Genetic Algorithms for Optimal Weight Design Problem of Frame Structure

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ABSTRACT: In this paper, we formulate an optimal weight design (OWD) problem of a 30 stories frame structure for a constrained allowable stress as a statically in determinate structure problem and solve it directly by keeping the constraints based on an improved genetic algorithm (GA). We discuss the efficiency between the proposed method and the discretized optimum criteria methods.

KEYWORDS: Frame structure, Genetic Algorithm

1.Introduction

Generally, the optimal design of a framed structure is executed by combining structure analysis with an optimization method and then the iterative retrieval method is used as the search technique.

This method, though, has a large number of design variables which increases the amount of calculation and hence increases the analysis time. One of the methods proposed to solve the above problem is the suboptimization method for frame structure analysis of multimembers.

D. Kavlie and J. Moe[1] simplified the analysis by gathering the cross section constants (flange cross section, web cross section, height of web girder) into 1 group and analyzing the structure using the penalty coefficient of the SUMT method (by doing the suboptimization of the edge force of each member, and by doing the optimization of the whole structure using the same penalty coefficient).

Ito[2] obtained the joint force of the substructure by whole structure analysis and did the suboptimization using the SUMT (Sequential Unconstrained Minimization Technique). In addition, they adopted an approximate solution of the whole structure by using the difference between the resultant value to the initial value. They also performed the whole optimization by carrying out the convergence decision in every suboptimization.

U. Kirsch, M. Reiss, and U. Shamir[3] adopted the calculation method which anticipated that the whole optimum is almost a cluster of the suboptimum. They set the boundaries conditions of the substructures using the balance of the force of the whole structure. Afterwards, they finished the optimization of the substructure independently. In addition, they performed the optimization of the whole structure, but not in all cases.

Y. Honma, *et al*.[4] proposed the two methods that follow:

1)They obtained the direction of the suboptimization. Next they obtained the variable of the substructure, and then performed the whole structure optimization.

2)They performed the suboptimization using the balance of the force of the whole structure and boundaries of the conditions of the displacement.

The application of genetic algorithms (GA) (which uses the stochastic method) effectively solves the combinatorial problem making it a remarkable new tool for solving these kind of problems [5].

In this paper, we formulate an optimal weight design OWD problem of 30 stories frame structure, which is a fully stress design without the deflection for a constrained allowable stress of frame member as a statically indeterminate structure problem, and are able to get a global solution and solve it directly by keeping the constraints by using improved GA [6]. As a result, the number of decision (design) variables does not increase and easily gets the best-compromised solution. We discuss the efficiency between the proposed method and a few discretized optimality criteria methods.

2. An Optimal Weight Design Problem of 30 stories frame

2.1 Formulation

Let us consider an OWD problem of 30 stories frame with a minimum weight which is the fully stress design without the deflection as shown in Fig. 1.

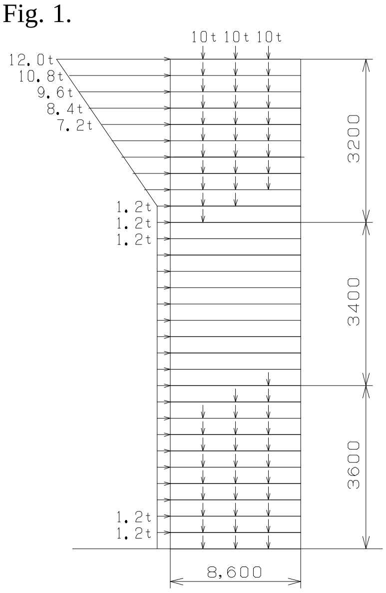

Fig. 1: An OWD problem of 30 stories frame

In Fig.1, the columns' lengths are $l_{1\text{-}10,31\text{-}40}$ = 360, $l_{11\text{-}20,41\text{-}50}$ = 340, $l_{21\text{-}30,51\text{-}60}$ = 320. The beams' length are $l_{61\text{-}90}$ = 860. The columns' cross sections/second moment of inertia are $A_{1\text{-}60}/I_{1\text{-}60}$. The beams' cross sections/second moment of inertia are $A_{61\text{-}90}/I_{61\text{-}90}$. Otherwise, the frame loads $f_{x_{1\text{-}30}}$ and $f_{y_{1\text{-}60}}$ act at each node and on beam respectively.

We formulate the optimal design problem with a satisfied allowable stress with a minimized 30 stories frame weight $W(A)$. It is the following linear programming (LP) problem:

$$\min \quad W(A) = r\sum_{i=1}^{90} l_i A_i \tag{1}$$

$$\text{s.t.} \quad G_i = s_i \le b_i, (i = 1,2,...,90) \tag{2}$$

$$s_i^L \le s_i \le s_i^U, (i = 1,2,...,90)$$

where

$$s_{i=M\max(i)} / Z_i, (i = 1,2,...,90) \tag{3}$$

Where $W(A)$ is the weight function, r is density of frames' members, $s_i (i = 1,...,90)$ is stress of frame's members, b_i is allowable stress of frames' members and s_i^L / s_i^U are lower/upper limits of allowable stress. $M_{\max(i)} / Z_i$ are maximum bending moment/section modulus of frame's members.

2.2 Design Variable

Honma *et al.* [4] reported a minimum weight design of a frame structure to cope with such a problem. That is, for a design variable, the following measure is introduced:

Table 1: design variable

$H/B = 1$	$A = 1.464 \times I^{0.4488}$	$Z = 0.9823 \times I^{0.7307}$
$1 \le H/B \le 2$	$A = 1.278 \times I^{0.4357}$	$Z = 0.9293 \times I^{0.7219}$
$2 \le H/B$	$A = 0.6468 \times I^{0.4789}$	$Z = 0.7002 \times I^{0.7346}$

Furthermore, they introduced a design variable with scaling $I^{1/4}$. H/B are hight /flange of a H shaped cross section. Z is the section modulus. We used this design variable with scaling $I^{1/4}$.

3. Representation and Evaluation Function

Yokota *et al.* [6] reported an IGA to cope with such a problem. That is, for the constraint condition, the following measure is introduced:

$$d_i = \begin{cases} 0; \\ G_i \le b_i, i = 1,2,...,90; \text{otherwize} \end{cases} \quad (4)$$

The measure d_i adopted here indicates how much the left-hand side of the expression corresponding to the constraint condition exceeds the right-hand side, and the evaluation function is defined as follows:

$$eval(V_k) = W(A_1, A_2, ...A_{90})\left(1 - \frac{1}{90}\sum_{i=1}^{90} d_i\right) \quad (5)$$

This IGA includes information about unfeasible chromosomes as near as possible to the feasible region in the evaluation function.

We can select the best chromosome with the following equation:

$$V^* = \arg\min_{V_k}\{eval(V_k)\} \quad (6)$$

Where the "arg" of argmin is an abbreviation for argument and argmin means that we adopt the chromosome whose evaluation returns the lowest value.

4 Algorithm

The procedure for solving the OWD problem of 30 stories frame by genetic algorithms is proposed in this section.

Step 1: Set population size *pop_size*, crossover probability P_c, mutation probability P_m, maximum generation $maxgen$, initial generation $gen = 0$, initial fitness value $maxeval = 0$.

Step 2: Generate initial population V_k $(k = 1,..., popsize)$ randomly.

Step 3: Calculate each chromosome's fitness value $eval(V_k)$ and set $gene = gene + 1$.

Step 4: If $gen < maxgen$, go to **Step 5**. If $gen = maxgen$, output $maxeval$ and terminate.

Step 5: Reproduce new chromosomes during arithmetic crossover and mutation process and perform Elitest Selection.

Step 6: Select any one chromosomes in the chromosome population, Calculate the fitness of the chromosome.

Step 7: From equation (6) determine the best chromosome, calculate the fitness, register them, and return to **Step 3**.

5. Numerical Example

In this paper, when we are given the design parameters, we discuss the optimal 30 stories frame structure design problem for minimizing the weight which is the fully stress design without the deflection as shown in Fig.1 and Table 2.

Table 2: The range of variable and coefficient values

Column: $l_{1\text{-}10,31\text{-}40} = 360$ $l_{11\text{-}20,41\text{-}50} = 340$ $l_{21\text{-}30,51\text{-}60} = 320$	beam: $l_{61\text{-}90} = 860$
Young modulus: E=2100	frame's density: r = 7.9
Column: $A_{1\text{-}60} = 1.464 \times I^{0.4488}$ $Z_{1\text{-}60} = 0.9823 \times I^{0.7307}$	beam: $A_{61\text{-}90} = 0.6468 \times I^{0.4789}$ $Z_{61\text{-}90} = 0.7002 \times I^{0.7346}$
Design variable:	$\lvert s \rvert \le 5.0$
Frame load f_x: $f_{x\,(1\text{-}20)}$ = 1.2; $f_{x\,(21\text{-}30)}$ = 1.2,24…,12.0	frame load f_y: $f_{y\,(1\text{-}60)} = 10$

We solve the OWD problem of by using our proposed algorithm. We set up the following initial parameters for the numerical example:

$$pop\_size = 20,\ P_c = 0.4,\ P_m = 0.1,\ maxgen = 100$$

We apply the proposed method to this problem and repeat the process for 100 generations. At the $99th$ generation of the chromosome and constraint design variables $x_{1-90} = I_{1-90}^{1/4}$, the following results are obtained:

$$V^{*}=\begin{bmatrix}x_1\\x_2\\x_3\\x_4\\x_5\\x_6\\x_7\\x_8\\x_9\\x_{10}\\x_{11}\\x_{12}\\x_{13}\\x_{14}\\x_{15}\\x_{16}\\x_{17}\\x_{18}\\x_{19}\\x_{20}\\x_{21}\\x_{22}\\x_{23}\end{bmatrix}=\begin{bmatrix}33.4\\28.4\\27.9\\26.2\\14.9\\30.2\\26.8\\26.5\\26.5\\26.5\\27.1\\27.2\\26.7\\26.5\\26.3\\26.6\\30.2\\28.8\\24.7\\24.8\\23.0\\21.8\\21.0\end{bmatrix},\quad\begin{bmatrix}x_{24}\\x_{25}\\x_{26}\\x_{27}\\x_{28}\\x_{29}\\x_{30}\\x_{31}\\x_{32}\\x_{33}\\x_{34}\\x_{35}\\x_{36}\\x_{37}\\x_{38}\\x_{39}\\x_{40}\\x_{41}\\x_{42}\\x_{43}\\x_{44}\\x_{45}\\x_{46}\end{bmatrix}=\begin{bmatrix}20.4\\19.5\\18.3\\16.8\\14.5\\10.2\\10.6\\35.2\\32.1\\30.7\\25.0\\38.0\\28.5\\30.6\\30.2\\29.9\\29.1\\28.0\\26.9\\26.8\\26.5\\26.3\\26.6\end{bmatrix},\quad\begin{bmatrix}x_{47}\\x_{48}\\x_{49}\\x_{50}\\x_{51}\\x_{52}\\x_{53}\\x_{54}\\x_{55}\\x_{56}\\x_{57}\\x_{58}\\x_{59}\\x_{60}\\x_{61}\\x_{62}\\x_{63}\\x_{64}\\x_{65}\\x_{66}\\x_{67}\\x_{68}\\x_{69}\end{bmatrix}=\begin{bmatrix}30.2\\28.8\\24.7\\25.0\\25.1\\25.7\\25.9\\25.4\\24.8\\24.0\\23.0\\21.8\\20.9\\19.8\\36.9\\39.6\\36.2\\43.1\\40.2\\39.1\\39.4\\39.4\\38.5\end{bmatrix},\quad\begin{bmatrix}x_{70}\\x_{71}\\x_{72}\\x_{73}\\x_{74}\\x_{75}\\x_{76}\\x_{77}\\x_{78}\\x_{79}\\x_{80}\\x_{81}\\x_{82}\\x_{83}\\x_{84}\\x_{85}\\x_{86}\\x_{87}\\x_{88}\\x_{89}\\x_{90}\end{bmatrix}=\begin{bmatrix}37.5\\35.6\\35.6\\35.6\\35.4\\35.3\\35.4\\38.0\\24.3\\27.5\\34.6\\34.5\\35.1\\35.7\\35.4\\34.7\\33.8\\32.7\\31.1\\29.0\\26.1\end{bmatrix}$$

The above values obtained are the best compromised solutions. The corresponding evaluation function value, that is, the minimized weight of the 30 stories frame structure, was $W(A_1, A_2, ..., A_{90}) = 184.136$.

6. Evaluation

By applying the method proposed in this paper to the numerical example described in the previous section, with population size equal to 20, crossover and mutation probabilities set to 0.4 and 0.1, respectively, and maximum number of generations set at 100, it was possible to obtain results shown in Table 3. And we also include the specification for the former method values in Table 3.

initial design :193.917 tons, IGA : 184.136 tons, [146.875 tons]

We obtained the design variables of each member, by doing the moments of the edge force of each member, and then performed the whole structure optimaization for fully stressed design.

After comparing initial design variables of the 30 stories frame structure figures and our calculation results, the weight of the 30 stories frame (*W*) improved by 9.78 tons (5%).

We obtained the design variables of each member (by doing the bending of the moments at the loading points on each beam).They improved by 47.04 tons (24.23%).

We infer this is due to the fact that the IGA includes information about unfeasible chromosomes as near as possible to the feasible region in the evaluation function.

We compare the results of IGA with another methods using SUMT method [4] and the Suboptimal method which applied SUMT method [4] as shown in Table 3 respectively. The SUMT method is a sequential unconstrained minimization technique that uses the objective function with penalty function. This method is not a universal methods for constraint handling, because, this getting the penalty technique contains the problem in which the how to get of the penalty value r is difficult. In these methods, the over half of the solution has exceeded the allowable stress in Table 3 [4].

It became clear that the scaling of the design variables used this time were specially effective for the simplification of the computation process for the member structure's optimization such as a cross-section of a H shaped steel beam.

Table 3: The result of calculation and former methods (design variables $x_{1-90} = I_{1-90}^{1/4}$)

	Initial Design		SUMT		Subopt		IGA		
	Beam	column	beam	column	beam	Column	beam	Column L	Column R
30			11.6	11.1	11.6	11.0	[20.7]26.1	20.9	19.8
29			14.1	12.1	14.0	11.9	[18.5]29.0	23.0	21.8
28	17.8	23.4	15.8	13.7	15.8	13.5	[21.1]31.1	24.8	24.0
27			17.2	14.9	17.1	14.8	[22.7]32.7	25.9	25.4
26			18.1	16.0	18.0	15.9	[23.9]33.8	25.1	25.7
25			18.9	17.0	18.8	16.9	[24.8]34.7	24.7	25.0
24			19.5	17.9	19.4	17.8	[25.6]35.4	30.2	28.8
23			19.8	18.7	19.8	18.6	[26.3]35.7	26.3	26.6
22	26.6	26.6	20.1	19.4	20.1	19.3	[27.0]35.1	26.8	26.5
21			20.3	20.1	20.2	20.1	[27.6]34.5	28.0	26.9
20			20.6	21.0	20.6	20.9	[32.8]34.6	29.9	29.1
19			21.0	21.6	20.9	21.6	[34.5]37.5	30.6	30.2
18			21.1	22.3	20.9	22.3	[37.5]24.3	38.0	28.5
17			21.2	23.0	21.1	23.0	[24.3]38.0	30.7	25.0
16	26.6	26.6	21.3	23.7	21.3	23.7	[38.0]35.4	35.2	32.1
15			21.2	24.5	21.2	24.4	[35.4]35.3	10.2	10.6
14			22.0	24.8	21.6	25.0	[35.3]35.4	16.8	14.5
13			20.7	26.2	21.5	25.7	[35.4]35.6	19.5	18.3
12			22.5	26.2	21.9	26.5	[35.6]35.6	21.0	20.4
11			21.3	27.3	22.1	27.1	[35.6]35.6	23.0	21.8
10	31.6	31.6	22.3	28.0	22.2	27.9	[28.1]37.5	24.7	24.8
9			22.4	28.7	22.2	28.0	[28.0]38.5	30.2	28.8
8			22.3	29.6	22.4	29.4	[30.1]39.4	26.3	26.6
7			23.1	30.0	22.4	30.4	[30.2]39.4	26.7	26.5
6			21.9	31.3	23.0	31.1	[30.6]39.1	27.1	27.2
5			24.7	30.9	24.5	30.8	[28.5]40.2	26.5	26.9
4	31.6	47.3	23.0	31.4	23.4	31.7	[31.6]43.1	26.8	26.5
3			15.8	35.5	13.2	35.7	[28.6]36.2	14.9	30.2
2			11.4	39.1	11.3	39.5	[30.6]39.6	27.9	26.2
1			10.2	42.2	11.0	42.5	[28.2]36.9	33.4	28.4

7. Conclusion

In this paper, we formulate an OWD problem of the 30 stories frame structure for a constrained allowable stress of the dimension of each member as a statically indeterminate structure problem and solve it directly by keeping the constraint by using IGA. We discuss the efficiency of the proposed method. As a result, the number of decision (design) variables did not increase, and easily got the best compromised solution.

When a chromosome is not contained in the feasible region, it will include information about the unfeasible region's chromosome in the evaluation function in order to improve its search efficiency.

As a result, we obtained a minimum weight of $W(A_1, A_2, ...A_{90}) = 184.136$ tons, then the weight of the 30 stories frame (W) improved by 9.78 tons (5%). and confirmed the efficiency of the proposed method.

The scaling of the design variables is specially effective for the simplification of the computation process for the member structure's optimization such as a cross-section of a H shaped steel beam.

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