FUZZY SET REPRESENTATIONS FOR IMAGE PROCESSING AND HOUGH TRANSFORM

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ABSTRACT: The paper contains several results due to 3-d image recognition by means of Hough transform and its extending. To describe uncertainty and imprecision of real images the probability model of contour image is introduced. It lets us consider the image as sample of probability distribution. On the family of probability measures set-theoretical operations are defined. As a result we get the method of contour image recognition that generalizes the standard Hough transform. This method may be interpreted in terms of fuzzy set theory and possibility theory.

KEYWORDS: pattern recognition, image processing, fuzzy set theory, possibility theory, Hough transform.

1. THE PROBLEM OF RECOGNITION GRAY LEVEL IMAGES. HOUGH TRANSFORM.

While analyzing gray level images the most informative features are contours represented with the set of points, in which large overfall of brightness is observed. Mathematically it means that for extracting contours it is necessary to determine point coordinates, in which function of brightness of the image has a large module of gradient or discontinue. Detectors of edges have got the broadest practical application in solving this problem. These detectors are based upon variational criteria offered by Canny Canny (1986). In the given work we expect that the problem of extracting edges is already solved. So it is necessary to analyze the contour image that can be presented by means of a characteristic function:

$$f(i,j)=\begin{cases}1, & \textit{if the pixel } (i,j) \textit{ belongs to contour},\\ 0, & \textit{otherwise.}\end{cases}$$

The problem of extracting edges of a certain form is very important in the recognition of contour images, because real contour image may be noisy or may have contour breakups. This may be caused by peculiarities of illuminating an image or objects may overlay each other in the field of the scene, etc. The method of solving this problem, known as a Hough transform Hecker (1994), is the least sensitive with respect to the mentioned peculiarities. It is widely used to extract different primitives of curved lines of various forms.

The idea of this transform consists in the following. Let points of a contour image be well approximated with a straight line, described by its normal form in Cartesian coordinates:

$$L\colon x\cos\Theta + y\sin\Theta = r\,.$$

The angle Θ determines the direction of the line, the positive number r is the distance from the straight line to origin. Let M be a set of points of image close to the line L. Degree of the closeness is set by a parameter $h > 0$, i.e.

$$M = \left\{ \bar{x} \in \mathrm{X} \,\middle|\, d\left(L,\bar{x}\right) \le h \right\},$$

where $X = \left\{ \bar{x}_k = \left(x_k^1, x_k^2\right) \right\}$ is a set of all points that describe the contour image in R^2, $d\left(L,\bar{x}\right)$ is the distance between the line L and point $\bar{x}$. In this case, under the appropriate choice of h the most part of points of the contour image will belong to the set M. This is just that simple principle that is a basis of the algorithm of extracting straight lines called the Hough transform. It consists in the following:

1) digitizing possible values of parameter Θ_i, $i=\overline{1,n}$, in order to get a solution satisfying the required precision;

2) permissible values of parameter r are divided into intervals: $[0, r_1), [r_2, r_3), \ldots, [r_{m-1}, r_m)$, where $0 < \rho_i < \rho_{i+1}$, $i = \overline{1, m-1}$;

3) calculate powers of sets $M(\Theta_i, r_{j-1}, r_j) = \left\{ \bar{x}_k \in \mathrm{X} \,\middle|\, x_k^1 \cos\Theta_i + x_k^2 \sin\Theta_i \in [r_{j-1}, r_j) \right\}$.

It is clear that the set $\left\{ \bar{x} \in R^2 \,\middle|\, x^1 \cos\Theta_i + x^2 \sin\Theta_i \in [r_{j-1}, r_j) \right\}$ is a band in R^2, i. e. the geometric set of points that lay between two straight lines satisfying the equations:

$$x^1 \cos\Theta_i + x^2 \sin\Theta_i = r_{j-1} \text{ and } x^1 \cos\Theta_i + x^2 \sin\Theta_i = r_j .$$

Hough transform is based on calculating the number of points of contour image situated between corresponding pair of lines. If $\left| M(\Theta_i, r_{j-1}, r_j) \right|$ got to be large for certain values of Θ_i, r_{j-1}, r_j, then we consider the straight line $x^1 \cos\Theta_i + x^2 \sin\Theta_i = \frac{r_{j-1} + r_j}{2}$ as an appropriate approximation of the set $M(\Theta_i, r_{j-1}, r_j)$.

2. THE CONTOUR IMAGE MODEL

The study of Hough transform and its modifications can not be realized without examination of contour image model. It is necessary to take into account impression and uncertainty of analyzed information. While constructing the probabilistic model we may consider the set X of points of contour image as a sample from a certain probabilistic distribution P. Consider now an example of how to choose a probabilistic law of the distribution.

Let a real contour image present a segment $\left\{ (x, y) \,\middle|\, x \in [-a, a],\ y = 0 \right\}$ of straight line. In fact we have a noisy sample $\mathrm{X} = \{\bar{x}_1, \ldots, \bar{x}_N\}$ of a given image, that presents a realization of succession $\{x_1, x_2, \ldots, x_N\}$ of evenly distributed random values. Each random value can be modeled in the following way:

1. Realize a random choice of point t_i in the segment $[-a, a]$;
2. Then add to t_i random vector $\bar{e}_i$. So we have

$$x_i = (t_i, 0) + \bar{e}_i,\ \bar{e}_i = (e_{1i}, e_{2i}) .$$

Let random value t_i have an even distribution upon the segment $[-a, a]$, with the density $f(t) = \frac{1}{2a}$, and random component $\bar{e}_i$ - a spherical normal distribution. In this case the set X will be an independent sample from probabilistic distribution P with density $\frac{1}{2a} \int_{-a}^{a} \frac{1}{2p s^2} e^{-\frac{(x-t)^2 + y^2}{2s^2}} dt$. There is a numeric characteristic $P\left\{ (x, y) \,\middle|\, -b \le y \le b \right\}$, that is very important while extracting straight lines with the help of Hough transform. The following lemma shows the way of its calculation.

Lemma 1. $P\left\{ (x, y) \,\middle|\, -b \le y \le b \right\} = \int_{-b}^{b} \frac{1}{\sqrt{2p}\, s} e^{-\frac{y^2}{2s^2}} dy$.

Now we generalize the obtained model for undefined contour images. Let the contour image be a curve L, parametrically defined. Usually, such parametric representation is chosen as follows:

1. $s \in [0, 2p]$, $r(0)$ is the first point of the curve, $r(2p)$ is the final one;
2. if l denotes the length of the curve, then the distance along the curve from the first point $r(0)$ to a current point is determined by $\frac{sl}{2p}$.

Then, realizing an even random choice of points on the curve and adding noise to the received sample, we obtain, that noisy image is described by a probabilistic distribution with the density $\frac{1}{2p}\int_{-p}^{p}\frac{1}{2ps^2}e^{-\frac{(x-x(s))^2+(y-y(s))^2}{2s^2}}ds$.

Let $d(L,\bar{x})$ be the distance from a curve L to a point $\bar{x}\in R$. When we consider a generalized Hough transform Hecker (1994) the following event

$$U=\left\{\bar{x}\in R \;\middle|\; d(L,\bar{x})\le h\right\}$$

is a focus of practical interest. The following lemma gives the evaluation of probability of this event.

Lemma 2. *Suppose that in each point of the curve* L *we can calculate a radius of curvature* $R(s)$ *and* $R=\inf_{s\in[0,2\pi]}R(s)\ge 3\sigma+h$ *, then* $\left[P(U)-2\Phi_0\left(\frac{h}{s}\right)\right]\in[-a,b]$, *where*

$$a=\frac{6\sigma}{l}+\left[\Phi_0\left(\frac{h}{\sigma}\right)-\Phi_0\left(\frac{h-\delta}{\sigma}\right)\right]2\Phi_0(3)+2\Phi_0\left(\frac{h}{\sigma}\right)(1-2\Phi_0(3)),$$

$$b=\Phi_0\left(\frac{h+d}{s}\right)+\Phi_0\left(\frac{h}{s}\right),\qquad \delta=\frac{9\acute{o}^2}{(R-h)+\sqrt{(R-h)^2-9\acute{o}^2}}.$$

This means that for evaluation of the event U it is possible to use a value $2\Phi_0\left(\frac{h}{s}\right)$ if both the length of the smallest radius R and the length of the curve relatively to the noise parameter s are large.

3. SET-THEORETICAL OPERATIONS ON IMAGES. THE INCLUSION MEASURE.

We will consider some generalization of Hough transform from the viewpoint of the possibility theory. Such a generalization is based on methods of classification of probabilistic distributions considered in the paper Bronevitch (1997).

Introduce the set-theoretical operations on contour images. Let F_1, F_2, F_3 be contour images, and they are not noisy. Let they be described by characteristic functions $f_1(\bar{x})$, $f_2(\bar{x})$, $f_3(\bar{x})$, where $\bar{x}\in R^2$. Then

1. $F_3=F_1\cap F_2$, *if* $\forall\bar{x}\in R^2$ $f_3(\bar{x})=f_1(\bar{x})\ \&\ f_2(\bar{x})$.
2. $F_3=F_1\cup F_2$, *if* $\forall\bar{x}\in R^2$ $f_3(\bar{x})=f_1(\bar{x})\vee f_2(\bar{x})$.
3. $F_1\subseteq F_2$, *if* $F_1\cap F_2=F_1$.

In these equations the standard logical operations "AND" (&) and "OR" ($\vee$) are used. Define similar set-theoretical operations on noisy contour images. Each noisy image F is described by $F=(f,P)$, where f is a characteristic function of a real contour image, P is a probabilistic measure that assigns a probabilistic law, describing a noisy image.

Let $d(f,\bar{x})$ be the distance from an arbitrary point $\bar{x}$ to contour image L described by a characteristic function f. Each noisy image F engenders events of the type

$$E(h)=\left\{\bar{x}\in R^2 \;\middle|\; d(f,\bar{x})<h\right\}.$$

Such events we will call as *minimal events.* Image $F=(f,P)$ we will call *a regular* one, if

$$\forall p\in[0,1]\exists h_p=\sup\{h\,|\,P\{E(h)\}\le p\},$$

$$P\left\{E(h_p)\right\}=p \text{ is truth.}$$

In this case each minimal event is determined by its probability uniquely and $A(p)=E(h_p)$ is the minimal event with the probability p. For regular images we introduce the set-theoretical operations, as well as the inclusion relation, like an extension of corresponding operations on minimal events:

1. $F_3 = F_1 \cap F_2, if \quad \forall p \in [0,1] \; A_3(p) = A_1(p) \cap A_2(p)$.
2. $F_3 = F_1 \cup F_2, if \quad \forall p \in [0,1] \; A_3(p) = A_1(p) \cup A_2(p)$.
3. $F_1 \subseteq F_2, if \quad \forall p \in [0,1] \; A_1(p) \subseteq A_2(p)$.

For images classification we can use the inclusion relations. However, it is not adapted for practical usage. Taking this in our account further consider inclusion measure. Let $p \in [0,1]$, then ð-local inclusion measure

$$y_p(F_1 \subseteq F_2) = P_1\{A_2(p) | A_1(p)\}$$

is a conditional probability of appearance of event $A_2(p)$ when observing an event $A_1(p)$ on measure P_1. However, it turns out to be more useful *the integral inclusion measure*

$$y(F_1 \subseteq F_2) = \int_0^1 (2p) y_p(F_1 \subseteq F_2) dp$$

that accounts values of ð–local measures for different *p* and satisfies all necessary requirements Bronevitch (1997). The practical calculation of inclusion measure is based on the following lemma.

Lemma 3. *The inclusion measure of event* F_2 *into event* F_1 *is determined as follows*

$$ø(F_1 \subseteq F_2) = 2\int_{R_2} \min[ì_1(\bar{x}), ì_2(\bar{x})] dP_1(\bar{x}),$$

where $ì_i(\bar{x}) = 1 - P\{\bar{y} \in R^2 \, | \, d(\bar{y},f) \le d(\bar{x},f)\}$.

A theoretical sense of the function $m(x)$ can be interpreted Bronevitch (1997) as a membership function of a fuzzy set, generated by probabilistic distributions. From the lemma 3, it is easy to derive a formula for statistical evaluation of inclusion measure $y(F_1 \subseteq F_2)$ using the points of noisy image $\{\bar{x}_1, ..., \bar{x}_N\}$:

$$y(F_1 \subseteq F_2) = \frac{2}{N} \sum_{i=1}^{N} \min[m_1(\bar{x}_i), m_2(\bar{x}_i)] \ .$$

The latest formula generalizes the known procedure of Hough transform. To get Hough transform one may assign $m_1(\bar{x}_i) = 1 \quad i = \overline{1, N}$, and fuzzy set F_2 bordering the real image must be an ordinary set in usual sense, i. e.

$$m_2(\bar{x}_i) \in \{0,1\} \quad \bar{x}_i \in R^2, \quad i = \overline{1, N}.$$

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