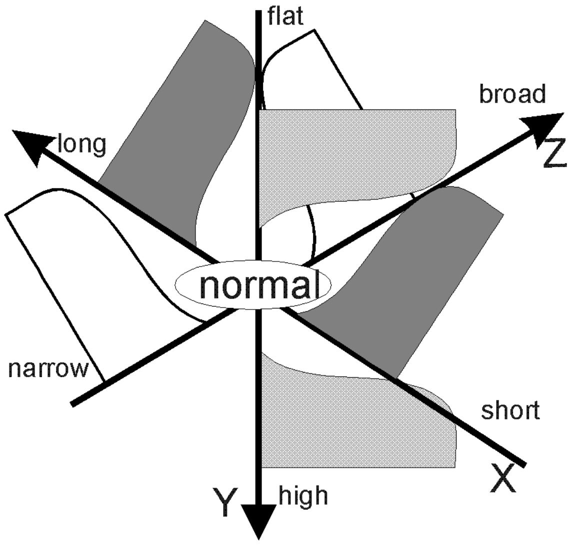


Figure 3: Three-dimensional presentation of the variables with respect to the axes of the brain atlas. The course of the sigmoid functions determines the membership to one single attribute. The central region for all attributes is regarded as 'normal'.

## RESULTS

The resulting memberships for the different attributes 'long', 'high' and 'broad' are given in Figure 4, the memberships for the attributes 'short', 'flat' and 'narrow' are given in Figure 5. It is clearly visible that in our population in attributes 'short', 'flat' and 'narrow' are predominating. In addition, it can be stated that even in those brains that receive one of the large attributes, none of the brains can be described as 'large'. There are on the contrary three brains, namely the numbers 1, 11, and 30, which can be called small (Figure 5).

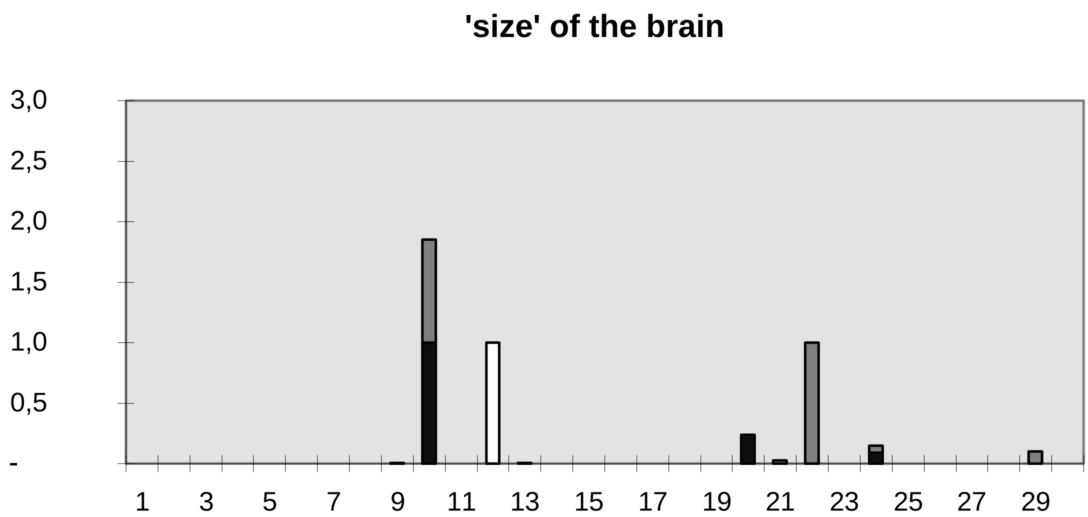


Figure 4: Resulting memberships for the different attributes 'long' (dotted), 'high' (striped) and 'broad'. The memberships are presented in the form of a stack, i.e. a maximum value of 3 can be reached.

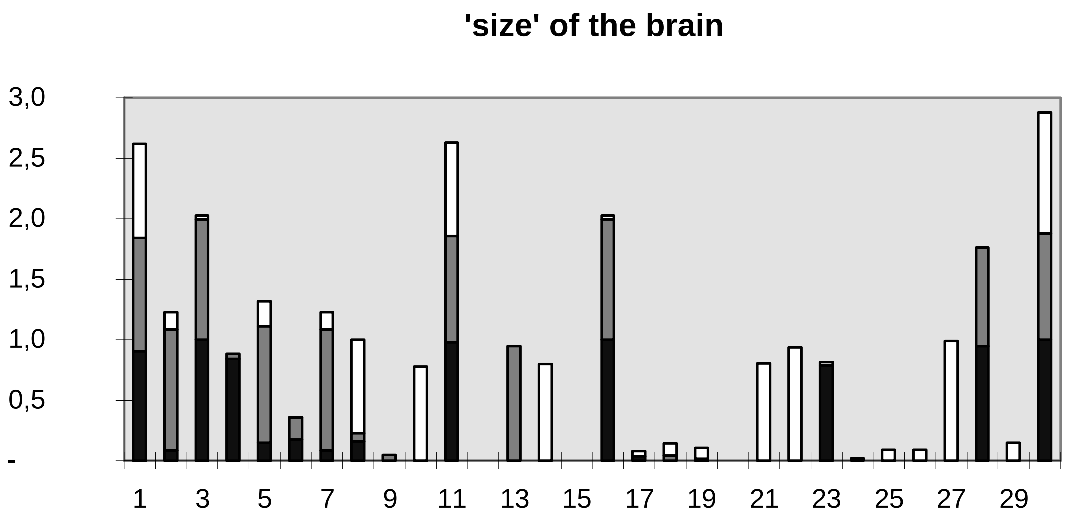


Figure 5: Resulting memberships for the different attributes ‘short’ (dotted), ‘flat’ (striped) and ‘narrow’.

## CONCLUDING REMARKS

Three further compound subsets are imaginable but here we have some linguistic difficulties. Solids usually have two remarkable extremities and one inconspicuous region in-between, and they are oriented orthogonal in space. In human brain pathology, acrocephaly and stenocephaly are medical terms of that kind. However, these terms describe malformations. That means, that there are no corresponding terms in normal anatomy for the described forms. Phrases like ‘oblate ellipsoid’ in the frontal, in the horizontal and in the sagittal plane sound very clumsy. Therefore it seems more suitable to use the intensifier ‘very’ in addition to two-word descriptions in these cases.

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