Multi-Target Assignment Algorithm Using "Soft" Multi-Criteria Decision-Making Methods

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ABSTRACT

Correctly associating target tracks to measurements is critical to the correct working of any multiple-target tracking system. The early assignment algorithms had poor performance because they did not account for missing tracks and observations. In order to raise the performance level, the modern approach is to include missing tracks and measurements. In this paper we present a new approach which raises the performance level without explicitly including missing tracks and measurements. In the new algorithm we generate a subset of high quality track-measurement pairs which optimally satisfy two criteria: (1) Each track-measurement pair has a high likelihood value and (2) The number of track-measurement pairs in the subset is close to the number of correct track-measurement pairs expected on the basis of the system parameters. The two criteria are defined using fuzzy membership functions and the optimal solution is found using "soft" decision making.

1. INTRODUCTION

Suppose that at a given time t there are M_{meas} measureements $\mathbf{Z}_{mi}$, $i \in \{1,2,...,M_{\mathrm{meas}}\}$ and M_{trk} predicted track vectors $\mathbf{Z}_{pj}$, $j \in \{1,2,...,M_{\mathrm{trk}}\}$. For each measurement-track pair we compute the likelihood $\lambda(i,j)$, $i \in \{1,2,...,M_{\mathrm{meas}}\}$, $j \in \{1,2,...,M_{\mathrm{trk}}\}$ that the ith measurement and the jth track form a true measurement-track pair by comparing $\mathbf{Z}_{mi}$ with $\mathbf{Z}_{tj}$. We may regard the likelihood values $\lambda(i,j)$ as forming an $M_{\mathrm{meas}}\times M_{\mathrm{trk}}$ matrix. Assignment algorithms generate the most probable set of measurement-track pairs, subject to two constraints: (1) a measurement can, at most, pair with one track and (2) a track can, at most, pair with one measurement. The performance of the early assignment algorithms was low because they did not take into account missing measurements and missing tracks. The reason for this was that the early assignment algorithms only used the information contained in the $M_{\mathrm{meas}}\times M_{\mathrm{trk}}$ likelihood matrix which, by definition, does *not* include information on missing measurements or tracks. In order to reduce the number of association errors, modern assignment algorithms use an *augmented* likelihood matrix λ_{aug} which includes information on missing measurements and tracks. The size of the augmented matrix λ_{aug} is $(M_{\mathrm{meas}}+M_{\mathrm{trk}})\times(M_{\mathrm{meas}}+M_{\mathrm{trk}})$ where rows 1 to M_{meas} correspond to existing measurements and rows $M_{\mathrm{meas}}+1$ to $M_{\mathrm{meas}}+M_{\mathrm{trk}}$ correspond to missing measurements, and columns 1 to M_{trk} correspond to existing tracks and columns $M_{\mathrm{trk}}+1$ to $M_{\mathrm{trk}}+M_{\mathrm{meas}}$ correspond to missing tracks (or false alarms).

We describe a new approach to solving the assignment problem which does not involve augmenting the likelihood matrix. Instead, we apply the methods of "soft" decision-making [31-38] to the $M_{\mathrm{meas}}\times M_{\mathrm{trk}}$ likelihood matrix λ to generate a *restricted* subset of *highly probable* measurement-track pairs.

2. MATHEMATICAL FORMULATION OF THE ASSIGNMENT PROBLEM

Suppose we may denote the measurement-track pairs using a binary matrix $\mathbf{A}_{\text{early}}$ of size $M_{\text{meas}} \times M_{\text{trk}}$: a one in row *i* and column *j*, ie. $A_{\text{basic}}(i, j) = 1$, denotes a pairing of the *i*th measurement with the *j*th track.

Mathematically, the optimum early solution $\mathbf{A}^*_{\text{early}}$ is the assignment matrix which maximizes

$$\sum_{i=1}^{M_{\text{meas}}} \sum_{j=1}^{M_{\text{trk}}} A(i, j)\lambda(i, j)$$

subject to the constraints

$$\sum_{i=1}^{M_{\text{meas}}} A(i, j) \le 1,\ j = 1,\ldots, M_{\text{trk}}$$

$$\sum_{j=1}^{M_{\text{trk}}} A(i, j) \le 1, i = 1,\ldots, M_{\text{meas}}$$

Similarly the optimum modern augmented solution $\mathbf{A}^*_{\text{aug}}$ is the matrix which maximizes

$$\sum_{i=1}^{M_{\text{meas}}+M_{\text{trk}}} \sum_{j=1}^{M_{\text{meas}}+M_{\text{trk}}} A_{\text{aug}}(i, j)\lambda_{\text{aug}}(i, j)$$

subject to the constraints

$$\sum_{i=1}^{M_{\text{meas}}+M_{\text{trk}}} A_{\text{aug}}(i, j) \le 1,\ j = 1,\ldots, M_{\text{meas}} + M_{\text{trk}}$$

$$\sum_{j=1}^{M_{\text{meas}}+M_{\text{trk}}} A_{\text{aug}}(i, j) \le 1, i = 1,\ldots, M_{\text{meas}} + M_{\text{trk}}$$

NEW SOLUTION

The main feature which distinguishes the new approach to solving the assignment problem from both the early approach and the modern augmented approach is that the number of measurement-track pairs is *not* specified beforehand. Instead, we use soft decision-making to define an *optimal* soft assignment $\mathbf{A}^*_{\text{soft}}$ in which the elements $A^*_{\text{soft}}(i, j) = 1$ denote the set of K^*_{soft} measurement-track pairs which *simultaneously* best satisfy the two constraints:

(1) The number of track-measurement pairs, K^*_{soft} is close to the expected number of track-measurement pairs

(2) The average likelihood of the K^*_{soft} track-measurement pairs is high.

We choose to describe each constraint using the language of fuzzy logic. The first constraint is represented by a membership function $\mu_K(\mathbf{A}_{\text{soft}})$ which we interpret to be the likelihood that, given the existing parameter values, the number of measurement-track pairs in the assignment $\mathbf{A}^*_{\text{soft}}$ is equal to the correct number of measurement-track pairs..

The second constraint is represented by the membership function $\mu_\lambda(\mathbf{A}_{\text{soft}})$

$$\mu_\lambda(\mathbf{A}_{\text{soft}}) = \frac{\bar{\lambda}(\mathbf{A}_{\text{soft}})}{\max(\bar{\lambda}(\mathbf{A}_{\text{soft}}))}$$

where

$$\bar{\lambda}(\mathbf{A}_{\text{soft}}) = \frac{\sum_{i=1}^{M_{\text{meas}}} \sum_{j=1}^{M_{\text{trk}}} A_{\text{soft}}(i,j)\lambda(i,j)}{\sum_{i=1}^{M_{\text{meas}}} \sum_{j=1}^{M_{\text{trk}}} A_{\text{soft}}(i,j)}$$

is the average track-measurement likelihood for a soft assignment $\mathbf{A}_{\text{soft}}$.

Suppose we label the different possible soft assignments as $\mathbf{A}_{\text{soft},1}$, $\mathbf{A}_{\text{soft},2}$, etc. Then the optimal assignment $\mathbf{A}^*_{\text{soft}}$ is the assignment which simultaneously has the largest $\mu_K(\mathbf{A}_{\text{soft}})$ and $\mu_\lambda(\mathbf{A}_{\text{soft}})$ pair of values:

$$\mathbf{A}^*_{\text{soft}} = \arg\max_k \left(\mu_K(\mathrm{A}_{\text{soft},k}) \text{ "AND" } \mu_\lambda(\mathrm{A}_{\text{soft},k})\right)$$

We measured the respective performances of the early, modern and new assignment algorithms by running each algorithm several thousand times in a Monte Carlo simulation.

RESULTS

Preliminary results show that the new solution has a significantly better performance than the early assignment algorithms and a slightly better performance than the modern augmented algorithm.