A Two-Level Sub-optimal Fuzzy-Based Prediction Control and its Application into PSS Design

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ABSTRACT: This paper proposes a two-level sub-optimal control using fuzzy prediction to control large scale systems. A class of large-scale linear systems composed of interconnected subsystems is investigated. The overall control problem that is posed as minimization of overall objective function, which is considered to be of quadratic form, is reduced to some optimization problems of lower order (sub)systems. The control input to each subsystem is composed of two signals. The First represents the local control signal (first level) and the second is the prediction signal (second level). In fact, the second signal is the prediction of interaction of other subsystems. Fuzzy logic theory is used for interaction prediction, where the prediction signal is constructed by a set of fuzzy sets with respect to state variables in an appropriate inference engine manner. Finally, the proposed method is applied to three-area power system for designing power system stabilizer (PSS).

KEYWORDS: Fuzzy Logic Theory, Large-Scale Systems, Power System Stabilizer, Multi-level Control.

INTRODUCTION

Hierarchical structures in large-scale systems such as complex industrial systems, management systems and power systems are theoretically investigated by Mesarovic (1970). The development of hierarchical control has grown by leaps and bounds in the past recent decades Jamshidi (1983) and Li (1993). This paper proposes a two-level sub-optimal control using fuzzy prediction to control large-scale systems. It is a special case of multilevel control where the complexity of large-scale control problems can be relaxed by solving a family of sub-problems that are of smaller dimensions and are more easily handled. Most of the above hierarchical control schemes have not the capability of on-line implementation. The control scheme used in this paper can be applied to the whole system in an on-line fashion. Because fuzzy logic theory can be used to coordinate the large-scale systems, Sadati (1995), this paper employs fuzzy logic theory to predict the interactions at the second level.

Consider the following large-scale linear system composed of N interconnected subsystems of the form

$$\dot{x}_i(t) = A_i x_i(t) + B_i u_i(t) + z_i(t) \qquad (1)$$

$$z_i(t) = \sum_{j \neq i}^{N} A_{ij} x_j(t) \qquad \text{i=1,..,}N$$

where, $u_i \in R^{p_i}$, $x_i, z_i \in R^{n_i}$ are respectively the control vector, state vector and interaction input signal, and A_i, B_i and A_{ij} are matrices of appropriate dimensions .

Assume that the overall performance index is of a quadratic form

$$J = \frac{1}{2}\int_0^\infty \left(\|x(t)\|_Q^2 + \|u(t)\|_R^2\right)dt \qquad (2)$$

where Q and R are symmetric matrices of appropriate dimensions. The main problem can be stated as follows:
Find a state feedback control law such that the objective function J to be minimized subject to (1), i.e.;

$$\min \quad J = \frac{1}{2}\int_0^{\infty}\left(\|x(t)\|_Q^2 + \|u(t)\|_R^2\right)dt$$

$$s.t. \quad \dot{x}_i(t) = A_i x_i(t) + B_i u_i(t) + z_i(t)\ , \quad z_i(t) = \sum_{j\neq i}^{N} A_{ij} x_j(t) \qquad for\ i = 1, \ldots, N. \tag{3}$$

DECOMPOSITION

Write the Lagrangian of overall optimization problem (3) as

$$L = \sum_{i=1}^{N}\left\{\int_0^{\infty}\left[\frac{1}{2}\|x_i(t)\|_{Q_i}^2 + \frac{1}{2}\|u_i(t)\|_{R_i}^2 + \rho_i(t)^T\left(z_i(t) - \sum_{j\neq i}^{N} A_{ij} x_j(t)\right)\right.\right.$$
$$\left.\left. + P_i(t)^T\left(-\dot{x}_i(t) + A_i x_i(t) + B_i u_i(t) + z_i(t)\right)\right]dt\right\}. \tag{4}$$

where $\rho_i(t)$ (i=1, 2, ..., N) is Lagrangian multiplier vector and $P_i(t)$(i=1, 2, ..., N) is adjoint vector.
the Lagrangian in (4) can be decomposed as the sum of N sub-Lagrangians, L_i , i. e. ;

$$L_i = \int_0^{\infty}\left[\frac{1}{2}\|x_i(t)\|_{Q_i}^2 + \frac{1}{2}\|u_i(t)\|_{R_i}^2 + \rho_i(t)^T z_i(t) - \sum_{j\neq i}^{N}\rho_j(t)^T A_{ji} x_i(t)\right.$$
$$\left. + P_i(t)^T\left(-\dot{x}_i(t) + A_i x_i(t) + B_i u_i(t) + z_i(t)\right)\right]dt \ , \quad \text{for } i = 1, 2, \ldots, N. \tag{5}$$

It is clear that the vectors $z_i(t)$ and $\rho_i(t)$ play the coupling role among the N sub-Lagrangians. At this moment assume that these vectors represent constant vectors; therefore all the N sub-Lagrangian problems can be solved independently. By defining the Hamiltonian of the ith subsystem as

$$H_i = \frac{1}{2}\|x_i(t)\|_{Q_i}^2 + \frac{1}{2}\|u_i(t)\|_{R_i}^2 + \rho_i(t)^T z_i(t) - \sum_{j\neq i}^{N}\rho_j(t)^T A_{ji} x_i(t)$$
$$+ P_i(t)^T\left(-\dot{x}_i(t) + A_i x_i(t) + B_i u_i(t) + z_i(t)\right). \tag{6}$$

the necessary conditions for the optimal control imply that

$$\dot{P}_i(t) = -Q_i x_i(t) - A_i^T P_i(t) + \sum_{j\neq i}^{N} A_{ji}^T \rho_j(t)$$
$$u_i(t) = -R_i^{-1} B_i^T P_i(t) \tag{7}$$

The above problem is similar to a tracking problem in optimal control theory. Assume that the adjoint vector $P_i(t)$ is of the following form

$$P_i(t) = K_i x_i(t) + g_i \tag{8}$$

By substituting (8) into (7), we have

$$-K_i A_i - A_i^T K_i + K_i B_i R_i^{-1} B_i^T K_i - Q_i = 0 \tag{9-a}$$

$$-\left(A_i^T - K_i B_i R_i^{-1} B_i^T\right)g_i - K_i z_i(t) + \sum_{j\neq i}^{N} A_{ji}^T \rho_j(t) = 0 \tag{9-b}$$

FIRST AND SECOND LEVELES PROBLEM FORMULATION

As one may see in the last section, the control input of *i*th subsystem is

$$u_i(t) = -R_i^{-1}B_i^T\left(K_i x_i(t) + g_i\right) \tag{10}$$

Where g_i and K_i come from (9). The necessary conditions for the optimal control imply that

$$z_i(t) = \sum_{j\neq i}^{N} A_{ij}x_j(t)\ ,\quad \rho_i(t) = -P_i(t) = -\left(K_i x_i(t) + g_i\right)\ \ i=1, ..., N. \tag{11}$$

In Jamshidi (1983) and Li (1993), the bounds [0,T] are considered for the integral of cost function (2); it is also considered that g is varied with time, then an off-line control procedure was proposed. In this paper g is considered fixed for each coordination sample time and Eq. (9-b) is used in an on-line control structure. Eq. (9-b) can be simplified as follows:

By substituting $z_i(t)$ and $\rho_i(t)$ from (11) into (9-b), we have

$$\left(A_i^T - K_iB_iR_i^{-1}B_i^T\right)g_i + K_i\sum_{j\neq i}^{N}A_{ij}x_j(t) + \sum_{j\neq i}^{N}A_{ji}^T g_i + \sum_{j\neq i}^{N}A_{ji}^T K_j x_j(t) = 0\ ,\ \ i=1, ..., N.$$

By combining the above equations, we may derive the following equation

$$\left(A - BR^{-1}B^TK\right)^T g + \left(A_z^TK + KA_z\right)x = 0 \tag{12}$$

where,

$$A = \begin{bmatrix} A_1 & A_{12} & \cdots & A_{1N} \\ A_{21} & A_2 & \cdots & A_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ A_{N1} & A_{N2} & \cdots & A_N \end{bmatrix},\ A_z = \begin{bmatrix} O & A_{12} & \cdots & A_{1N} \\ A_{21} & O & \cdots & A_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ A_{N1} & A_{N2} & \cdots & O \end{bmatrix}, g = \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_N \end{bmatrix},\ x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix},\ K = diag\{K_i\}\ ,\ B = diag\{B_i\}.$$

From above it is easily to see that the vector g is related to state vector x as:

$$g = -\left(A - BR^{-1}B^TK\right)^{-T}\left(A_z^TK + KA_z\right)x \tag{13}$$

It is clear that the g is dependent to time varying state vector x , however It is assume that g is fixed in each coordination sample time. Practically we cannot implement Eq. (13) because x is not available. Therefore the prediction of x , $\hat{x}$, is used in (13), i. e. ;

$$g = -\left(A - BR^{-1}B^TK\right)^{-T}\left(A_z^TK + KA_z\right)\hat{x} \tag{14}$$

So prediction of x is done at the second level, then vector g is constructed and will be sent to the local units. Local controllers generate the control input of each subsystem by (10). This structure is shown in Figure 1.

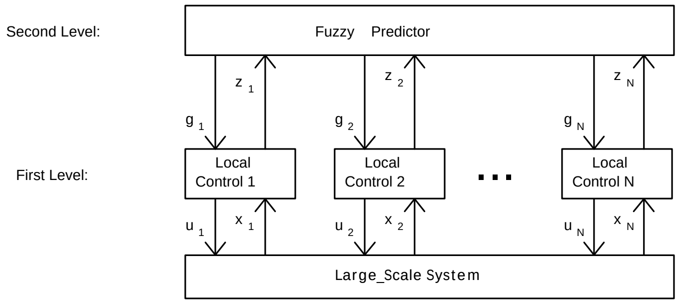

Figure 1: Hierarchical control structure of the proposed scheme

FUZZY PREDICTION

Assume that x_t is the state vector at time *t*. The goal is to find a fuzzy-based prediction $\hat{x}_{t+1}$ from x_t and x_{t-1} . For *i* = -*k*, ..., 0, ..., *k*, let the fuzzy terms E_i , F_i and G_i are considered for variables x_t , x_{t-1} and x_{t+1}, respectively with the following triangular membership functions shown in Fig. 2.

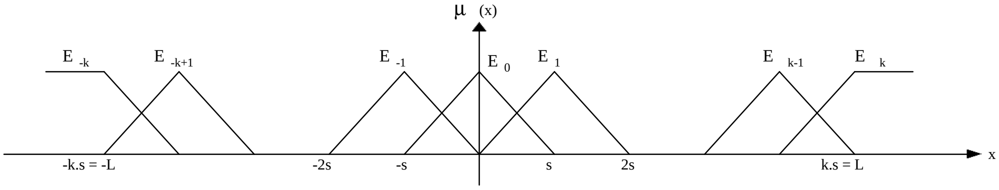

Figure 2: Membership functions of x_t

Let x_t and x_{t-1} belong to the interval [-*L* , *L*]. If we assume that the values of vector *x* vary approximately linearly, we may have the best estimation of *x* as:

$$\hat{x}_{t+1} = 2x_t - x_{t-1}$$

In Fuzzy case, if x_t belongs to E_i and x_{t-1} belongs to F_j then $\hat{x}_{t+1}$ will belong to G_{2i-j}. The areas R_0 to R_8 with respect to x_t and x_{t-1} may be partitioned as shown in Fig. 3.

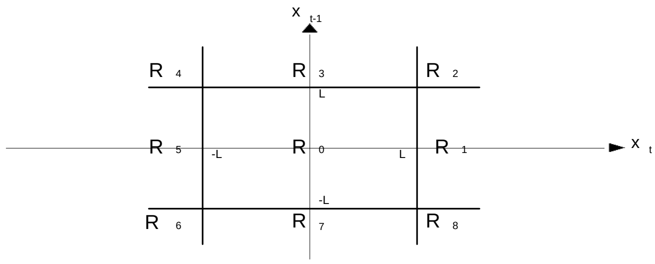

Figure 3: Different areas with respect to x_t and x_{t-1}

AREA $\mathbf{R_0}$

In this area it is obvious that any x_t (or x_{t-1}) intersects with two fuzzy sets E_i and E_{i+1} (or F_j and F_{j+1}). Therefore only the following four rules are fired in this area:

1. If x_t is E_{i+1} and x_{t-1} is F_{j+1} then x_{t+1} is G_{2i-j+1} .

2. If x_t is E_{i+1} and x_{t-1} is F_j then x_{t+1} is G_{2i-j+2} .

3. If x_t is E_i and x_{t-1} is F_{j+1} then x_{t+1} is G_{2i-j-1} .

4. If x_t is E_i and x_{t-1} is F_j then x_{t+1} is G_{2i-j} .

For convenience, consider two functions f_1 and f_2 as follow

$$f_1(x,i) = \frac{x}{s} + 1 - i \quad \text{and} \quad f_2(x,i) = -\frac{x}{s} + 1 + i$$

If the membership value of x_t to E_i is equals to μ_1 and the membership value x_{t-1} of to F_j is equals to μ_2 , then the corresponding output fuzzy set , G_{2i-j} , will be multiplied by $\alpha = \min(\mu_2, \mu_2)$ and the prediction is done by defuzzyfying the overall output fuzzy set. Let

$$\begin{matrix} \alpha_1 = \min\{f_1(x_t, i+1), f_1(x_{t-1}, j+1)\} \\ \alpha_2 = \min\{f_1(x_t, i+1), f_2(x_{t-1}, j)\} \end{matrix} \quad \text{and} \quad \begin{matrix} \alpha_3 = \min\{f_2(x_t, i), f_1(x_{t-1}, j+1)\} \\ \alpha_4 = \min\{f_2(x_t, i), f_2(x_{t-1}, j)\} \end{matrix} ,$$

then the output fuzzy set G_{out} becomes

$$G_{out} = \alpha_1 G_{2i-j+1} \oplus \alpha_2 G_{2i-j+2} \oplus \alpha_3 G_{2i-j-1} \oplus \alpha_4 G_{2i-j} ,$$

where $\oplus$ represents the sum operation in fuzzy sets. After applying the center of gravity defuzzyfication to the fuzzy set G_{out}, we will have

$$x_{t+1} = \frac{\alpha_1(2i-j+1)s^2 + \alpha_2(2i-j+2)s^2 + \alpha_3(2i-j-1)s^2 + \alpha_4(2i-j)s^2}{\alpha_1 s + \alpha_2 s + \alpha_3 s + \alpha_4 s} = \left(2i - j + \frac{\alpha_1 + 2\alpha_2 - \alpha_3}{\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4}\right)s$$

OTHER AREA

Following the procedure given in last section we may derive the following results for the other areas (area R2 to area R9).

- **In area R_1 :** $x_{t+1} = 2L - x_{t-1}$
- **In area R_2 :** $x_{t+1} = L$
- **In area R_3 :** $x_{t+1} = -L + 2x_t$
- **In area R_4 :** $x_{t+1} = -3L$
- **In area R_5 :** $x_{t+1} = -2L + 2x_{t-1}$
- **In area R_6 :** $x_{t+1} = -L$
- **In area R_7 :** $x_{t+1} = L + 2x_t$
- **In area R_8 :** $x_{t+1} = 3L$

Therefore, the fuzzy prediction of x_{t+1} is obtained from the above equations. k is an arbitrary number which specifies the fuzzy sets, for example k=4 specifies the fuzzy sets of Zero, Small, Medium, Large, and Very Large. Also, the value of L is considered as a multiplicative of x_t (i.e., $L = \alpha x_t$, where α is an arbitrary scalar, for example $\alpha = 1.2$), this consideration reduces the error of prediction. The reason for the error reduction can be stated as follows. If the value of L is very high (with respect to x_t and x_{t-1}) then the prediction error will grow up, because the variation of x_t with respect to fuzzy set domains is very low and its effect will not be observed for error reduction. Note that the above fuzzy inference is applied independently to each element of vector *x*.

In addition to the above rules, the following two improving rules are considered as follows:

R1: If $|x_t| < \varepsilon$ and $|x_{t-1}| < \varepsilon$ then x_{t+1} is 0.

R2: If $\|x_t - x_{t-1}\| < \eta$ then $x_{t+1} = \gamma . x_{t+1}$.

Rule R1 prevents the prediction of states from oscillation about zero when the system tends to its steady state (ε is an arbitrary small positive real number), and rule R2 conducts the system to its steady state by applying a descent factor γ to the state variables.

SIMULATION RESULTS

The proposed scheme is used to design a PSS for three-area power system. The specifications of each machine and network admittance matrix is considered as Anderson (1981).

The state variables of each machine are $x_1 = \Delta\omega$, $x_2 = \Delta\delta$, $x_3 = \Delta e'_q$, x_4 equal to output deviation of *AVR* , and x_5 and x_6 represent the state variables of governor. Our goal is the design of *PSS* where its inputs are frequency deviation ($\Delta\omega$) of each machine and its outputs are control signals into governor and *AVR*. To construct the proposed two level controller we linearize the system as Yu (1983).

The following figure compares the torque angle deviations of subsystem 1 for three kinds of *PSS* (centralized, decentralized and the proposed method) for thefollowing initial state X_0.

$$X_0 = [-.03 .05\ -.02 .0200\ -.02 -.05 -.02 .05 .05 .03 .05 .04 .025 -.05 .03 -.05 .02]$$

As observed from figures 4 to 6 the proposed method outperforms the other two methods control has the better response. Note that the coordination sample time is 100 ms.

As we can see from simulation results (compare Fig. 5 and Fig. 6) the performance of the proposed controller may be enhance by reducing the coordination sample time.

Figure 7 shows how good the actual state can be predicted by the proposed fuzzy estimator. The same results have been obtained for the other two subsystems and for the sake of brevity we didn't show them here.

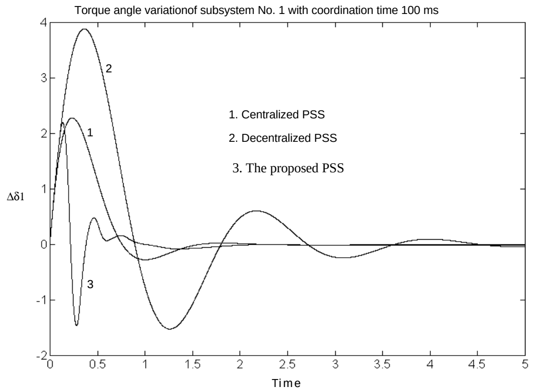

Fig.ure 4: Torque angle deviation of subsystem 1 for three kind of PSS

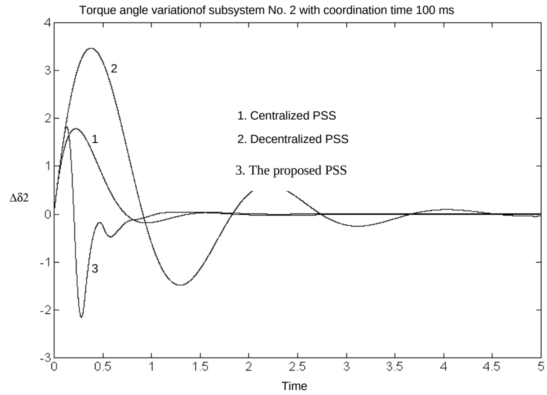

Fig.ure 5: Torque angle deviation of subsystem 2 for three kind of PSS

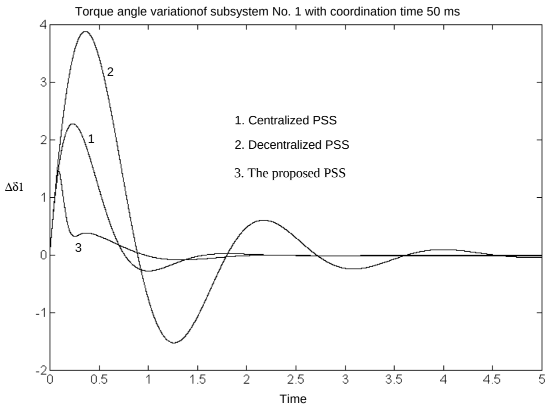

Figure 6: Torque angle deviation of subsystem 1 for three kind of PSS

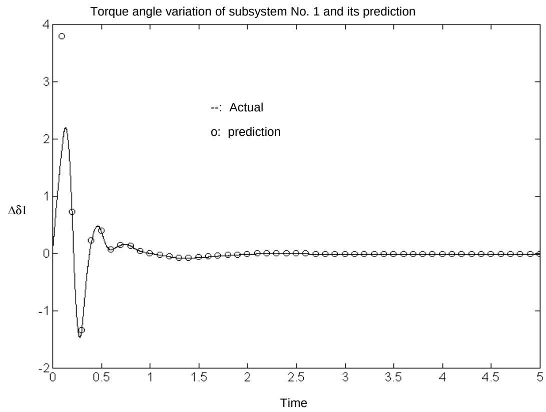

Fig.ure 7: prediction of load angle of subsystem 1

CONCLUSION

In this paper a two-level sub-optimal control using fuzzy prediction was developed to control large-scale systems. The proposed controller has been applied to three-area power system. The results of our proposed controller have been compared with those of the other two PSS controllers frequently cited in the literature known as decentralized and centralized controller. Simulation results easily highlight the merit of our method.

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