MULTIOBJECTIVE TRANSPORTATION PROBLEM WITH LINGUISTIC VARIABLES

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ABSTRACT: Part 1 of this paper contains the problem statement. Multiobjective transportation model with linguistic variables in objective functions is described here. The comparison, addition and subtraction operations of linguistic variables are defined in part 2. Also a numerical example of such problem solving is given in part 3. It is based on the algorithm of construction of the spanning subgraph with fuzzy numbers.

KEYWORDS: transportation problem, linear programming, linguistic variables.

1. PROBLEM STATEMENT

In the general case a transporting problem has the following statement [Lyashenko (1975)]. A homogeneous product is to be transported from each of m sources to n destinations. Maximal capacities of sources are equal to a_i, i=1,...,m accordingly. This product is used by n destinations, which have requirements b_j, j=1,...,n. Transportation costs of product unit from i source to j destination are given to all i=1,2,...,m and all j=1,2,...,n and are equal to c_{ij}. It is required to find such transportation volumes X_{ij} from each sources to each destinations so that the total transportation costs would be minimum and requirements of all destinations would be covered.

In most cases transportation costs of product unit are the crisp values. But there are many tasks with fuzzy dates. For example, demand, supply or others parameters could be fuzzy. Any criteria may also be used, for example transportation cost, transportation time etc.

In this paper we will consider one case of transport optimal plan determination, considering that we have to optimize p criteria.

Then this problem can be modeled as follows:

$$\textit{minimize} \quad \sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{c}_{ij}^{1}$$

...

$$\textit{minimize} \quad \sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{c}_{ij}^{p}$$

$$\textit{such that} \quad \sum_{j=1}^{n} x_{ij} = a_i, \quad i = 1,\dots,m,$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \quad j = 1,\dots,n,$$

$$x_{ij} \geq 0.$$

Let we make an optimization on criteria C_k (k=1,2,...,p). Transportation parameters are represented as linguistic variables $(y_k, T_k, U_k, G_k, \tilde{M}_k)$ [Zimmermann (1991)], where

y_k – name of linguistic variable;
T_k – term set of linguistic variable y_k;
U_k – domain of linguistic variable values;
G_k – syntactic rule which generates the terms in the term set;
M_k – semantic rule for associating with each y_k its meaning, $\tilde{M}(Y)$ which is a fuzzy subset of U_k.

This means the transportation parameters can not be negative, then left border of all domains have to be not less then zero, i.e.

$U_1 = \{0, ..., \max_{i,j} c_{ij}^1\}, ..., U_p = \{0, ..., \max_{i,j} c_{ij}^p\}$.

$T_k = \{T_l^k\}$ – term set of the linguistic variable y_k.,

k=1,...,p; l=1,...,q, where q_k – number of terms of linguistic variable y_k..

For each transportation from i sources to j destinations the fuzzy situation can be constructed.

A fuzzy situation is characterized by linguistic variables, the terms of which have varies degrees of truth or belonging to the power set [Zadeh (1975)]:

$$\tilde{s} = \{(m_s(y_k)/y_k)\},\ y_k \in Y,$$

where $m_s(y_k) = \{(m_{ms(yk)}(T_l^k)/T_l^k)\}$, k = 1, ..., p; l = 1, ..., q_k.

$m_{ms(yk)}(T_l^k)$ - value of membership degree to term T_l of linguistic variable y_k.

Then the values in position s_{ij} (i=1,2,…,m; j = 1,2,…,n) in the transportation matrix can be represented as terms of linguistic variable, for example:

$$\begin{aligned}
\tilde{s}_{11} = \{&((m_{ms_{11}(y_1)}(T_1^1)/T_1^1), (m_{ms_{11}(y_1)}(T_2^1)/T_2^1),..., (m_{ms_{11}(y_1)}(T_{q_1}^1)/T_{q_1}^1) / y_1),\\
..., \quad &((m_{ms_{11}(y_k)}(T_1^k)/T_1^k), (m_{ms_{11}(y_k)}(T_2^k)/T_2^k),..., (m_{ms_{11}(y_k)}(T_{q_k}^k)/T_{q_k}^k) / y_k),\\
..., \quad &((m_{ms_{11}(y_p)}(T_1^p)/T_1^p), (m_{ms_{11}(y_p)}(T_2^p)/T_2^p),..., (m_{ms_{11}(y_p)}(T_{q_p}^p)/T_{q_p}^p) / y_p)\}.\\
...&\\
\tilde{s}_{mn} = \{&((m_{ms_{mn}(y_1)}(T_1^1)/T_1^1), (m_{ms_{mn}(y_1)}(T_2^1)/T_2^1),..., (m_{ms_{mn}(y_1)}(T_{q_1}^1)/T_{q_1}^1) / y_1),\\
..., \quad &((m_{ms_{mn}(y_k)}(T_1^k)/T_1^k), (m_{ms_{mn}(y_k)}(T_2^k)/T_2^k),..., (m_{ms_{mn}(y_k)}(T_{q_k}^k)/T_{q_k}^k) / y_k),\\
..., \quad &((m_{ms_{mn}(y_p)}(T_1^p)/T_1^p), (m_{ms_{mn}(y_p)}(T_2^p)/T_2^p),..., (m_{ms_{mn}(y_p)}(T_{q_p}^p)/T_{q_p}^p) / y_p)\}.
\end{aligned}$$

One such situation can be used for each position in transportation matrix.

2. PROBLEM SOLVING

We will use an algorithm of construction of spanning subgraph in the fuzzy bipartite graph [Bershtein, Dzuba (1998)] for such a problem solving. Also in this case we have to determine comparison, addition and subtraction operations of linguistic variables.

2.1. LINGUISTIC VARIABLE COMPARISON

For comparison we can use the degree of fuzzy equality or the degree of fuzzy inclusion. If we have a set of linguistic variables, then we can operate with fuzzy situation. Then we have to describe each situation and define standard description of object states as a fuzzy situations. Standard situation will be such a situation, linguistic variable values of which lead to the best result, i.e. during minimization of objective function we will receive the following set:

S_{st} = {($(m_{ms_{st}(y_1)}(T_1^1)/T_1^1), (m_{ms_{st}(y_1)}(T_2^1)/T_2^1), \ldots, (m_{ms_{st}(y_1)}(T_{q_1}^1)/T_{q_1}^1)$ / y_1),

..., ($(m_{ms_{st}(y_k)}(T_1^k)/T_1^k), (m_{ms_{st}(y_k)}(T_2^k)/T_2^k), \ldots, (m_{ms_{st}(y_k)}(T_{q_k}^k)/T_{q_k}^k)$ / y_k),

..., ($(m_{ms_{st}(y_p)}(T_1^p)/T_1^p), (m_{ms_{st}(y_p)}(T_2^p)/T_2^p), \ldots, (m_{ms_{st}(y_p)}(T_{q_p}^p)/T_{q_p}^p)$ / y_p)},

where

$m_{ms_{st}(y_k)}(T_1^k) = 1,\ m_{ms_{st}(y_k)}(T_l^k) = 0$, (k=1,...,p; l=2,...,$q_p$) that means we make parameter minimization.

Now it is possible to determine the degree of fuzzy equality each situation with this standard situation. We will consider that set the degree of fuzzy equality with standard situation of which is maximum as minimum.

The degree of fuzzy equality $\mu(\tilde{s}_i, \tilde{s}_j)$ of situations $\tilde{s}_i$ and $\tilde{s}_j$ can be defined as follows [Melikhov, Bershtein, Korovin (1987)]:

$$\mu(\tilde{s}_i, \tilde{s}_j) = v(\tilde{s}_i, \tilde{s}_j) \,\&\, v(\tilde{s}_j, \tilde{s}_i),$$

$$v(\tilde{s}_i, \tilde{s}_j) = \underset{y \in Y}{\&} \left(\mu_{\tilde{s}_i}(y) \rightarrow \mu_{\tilde{s}_j}(y) \right)$$

where & - minimum operator,

$$m_{\tilde{S}_i}(y) \rightarrow m_{\tilde{S}_j}(y) = \max\left(1 - m_{\tilde{S}_i}(y), m_{\tilde{S}_j}(y)\right).$$

Or we can use the fuzzy inclusion degree, which is defined as follows:

$$v(\tilde{s}_i, \tilde{s}_j) = \underset{y \in Y}{\&} \left(\mu_{\tilde{s}_i}(y) \rightarrow \mu_{\tilde{s}_j}(y) \right)$$

In any case full order is not provided (two or more sets could have an equal degree of fuzzy equality or fuzzy inclusion with standard situation). Therefore, we must use two criteria for the situation ordering. If it will be not enough either, then as a minimum set the set with minimum rejection from standard situation can be chosen. As a rejection can be used for example, Hamming distance or others.

2.2. LINGUISTIC VARIABLE SUBTRACTION

We will calculate the difference of two linguistic variables, using the same operation as for the fuzzy sets:

$$\tilde{s}_i \setminus \tilde{s}_j = \tilde{s}_i \cap \neg \tilde{s}_j,$$

where $\tilde{s}_i \cap \neg \tilde{s}_j = \min\left(m_{\tilde{S}_i}(y), 1 - m_{\tilde{S}_j}(y)\right)$.

For fuzzy situations subtraction operation is carried out for each value of each linguistic variable. As a result we will also receive the fuzzy situation.

2.3. LINGUISTIC VARIABLE ADDITION

In some cases we must add two situations. There are many addition operations, but in this case it is important that after addition of two situations which have larger transportation parameters then another we could obtain a result value which will be bigger than the addition result of two situations which have smaller transportation parameters. Therefore, we can use the algebraic sum [Zimmermann (1991)] :

$$\tilde{s}_i + \tilde{s}_j = m_{\tilde{s}_i}(y) + m_{\tilde{s}_j}(y) - m_{\tilde{s}_i}(y) \cdot m_{\tilde{s}_j}(y).$$

All those operations will be necessary to transportation problem solving.

3. EXAMPLE

Let there be a transportation task, which is represented as following matrix.

	1	**2**	**3**	$\mathbf{a_i}$
A	$\tilde{s}_{11}$	$\tilde{s}_{12}$	$\tilde{s}_{13}$	30
S = B	$\tilde{s}_{21}$	$\tilde{s}_{22}$	$\tilde{s}_{23}$	70
$\mathbf{b_j}$	20	40	40	

$a_1 = 30$, $a_2 = 70$ – supplies,
$b_1 = 20$, $b_2 = 40$, $b_3 = 40$ – demands.

The numbers in positions s_{ij} are linguistic variables. Optimisation is fulfilled by 2 criteria:

1) minimisation of total transportation cost;
2) minimisation of total transportation time.

Therefore, we can build a fuzzy situations with two linguistic variables:
y_1 = "Transportation cost"; T_1 = {small, average, large}, U_1 = [0,...,100].
y_2 = "Transportation time"; T_2 = {small, average, large}, U_2 = [0,...,30].

Then we receive following situations:

$\tilde{s}_{11}$ = { ((0,3/small), (0,8/average), (0/large) / y_1),
((0,6/small), (0/average), (0/large) / y_2) }.

$\tilde{s}_{12}$ = { ((0,7/small), (0,5/average), (0/large) / y_1),
((1/small), (0/average), (0/large) / y_2)}.

$\tilde{s}_{13}$ = { ((0/small), (0/average), (0,8/large) / y_1),
((0/small), (0/average), (1/large) / y_2) }.

$\tilde{s}_{21}$ = { ((0/small>, (1/average), (0/large) / y_1),
((0/small), (0,8/average), (0/large) / y_2) }.

$\tilde{s}_{22}$ = { ((0/small), (0/average), (1/large) / y_1),
((0/small), (0/average), (1/large) / y_2) }.

$\tilde{s}_{23}$ = { ((0,8/small), (0,4/average), (0/large) / y_1),
((0,3/small), (0,1/average), (0/large) / y_2) }.

That is we minimize two parameters, then we have such a standard situation.
$\tilde{s}_{st}$ = { ((1/small), (0/average), (0/large) / y_1),
((1/small), (0/average), (0/large) / y_2) }.

<u>Solving</u>
We will use an algorithm of construction of spanning subgraph in the fuzzy bipartite graph [Bershtein, Dzuba (1998)] for problem solving.

	1	**2**	**3**	$\mathbf{a_i}$
A	$\tilde{s}_{11}$	$\tilde{s}_{12}$	$\tilde{s}_{13}$	30
S = B	$\tilde{s}_{21}$	$\tilde{s}_{22}$	$\tilde{s}_{23}$	70
$\mathbf{b_j}$	20	40	40	

	20	10		
$\mathbf{X_0}$ =			40	30
		30		

Q = 20.

	(+)		+	
	0*	0'	$\tilde{s}^{(1)}_{13}$	+
$\mathbf{S_1}$ = x	$\tilde{s}^{(1)}_{21}$	$\tilde{s}^{(1)}_{22}$	0	-h
			+h	

$h = \min\{\tilde{s}^{(1)}_{21};\ \tilde{s}^{(1)}_{22}\} = \tilde{s}^{(1)}_{21}$
$v(\tilde{s}^{(1)}_{21}, \tilde{s}_{st}) = 0{,}2;$
$v(\tilde{s}^{(1)}_{22}, \tilde{s}_{st}) = 0.$

For the situation comparison we will use the degree of fuzzy inclusion $v(s_{ij}, s_{st})$ each situation to standard situation.

$v(\tilde{s}_{11}, \tilde{s}_{st}) = 0{,}2;$ $v(\tilde{s}_{21}, \tilde{s}_{st}) = 0;$
$v(\tilde{s}_{12}, \tilde{s}_{st}) = 0{,}5;$ $v(\tilde{s}_{22}, \tilde{s}_{st}) = 0;$
$v(\tilde{s}_{13}, \tilde{s}_{st}) = 0;$ $v(\tilde{s}_{23}, \tilde{s}_{st}) = 0{,}6;$

$\tilde{s}_{21} > \tilde{s}_{11};$ $\tilde{s}_{22} > \tilde{s}_{12};$ $\tilde{s}_{13} > \tilde{s}_{23}.$

$\tilde{s}^{(1)}_{13} = \tilde{s}_{13} - \tilde{s}_{23}$={((0/small), (0/average), (0,8/large)/y_1),
((0/small), (0/average), (1/large)/ y_2)}.

$\tilde{s}^{(1)}_{21} = \tilde{s}_{21} - \tilde{s}_{11}$={((0/small), (0,2/average), (0/large)/y_1),
((0/small), (0,8/average), (0/large)/ y_2)}.

$\tilde{s}^{(1)}_{22} = \tilde{s}_{22} - \tilde{s}_{12}$={((0/small), (0/average), (1/large)/y_1),
((0/small), (0/average), (1/large)/ y_2)}.

$\tilde{s}^{(2)}_{22} = \tilde{s}^{(1)}_{22} - \tilde{s}^{(1)}_{21}$={((0/small), (0/average), (1/large)/y_1),
((0/small), (0/average), (1/large)/ y_2)}.

$\tilde{s}^{(2)}_{13} = \tilde{s}^{(1)}_{13} + \tilde{s}^{(1)}_{21}$={((0/small), (0,2/average), (0,8/large)/y_1),
((0/small), (0,8/average), (1/large)/ y_2)}.

$S_2 =$ x

(+)		(+)	
0*	0’	$\tilde{s}^{(2)}_{13}$	+
0’	$\tilde{s}^{(2)}_{22}$	0*	

$X_1 =$

	30		
20		40	10
	10		

Q = 10.

$S_3 =$ x

+		+	
0	0’	$\tilde{s}^{(2)}_{13}$	+
0	$\tilde{s}^{(2)}_{22}$	0	-h
+h		+h	

$h = s^{(2)}_{22}$

$S_4 =$ x

(+)		(+)	
$\tilde{s}^{(3)}_{11}$	0’	$\tilde{s}^{(3)}_{13}$	+
0*	0’	0*	+

$\tilde{s}^{(3)}_{13} = \tilde{s}^{(2)}_{13} + \tilde{s}^{(2)}_{22}$={((0/small), (0,2/average), (1/large)/y_1), ((0/small), (0,8/average), (1/large)/ y_2)}.

$\tilde{s}^{(3)}_{11} = \tilde{s}^{(2)}_{22}$={((0/small), (0/average), (1/large)/y_1), ((0/small), (0/average), (1/large)/ y_2)}.

$X_2 =$

	30		
20	10	40	= X_{opt}

This algorithm takes into account fuzzy transportation parameters, but that is the supplies and the demands were crisp values then we obtain a crisp optimal solution of transportation problem.

CONCLUSION

This approach can be used in multiobjective transporting problem if we have the linguistic uncertainty in objective functions and the crisp constrains. It does not require any preparatory calculations and the operations with fuzzy situations are simple. If we have more sources and more destinations, then it will be not more then *mn* comparison operation for each algorithm stage. If we have more than two criteria, then we receive only more linguistic variables for each fuzzy situation, but not more situations. It proofs that the using of situational approach simplifies this task solving.

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