A GENETIC ALGORITHM TO TRANSIT FROM FUZZY ENVIRONMENT TO EVIDENTIAL ENVIRONMENT

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Abstract

This paper is intended to deal with the transition from a fuzzy environment to an evidential environment. The transition process uses genetic algorithm to search an optimal consonant belief function which can be associated to the initial membership function. The population fitness value is calculated based on two parameters: the belief function information quantity and an error, characterizing the difference between the original membership function and a new membership function defined from the desired consonant belief function.

Key words:

fuzzy sets, theory of evidences, genetic algorithm, membership function, belief function.

1. Introduction

The problem of transition from fuzzy environment to evidential environment has received attention of many searchers and continues to be attractive [Dubois and Prade 1990, Dubois, Prade and Sandra 1993, Wang ,1982]. Researches in this way can be useful, particularly in an heterogeneous uncertain and imprecise environment where different sources of uncertain information are involved. Decision making in such environment needs to develop decision support system where many computing methods can contribute together to solve decision problems.

Developing a multi-method reasoning decision support system engages designers to build bridges between those methods. A special interest is given by this paper to the transition from Fuzzy sets to the theory of evidences, two famous and known uncertainty computing methods which are frequently used in decision support systems.

A fuzzy set is characterized by a membership function for which a consonant belief function can be associated [Dubois D. and Yager R., 1992]. We use a genetic algorithm optimization technique based on the maximization of consonant belief function information quantity and the minimization of the error produced from the difference between the original membership function and a new membership function which is derived from the equivalent consonant belief function.

2. fuzzy set theory

Fuzzy sets theory is due to Lotfi Zadeh, professor in Berkeley university[Zadeh L. 1965]. In his original paper, Zadeh formally defined fuzzy sets theory as an extension of the traditional set theory to resolve problems sometimes generated by hard and rigid "nothing or all" classifications of Aristotelian logic. In fuzzy logic sets can be defined qualitatively using linguistic terms (low, medium, high, and so on). For each element of a fuzzy set, a degree of membership is assigned. This degree of membership is defined by a function, appropriately called a membership function. The domain of membership function (that it what goes into a membership function) is the set of possible values for a given variable.

For a fuzzy subset A of the universe of discourse Ω, a membership function is defined as follows:

$$\mu_A: \quad \Omega \longrightarrow [0,1]$$
$$x \longrightarrow \mu_A(x)$$

where $\mu_A(x)$ is the degree of membership of the element x to A.

3. Theory of evidences

This theory started by Dempster on his own attempting to model uncertainty by a range of probabilities instead of a single probabilistic number [Dempster, 1976]. Shafer then extended Dempster's work in a book of his own entitled "A Mathematical Theory of Evidence," [Shafer 1976].

As defined by Shafer, uncertainty in this environment is modeled by a range of probabilities rather than a single probabilistic number. Basic probability assignments are called masses and they are linked to elements of environment. An element of the environment (or frame of discernment) is called focal element and it can not be dissociated.

The mass is formally expressed as a function which maps each element of the power set into a real number in the interval [0,1]. This mapping is formally stated as follows:

$$m: \quad P(\Omega) \longrightarrow [0,1]$$
$$A \longrightarrow m(A)$$

With $m(\varnothing) = 0$ and $\sum_{A \notin P(\Omega)} m(A) = 1$

where $0 < m(A) < 1$.

The belief in an evidence is the sum of masses of focal elements which fully support it. It is formulated as follows:

$\text{Bel}[A] = \sum_{B \subseteq A} m(B)$, $\text{Bel}(\varnothing) = 0$ and $\text{Bel}(\Omega) = 1$.

Plausibility measure is the dual of the belief and it is interpreted as the sum of masses of focal elements which partially or fully support it:

$$\text{Pl}[A] = \sum_{A \cap B \neq \varnothing} m(B)$$

with: $\text{Pl}(A) = 1 - \text{Bel}(\overline{A})$ and $\text{Pl}(A) >= \text{Bel}(A)$.

The ignorance is modeled by the difference:

$$\text{Bel}(A) - \text{Pl}(A).$$

It is interpreted as the degree to which the mass supports A and $\overline{A}$ in the same time.

Note that : $\text{Bel}(A) + \text{Bel}(\overline{A}) <= 1$

which means that a lack of belief in $x \in A$ does not imply a strong belief in $x \in \overline{A}$ [Dubois, Prade 1980].

4. Passage from Fuzzy Set Theory to Dempster-Shafer Theory

Zadeh proposed the consonant belief function as a solution to this problem [Dubois D., Prade H, 1990]. A consonant belief function is a belief function with nested focal elements.

Given a membership

$$m_A : \Omega \longrightarrow [0,1]$$
$$x \longrightarrow m_A(x)$$

we can fix n real number $a_1, a_2, \ldots, a_n$ with :

$$0 < a_i <= 1.$$

For each a we associate a level cut:

$$A_a = \{x / m_A(x) >= a\}.$$

It is easy to check that the $A_{\alpha i}$ are nested:

$$A_{a_1} \subset A_{a_2} \subset \ldots \subset A_{a_n}$$

The consonant random set $R_A = \{(A_{a_i}, m_i), i=1,n\}$ such that $m_i = a_i - a_{i+1}$, is equivalent to a random set A in the sense that m_A and mi are bijectively linked that is [Dubois and Prade, 1990]:

$$m_F(x) = \sum_{x \in Ai} m(Ai)$$

5. Genetic Algorithms

John Holland, Professor at the university of Michigan, developed Genetic algorithms (GAs) based on Darwin's basic natural principle of survival of fittest, mutation, and cross-breading. [Goldberg 1989].

In GAs, the natural parameter set of the optimization problem is needed to be coded as a finite-length string over some finite alphabet. The smallest unit of a GAs is called a gene. A gene represents a unit of information and a series of genes or sting is called chromosome., each string is called chromosome.

To determine if a chromosome is a good solution for a particular problem or not, the GAs uses a decoding module which decodes a chromosome into a solution for the problem. Another module called fitness function determines which chromosome solutions are good or not. The fitness function does the same thing as the objective function in a traditional optimization method.

GA creates an initial population of different chromosomes and begins an iterative process of information refinement in the chromosomes using three operations: selection, crossover, and mutation. These operations produce new chromosomes which form a new population. Each new population is called generation.

The selection or reproduction allows the GA to choose ‘fitter’ chromosomes to remain and multiply in the next generation, but eliminates the poor ones.[Goldberg 1997].

During the crossover two chromosomes basically swap some of their information gene by gene. It allows the combination of the elements within one solution with those of another (example from 1 to 0 or from 0 to 1). This allow GA ‘to share the wealth” (“or spread the misery”).

The mutation operator changes the value of a gene from its current setting to a different one. Mutation can introduce new solutions that may not have existed in the current population.

6 The use of Genetic Algorithm to transit from Fuzzy environment to Evidential Environment

The basic idea of this passage is to construct a belief structure:

(A1; m1, A2; m2,…,An; mn)

from a membership function :

$$m_{1A}: \Omega \longrightarrow [0,1]$$
$$x \longrightarrow m_{1A}(x)$$

where m is defined as follows:

$$m: P(\Omega) \longrightarrow [0,1]$$
$$A \longrightarrow m(A)$$

$$\text{with} \quad m(Ai) = \sum_{x \in Ai} m_{1A}(x) / |Ai|$$

A new membership function is to be constructed from this new belief structure:

$$m_{2A}: \Omega \longrightarrow [0,1]$$
$$x \longrightarrow m_{2A}(x)$$

$$\text{with:} \quad m_{2A}(x) = \sum_{x \in Ai} m(Ai).$$

To decide whether this passage is acceptable or not, we have to minimize the least square error:

$$\text{Error} = \sum_{x \in \Omega} (m_{1A}(x) - m_{2A}(x))^2$$

if the error is not acceptable (it is great than a fixed threshold) then we have to go back and construct a new belief function , restart a new iteration and so on.

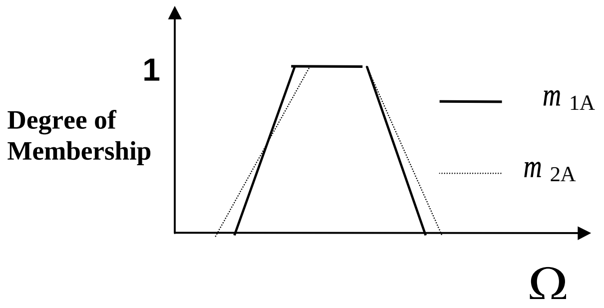

Figure 6.1: Membership Function Approximation

6.1 Coding Technique

A population is a set of chromosomes, where each chromosome (string of binary digits) represents a belief structure (A_0,m_0, A_1;m_1, …,A_{m-1}; m_{m-1}) . A chromosome is coded as follows:

Bel_k : is a belief structure:

	A_{m-1}	…	A_i	…	A_0
$Bel_{k=0,m-1}$	b_{m-1}	…	b_i	…	b_0

where

$$b_{i=0,m-1} = \begin{cases} 1 \text{ if } A_i \in Bel_k \quad /^*Bel_i \text{ is a belief structure} \\ 0 \text{ else } /^* A_i \notin Bel_k \end{cases}$$

with $m = 2^n$, $A_{i=1,m} \in 2^{\Omega}$ and $\Omega = \{x_1, x_2, …,x_n\}$, A_i is a focal element.

Ai is coded as follows:

	x_{n-1}	…	x_j	…	x_0
Ai	c_{n-1}	…	c_j	…	c_0

where

$$c_{j=0,n-1} = \begin{cases} 1 \text{ if } x_j \in Ai \text{ (focal element)} \\ 0 \text{ else } /^* x_j \notin A_i \end{cases}$$

6.2 Population Fitness Value

population fitness value to be maximized is the sum of two values:

- the consonant information content in the desired belief function Infb and
- $n - \sum_{x \in \Omega} (m_{1A}(x) - m_{2A}(x))^2$.

where n is the universe of discourse size (our discussion concerns a finite size) .

A measure of the information content of an evidence seems to be a appropriate mean to refine our search of optimal consonant belief function. Philippe

Smets in his paper « Information content of an Evidence introduced this measure Infb of information content of a belief function» [Smets, 1985, Klir, 1987]. He defined the information content Infb of an evidence of a belief function as :

$$Infb = - \sum_{A \subseteq X} \log Q(A),$$

$$\text{with } Q(A) = \sum_{A \subseteq B} m(B).$$

7. Application for marketing strategic decision

Commercialization of Tunisian dates is organized by the GID (Groupement Interprofessionnel des dattes) which is a public organism charged by the regulation of Tunisian date price fluctuation. The GID decision makers mentioned that Tunisian exporters didn't respect this fixed minimum selling Tunisian date price (MSTDP) and belief that is due to its non realistic aspect. In a multi-method reasoning executive support system which we are developing , we apply this genetic algorithm to transit from a fuzzy expert thinking to an evidential expert thinking.

The universe of discourse is:

$$\Omega = \{x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7\}$$

where x_i is the price of 1kg of dates in Tunisian Dinar (TD).

Figure 7.1 shows that the expert MTDSP membership function is a trapezoidal function.

Figure 7.1 : Realistic MSTDP membership function

Variable	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7
value (TD)	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9
$\mu(x_i)$	0.25	0.75	1	1	1	0.75	0.5	0.25

Table 7.1: MTDSP membership function values

The result of genetic algorithm research of optimal consonant belief function is given by the Table 7.2.

Focal Element (MSTDP is realistic)	Masse
{x6 }	0.0263
{x5 x6 }	0.0614
{x4 x5 x6 }	0.0965
{x3 x4 x5 x6 }	0.1316
{x3 x4 x5 x6 x7 }	0.1404
{x2 x3 x4 x5 x6 x7 }	0.1667
{x1 x2 x3 x4 x5 x6 x7 }	0.1842
{x0 x1 x2 x3 x4 x5 x6 x7 }	0.1930

Table 7.2: Expert equivalent consonant belief function of µ1 .

8. Conclusion

The use of genetic algorithm to transit from a fuzzy environment to an evidential environment demonstrated a remarkable efficiency to reach the optimal solution. During the research process, new populations have been created and poor populations have been dropped. A good choice of the initial population can give better results. We notice also that the genetic algorithm makes time to reach the optimal solution. This problem can be resolved by using advanced operators and techniques of genetic algorithms.

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