A Fuzzy Multiattribute Evaluation Method

Cengiz Kahraman* **Ethem Tolga****

*Istanbul Technical University
Department of Industrial Engineering
80680 Macka-Istanbul-Turkey
E-mail: cengiz2@ayasofya.isl.itu.edu.tr
**Galatasaray University
Faculty of Engineering and Technology
80840 Ortakoy-Istanbul-Turkey
E-mail: tolga@gsunv.gsu.edu.tr

ABSTRACT: In crisp multiple attribute decision making models it is usually assumed that the final judgements of the alternatives are expressed as real numbers. Fuzzy models are sometimes justified by the argument that the goals, g_j , or their attainment by the alternatives, x_i , respectively, can not be defined or judged crisply but only as fuzzy sets. The final judgements are also represented by fuzzy sets which have to be ordered to determine the optimal alternative. In the paper, two computer integrated manufacturing (CIM) systems are evaluated by using a fuzzy multiple attribute method with a hierarchical procedure for determining the weights of the attributes. This is done by computing the eigenvectors of the matrix of the relative weights of subjective estimates. The subjective attributes considered are quality of results, ease of use, competitive advantage, adaptability, and expandability. The objective attribute considered is net present worth. The optimal CIM alternative is the one with the highest degree of membership in the fuzzy set decision.

KEYWORDS: fuzzy sets, hierarchy, multiple attribute, computer integrated manufacturing

1. INTRODUCTION

The crisp weighted evaluation method involves first transforming any objective (economic) measure of merit for each alternative into a score between 0 and 1 so that the sum of the objective scores totals 1. For subjective attributes, nominal ratings are transformed into numerical scores, and the relative importances of the attributes are weighted so that the scores of each attribute over all attributes and the weighted subjective scores over all attributes each sum to 1. Next, weights are assigned to the objective and the subjective scores such that those two weights sum to 1. The combined weighted scores (which also sum to 1) show the relative desirability for each alternative (Canada and Sullivan, 1989). The combined measure of weighted evaluation for each alternative k, We_k, is

$$WE_k = (a)(OM_k) + (1-a)(SM_k) \quad (1)$$

where OM_k :objective measure for alternative k; SM_k : subjective measure for alternative k. a : relative importance weighting for OM_k and $0 \leq OM_k, SM_k, a \leq 1$.

Klee (1971) was among the first researchers to propose a linear additive model for scoring competing alternatives. This model involves weighting attributes. Soni *et al.* (1992) propose the Pearson product-moment coefficient of correlation to be used for the situations where the assumptions for linear additive methodology are grossly violated. Canada (1986) use a non-traditional method for evaluating CIM opportunities, assigning weights to intangibles. Park *et al.* (1990) use an integrated economic and strategic approach for investment decision, assigning weights to strategic factors.

2. FUZZY MULTIATTRIBUTE EVALUATION METHOD

The concept of a fuzzy set deals with the representation of classes whose boundaries are not quite determined. It uses a characteristic function, taking values usually in the interval $[0,1]$. A major contribution of fuzzy set theory is its capability of representing vague knowledge. Zadeh (1965) first introduced the fuzzy set theory, which was oriented to the rationality of number is a normal and convex fuzzy set with membership function $m(x)$ which both satisfies normality: $m(x)=1$ for at least one $x \in$ R, and convexity: $m(x') \geq m(x_1) \Lambda m(x_2)$ where Λ stands for min operator and $m(x) \in [0,1]$ and $\forall x' \in [x_1, x_2]$.Fuzzy numbers are very useful in promoting the representation and processing of information under fuzzy environment. A trapezoidal fuzzy number can be denoted by (a,b,c,d) while a triangular fuzzy number can be denoted by (a',b',c'). A trapezoidal fuzzy number and a triangular fuzzy number are depicted in Figure 2 and Figure 3 respectively. a and a' denote the smallest possible value. d and c' denote the largest possible value. $[b,c]$ and b' denote the most promising value. The membership functions of a trapezoidal fuzzy number and a triangular fuzzy number are given in Eq.(2) and Eq.(3) respectively. $f_1(.)$ is the function of the left side representation of a fuzzy number while $f_2(.)$ is the function of the right side representation of a fuzzy number.

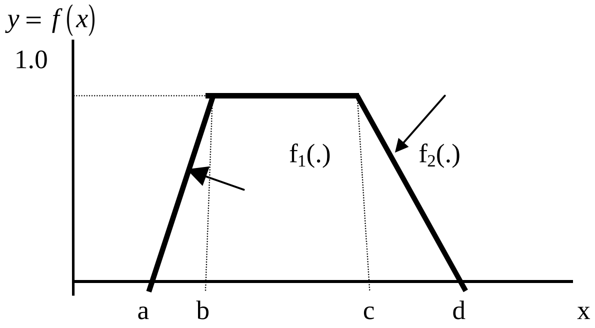

Figure 2. A Trapezoidal Fuzzy Number

$$f(x) = \begin{cases} 0 & , x \leq a \\ (x-a)/(b-a) & , a \leq x \leq b \\ 1 & , b \leq x \leq c \\ (x-d)/(c-d) & , c \leq x \leq d \\ 0 & , x \geq d \end{cases} \quad (2)$$

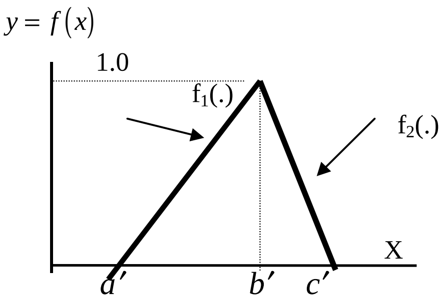

Figure 3. A Triangular Fuzzy Number

$$\begin{array}{ll} 0 & , x \leq a' \\ (x-a')/(b'-a') & , a' \leq x \leq b' \end{array}$$

$$f(x) = \begin{cases} (x - c')/(b' - c'), & b' \le x \le c' \\ 0, & x \ge c' \end{cases} \quad (3)$$

Algebraic operations on fuzzy numbers are given in the appendix. ' ~ ' will be used above the symbol indicating a fuzzy number from now on.

Liao (1996) presents a fuzzy multi-criteria decision-making method developed to support material selection decisions in engineering design applications. The methodology first asks the decision maker - the design engineer- to provide the necessary information on relevant material properties, their desired values, and their respective importance weights. Saaty's procedure is used to obtain a ratio scale for group of elements based on a paired comparison of each of the elements. Kahraman *et al.* (1996) develop a fuzzy economic and strategic model to evaluate motorcars. Kahraman and Tolga (1996) present a multi-attribute fuzzy scoring model to select the best technology.

Fuzzy Subjective Measure

Subjective attribute weights are assigned with a hierarchical procedure by computing the eigenvectors of the matrix of relative weights of subjective estimates. The procedure is as in the following:

Let $X = \{x_1, \ldots, x_n\}$ be a set of alternatives. The goals are represented by the fuzzy sets $\tilde{G}_j$, $j = 1, \ldots, m$. The '*importance'* (weight) of goal *j* is expressed by w_j. The '*attainment'* of goal $\tilde{G}_j$ by alternative x_i is expressed by the degree of membership $m_{\tilde{G}_j}(x_i)$. The decision is defined as the intersection of all fuzzy goals, that is,

$$\tilde{D} = \tilde{G}_1^{w_1} \cap G_2^{w_2} \cap \ldots \cap G_m^{w_m} \quad (4)$$

and the optimal alternative is defined as that achieving the highest degree of membership in $\tilde{D}$. The weights are used as exponents to express the importance of a goal. The higher the importance of a goal the larger should be the exponent of its representing fuzzy set, at least for normalized fuzzy sets and when using the min-operator for the intersection of the fuzzy goals (Zimmermann, 1994).

Yager (1982) concentrates on the problem of determining the weights of the goals. As a solution that problem he suggests Saaty's hierarchical procedure for determining weights by computing the eigenvectors of the matrix *M* of relative weights of subjective estimates. Yager mentions that *the membership grade in all objectives having little importance (w>1) become smaller. This has the effect of making the membership function of the decision subset D, which is the min value of each X over all objectives, being more determined by the important objectives, which is at it should be. Furthermore, this operation (min) makes particularly small those alternatives that are bad in important objectives, therefore when we select the* x_i *that maximizes D, we will be very unlikely to pick one of these.*

The solution procedure can be described as follows: Given the set $X = \{x_1, \ldots, x_n\}$ and the degrees of membership $m_{\tilde{G}_j}(x_i)$ of all x_i in the fuzzy sets $\tilde{G}_j$ representing the goals (Zimmermann, 1994).

1. Establish by pairwise comparison the relative importance, a_i, of the goals among themselves. Arrange the a_i in a matrix *M*.

$$M = \begin{bmatrix} \frac{a_1}{a_1} \frac{a_1}{a_2} \cdots \frac{a_1}{a_n} \\ \frac{a_2}{a_1} \frac{a_2}{a_2} \cdots\cdots \\ \cdots\cdots\cdots\cdots \\ \frac{a_n}{a_1} \frac{a_n}{a_2} \cdots \frac{a_n}{a_n} \end{bmatrix} \quad (5)$$

2. Determine consistent weights w_j for each goal by employing Saaty's eigenvector method.

3. Weight the degrees of goal attainment, $m_{\tilde{G}_j}(x_i)$ exponentially by the respective w_j. The resulting fuzzy sets are $\left(\tilde{G}_j(x_i)\right)^{w_j}$.

4. Determine the intersection of all $\left(\tilde{G}_j(x_i)\right)^{w_j}$:

$$\tilde{D} = \left\{ \left(x_i, \min_j \left(m_{\tilde{G}_j}(x_i) \right)^{w_j} \right) \middle| i = 1, \ldots, n;\ j = 1, \ldots, m \right\} \quad (6)$$

5. Select the x_i with largest degree of membership in $\tilde{D}$ as the optimal alternative.

Fuzzy Objective Measure

The fuzzy objective measure for each alternative *k*, $FO\tilde{M}_k$, can be calculated by using the following fuzzy present worth-based formula,

$$FO\tilde{M}_k = \frac{FP\tilde{W}_k}{Max\left(FP\tilde{W}_k,\ k = 1,2,\ldots,K\right)} \quad (7)$$

where *K* is the total number of alternatives and $FP\tilde{W}_k$ is the fuzzy present worth of alternative *k*. $FP\tilde{W}_k$ can be calculated by using the following fuzzy present worth formula:

$$f_s\left(y \middle| FP\tilde{W}_k\right) = \sum_{n=1}^{N} f_s\left(y \middle| \tilde{C}_n\right)\left[1 + f_{3-s}\left(y \middle| \tilde{i}\right)\right]^{-n} \quad (8)$$

where y shows that $FP\tilde{W}_k, \tilde{C}_n, \tilde{i}$ are the functions of membership degrees. $\tilde{C}_n$ is the fuzzy net cash flow of alternative *k*. $\tilde{i}$ is the fuzzy discount rate assumed to be constant during the project life. *N* is the non-fuzzy project life. *s* shows the left side representation of a triangular fuzzy number when *s=1* while '*s=2*' shows the right side representation.

3. RANKING METHODS FOR FUZZY NUMBERS

There are a number of methods that are devised to rank mutually exclusive projects such as Chang's Method (1981), Jain's Method (1976), Dubois and Prade's Method (1983), Yager's method (1980), Baas and Kwakernaak's Method (1977). However, certain shortcomings of some of the methods have been reported in Bortolan and Degani (1985), Chen (1985), and Kim and Park (1990). Because the ranking methods might give different ranking results, they must be used together to obtain the true rank. Chui and Park (1994) compare some ranking methods by using a numerical example and determine which methods give the same or very close results to one another. Chui and Park's (1994) weighting method and Kaufmann and Gupta's (1988) method are the two of these methods and these methods are easy to calculate and require no graphical representation.

Chiu and Park's (1994) weighted method for ranking TFNs with parameters (a, b, c) is formulated as

$$\left((a+b+c)/3\right)+wb$$

where *w* is a value determined by the nature and the magnitude of the most promising value. Kaufmann and Gupta (1988) suggest three criteria for ranking TFNs with parameters (a,b,c). The dominance sequence is determined according to priority of:

1. Comparing the ordinary number (a+2b+c)/4
2. Comparing the mode, (the corresponding most promise value), b, of each TFN.
3. Comparing the range, c-a, of each TFN.

The preference of projects is determined by the amount of their ordinary numbers. The project with the larger ordinary number is preferred. If the ordinary numbers are equal, the project with the larger corresponding most promising value is preferred. If projects have the same ordinary number and most promising value, the project with the larger range is preferred.

4. A NUMERIC EXAMPLE

Alternative two computer integrated manufacturing systems (CIM) are expected to have the following fuzzy first costs and fuzzy net cash flows:

Table 1. Fuzzy Cash Flows ($\$x10^4$)

Year	CIM_I	CIM_2
0	(-12,-10,-9)	(2.5, 3.5, 5)
1	(-8, -7.5,-5.5)	(4, 6, 6.5)
2	(7, 9, 10.5)	(5.5, 7, 8.5)

$\tilde{i} = (0.07 + 0.02y, 0.12 - 0.03y)$

For the same alternative CIM systems for which fuzzy cash flows are shown above, Table 2 gives evaluation ratings for each alternative regarding the attributes. Membership degrees are used for ratings. So, the sum does not need to be equal to 1 for each attribute.

Table 2. Evaluation Ratings with Membership Degrees

Subjective Attribute	Alternatives	
	CIM_I	CIM_{II}
QR	0.7	0.4
EU	0.55	0.65
CA	0.8	0.4
A	0.4	0.7
E	0.45	0.75

The matrix of relative weights of subjective estimates is given in the following :

Table 3. The Matrix of Subjective Attribute Weights

	QR	EU	C	A	E
QR	1	3	7	7	9
EU	1/3	1	3	3	7
C	1/7	1/3	1	1	5
A	1/7	1/3	1	1	5
E	1/9	1/7	1/5	1/5	1

Using the matrix of subjective attribute weights in Table 3 and Saaty's method, the following vector is obtained:

$$w_{aw}=(0.537, 0.230, 0.100, 0.100, 0.033)$$

Exponential weighting of the attributes by their respective weight yields

$$Q\tilde{R}_{I,II}^{0.537} = \{(I,0.826),(II,0.611)\}$$
$$E\tilde{U}_{I,II}^{0.230} = \{(I,0.872),(II,0.906)\}$$
$$C\tilde{A}_{I,II}^{0.100} = \{(I,0.978),(II,0.912)\}$$
$$\tilde{A}_{I,II}^{0.100} = \{(I,0.912),(II,0.965)\}$$
$$\tilde{E}_{I,II}^{0.033} = \{(I,0.974),(II,0.991)\}$$

The fuzzy set decision $\tilde{D}$, as the intersection of the sets above becomes

$$\tilde{D} = \{(I,0.826),(II,0.611)\}$$

and the optimal alternative regarding subjective measures is alternative I with a degree of membership in $\tilde{D}$ of $m_{\tilde{D}}(I) = 0.826$.

Fuzzy present worths are calculated as in the following:

$$f_1\left(y \middle| FP\tilde{W}_I\right) = (20{,}000y - 120{,}000) +$$
$$(20{,}000y + 40{,}000)(1.12 - 0.03y)^{-1} +$$
$$(20{,}000y + 70{,}000)(1.12 - 0.03y)^{-2}$$

$$f_2\left(y \middle| FP\tilde{W}_I\right) = (-10{,}000y - 90{,}000) +$$
$$(65{,}000 - 5{,}000y)(1.07 + 0.02y)^{-1} +$$
$$(105{,}000 - 15{,}000y)(1.07 + 0.02y)^{-2}$$

$$f_1\left(y\middle|FP\tilde{W}_{II}\right) = \left(5{,}000y - 80{,}000\right) +$$
$$\left(10{,}000y + 25{,}000\right)\left(1.12 - 0.03y\right)^{-1} +$$
$$\left(15{,}000y + 55{,}000\right)\left(1.07 + 0.02y\right)^{-2}$$

$$f_2\left(y\middle|F\tilde{P}_{II}\right) = \left(-20{,}000y - 55{,}000\right) +$$
$$\left(50{,}000 - 15{,}000y\right)\left(1.07 + 0.02y\right)^{-1} +$$
$$\left(85{,}000 - 15{,}000y\right)\left(1.07 + 0.02y\right)^{-2}$$

$FP\tilde{W}_I$ and $FP\tilde{W}_{II}$ are obtained as triangular fuzzy numbers:

$$FP\tilde{W}_I = \left(-\$28{,}482.15;\$30{,}797.07;\$62{,}458.72\right)$$

$$FP\tilde{W}_{II} = \left(-\$13{,}832.91;\$16{,}027.69;\$65{,}971.26\right)$$

Chui and Park’s ranking method (*w*=0.3):

For alternative I:

((a+b+c)/3)+wb=30,830.33

For alternative II:

((a+b+c)/3)+wb=27,530.32

According to Chui and Park’s ranking method, alternative I is selected. Using the priority of Kaufmann and Gupta’s ranking method:

For alternative I:

(a+2b+c)/4=23,892.677

For alternative II:

(a+2b+c)/4=21,048.432

are obtained. Alternative I with the larger ordinary number is selected.

The results of both objective measure and subjective measure indicate that CIM_I is superior to CIM_{II}.

Alternatively using eq. (7), we obtain

$$FO\tilde{M}_I = \frac{FP\tilde{W}_I}{FP\tilde{W}_I} = 1$$

and

$$F\tilde{PW}_{II} = \frac{F\tilde{PW}_{II}}{F\tilde{PW}_{I}} \prec \tilde{1}$$

where $\tilde{1}$ is $(1,1,1)$.

Because $FO\tilde{M}_I \succ FO\tilde{M}_{II}$, CIM_I is selected.

5. Conclusions

Weighting of attributes involves quantifying the relative importance of each attribute and the most important usual theoretical restriction in the naming of attributes is that they be independent of one another. Assigning numerical values regarding the degree to which each alternative satisfies each attribute is generally a difficult judgment task using an arbitrary scale. Using fuzzy numbers instead of ordinal numbers may prevent decision-makers to make some erroneous assignments for objective measures. The hierarchical procedure used in the paper is appropriate to select the best alternative when the evaluating process has more than one attributes. Fuzzy sets theory is a significant tool for the problems with multi-attributes which require some subjective evaluations. Because advanced manufacturing technologies (AMTs)have many subjective and strategic attributes like competitive advantage, quality improvement, flexibility, adaptability, and expandability, the justification of AMTs is one of the areas which fuzzy sets theory can be applied successfully.

Appendix

In the following, some of the extended algebraic operations of triangular fuzzy numbers are expressed. Assume P=(a,b,c) and Q=(d,e,f). a,b,c,d,e,f are all positive numbers.

<u>Changing Sign</u>

-(a, b, c) = (-c, -b, -a)
or -(d, e, f) = (-f, -e, -d)

<u>Addition</u>

P⊕Q=(a+d, b+e, c+f)
and k ⊕ (a, b, c) = (k+a, k+b, k+c)
or k ⊕ (d, e, f) = (k+d, k+e, k+f)
if k is an ordinary number (a constant).

<u>Subtraction</u>

P-Q= (a-f, b-e, c-d)
and (a, b, c) - k = (a-k, b-k, c-k)
or (d, e, f) - k = (d-k, e-k, f-k)
if k is an ordinary number.

<u>Multiplication</u>

P⊗Q= (ad, be, cf)
and k ⊗ (a, b, c) = (ka, kb, kc)
or k ⊗ (d, e, f) = (kd, ke, kf)
if k is an ordinary number.

<u>Division</u>

$P \varnothing Q = (a/f, b/e, c/d)$

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