

# OPTIMIZATION of STRUCTURE of PORTFOLIO under CONDITIONS UNKNOWN PROBABILITY DISTRIBUTION

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The plenty of researches is devoted to the analysis and methods of shaping of an optimum (rational) portfolio, the most beneficial plan of distribution of the investments, for example in [1, 2, 3, 4].

In majority of the proceedings the hypothesis, in the correspondence with which the incomes or profit rates of securities  $R_i$  ( $i=1, \dots, k$ ) consider as the normally distributed casual values. This hypothesis is accepted even then, when for this purpose there are no basis, when there is no sufficient statistic information concerning dynamics(changes) of the profit for the previous period, when there is no convincing and comprehensive expert information.

Let's try to refuse a hypothesis (when in it there is a sense), that the profits or profit rates of securities are normally distributed casual values.

Let's enter such labels:

$k$  - an amount of kinds securities, which are attracted in a portfolio numbered from 1 up to  $k$ ;

$\mathbf{X} = (X_1; \dots; X_k)$  — an acceptable portfolio, where  $X_i$  - share of  $i$ -th paper ( $i=1, \dots, k$ );

$n$  - amount of possible values of profit rate, available for observation;

$r_{ij}$  — the possible values of profit rate of  $i$ -th paper;

$q_j$  — probability of appropriate possible values of profit rates ( $j=1, \dots, n$ );

$m_i$  — expected profit rate of  $i$ -th paper ( $j=1, \dots, n$ );

$\mathbf{S}_i$  — a risk (expressed as standard deviation of profit rate) of  $i$ -th paper ( $j=1, \dots, n$ );

$r^{il}$  — a coefficient of correlation of profit rates of  $i$ -th and  $l$ -st securities ( $i=1, \dots, k, l=1, \dots, k$ );

$R = R_{k \times n} = (r_{ij})$  — a functional of an estimation (matrix) by a size  $k \times n$ .

Profit rates of  $i$ -th paper  $R_i$  ( $i=1, \dots, k$ ) and portfolio securities  $R_x$  are casual values. It is obvious, that

$$m_i = \mathbf{M}(R_i) = \sum_{j=1}^n r_{ij} q_j \quad (i=1, \dots, k), \quad (1)$$

$$\mathbf{S}_i^2 = \mathbf{D}(R_i) = \sum_{j=1}^n (r_{ij} - m_i)^2 q_j = \sum_{j=1}^n r_{ij}^2 q_j - m_i^2 \quad (i=1, \dots, k), \quad (2)$$

$$\mathbf{S}_i = \sqrt{\mathbf{D}(R_i)} \quad (i=1, \dots, k), \quad (3)$$

$$r^{il} = \frac{\sum_{j=1}^n (r_{ij} - m_i)(r_{lj} - m_l) q_j}{\mathbf{S}_i \mathbf{S}_l} = \frac{\sum_{j=1}^n r_{ij} r_{lj} q_j - m_i m_l}{\mathbf{S}_i \mathbf{S}_l} \quad (i=1, \dots, k, l=1, \dots, k). \quad (4)$$

Profit rate of the portfolio securities is equal

$$R_x = \sum_{i=1}^k R_i X_i, \quad \sum_{i=1}^k X_i = 1, \quad X_i \geq 0 \quad (i=1, \dots, k).$$

Expected profit rate of a portfolio can be calculated by the formula:

$$m_x = \mathbf{M}(R_x) = \mathbf{M}\left(\sum_{i=1}^k R_i x_i\right) = \sum_{i=1}^k \mathbf{M}(R_i) x_i = \sum_{i=1}^k m_i x_i$$

And its risk - variance (variation) of a portfolio

$$\mathbf{s}_x^2 = \mathbf{D}(R_x) = \sum_{i=1}^k \sum_{l=1}^k \mathbf{s}_i \mathbf{s}_l \mathbf{r}_{il} x_i x_l = \sum_{i=1}^k \mathbf{s}_i^2 x_i^2 + 2 \sum_{i=1}^{k-1} \sum_{l=i+1}^k \mathbf{s}_i \mathbf{s}_l \mathbf{r}_{il} x_i x_l$$

or as standard deviation of a portfolio  $\mathbf{s}_x = \sqrt{\mathbf{D}(R_x)}$ .

Let's assume, that the probability's distribution  $\mathbf{Q} = (q_1, \dots, q_n)$  is not known. According to a of Gibbs-Janes maximum principle [5], the most typical of probability distributions of uncertain medium conditions are such, which maximize the elected measure of indeterminacy in conditions of the given information about a conduct of a medium. It's used a formalism of reproduction of unknown values here due to the law of distribution of an casual value under restrictions of maximum of a Shannon's entropy

$$H_e(\mathbf{Q}) = -\sum_{j=1}^n q_j \ln q_j$$

For an information situation  $I_4$  (see, for example, [5, 6]), which is characterized by unknown distribution  $\mathbf{Q} = (q_1, \dots, q_n)$  and the absence of an active counteraction of a medium of a possibility of the decision making by the subject of management, the mathematical model of a problem of choice of an optimum structure of portfolio looks like

$$m_x = \mathbf{M}(R_x) = \sum_{i=1}^k m_i x_i \rightarrow \max \quad (5)$$

$$\mathbf{s}_x^2 = \mathbf{D}(R_x) = \sum_{i=1}^k \sum_{l=1}^k \mathbf{s}_i \mathbf{s}_l \mathbf{r}_{il} x_i x_l \rightarrow \min \quad (6)$$

$$H_e(\mathbf{Q}) = -\sum_{j=1}^n q_j \ln q_j \rightarrow \max \quad (7)$$

under the system of restrictions

$$\sum_{i=1}^k x_i = 1 \quad (8)$$

$$\sum_{j=1}^n q_j = 1 \quad (9)$$

$$x_i \geq 0 \quad (i=1, \dots, k), \quad (10)$$

$$q_j \geq 0 \quad (j=1, \dots, n). \quad (11)$$

Thus, it is necessary to determine shares  $x_i$  ( $i=1, \dots, k$ ), probability  $q_j$  ( $j=1, \dots, n$ ), which it will maximize expected value of profit rate  $R_x$  of a portfolio  $\mathbf{x} = (x_1, \dots, x_k)$ , to minimize its risk and to maximize a Shannon's entropy. Let's remark, that the expected profit rates of return (1) and (5), coefficients of correlation (4), risks (2), (3), (6), Shannon's entropy (7) are depended on probabilities  $q_j$  ( $j=1, \dots, n$ ) of realization of economic medium conditions.

In [7] is considered the multicriteria method of optimization offered by the Italian economist V.Pareto. Let's introduce methods of optimization of the portfolio's structure of securities under condition of unknown probability distribution.

Let  $I_1, I_2, I_3$  — strictly positive numbers, such that  $I_1 + I_2 + I_3 = 1$ .

We formulate a new criterion

$$\begin{aligned} f(\mathbf{x}_1, \dots, \mathbf{x}_k; \mathbf{q}_1, \dots, \mathbf{q}_n) &= \lambda_1 m_x - \lambda_2 \sigma_x^2 + \lambda_3 H_e(\mathbf{Q}) = \\ &= I_1 \sum_{i=1}^k m_i x_i - I_2 \sum_{i=1}^k \sum_{l=1}^k s_i s_l r_{il} x_i x_l - I_3 \sum_{j=1}^n q_j \ln q_j \end{aligned}$$

and solve such problem of mathematical programming

$$f(\mathbf{x}_1, \dots, \mathbf{x}_k; \mathbf{q}_1, \dots, \mathbf{q}_n) \rightarrow \max$$

under system of restrictions (8), (9), (10), (11).

The solution of this problem  $(\mathbf{x}_1^*, \dots, \mathbf{x}_k^*; \mathbf{q}_1^*, \dots, \mathbf{q}_n^*)$  is a Pareto vector, and the portfolio  $\mathbf{x}^* = (\mathbf{x}_1^*, \dots, \mathbf{x}_k^*)$  is effective.

Let's consider other method. Applying a Gibbs-Janes maximum uncertainty principle under condition of an information

situation  $I_4$ , we can determine a dot evaluation  $q_1^* = q_2^* = \dots = q_n^* = \frac{1}{n}$ . Substituting this value in the formulas (10), (2), (3), (4), (7), we shall receive

$$\begin{aligned} m_i &= \frac{1}{n} \sum_{j=1}^n r_{ij} = \bar{r}_i \quad (i=1, \dots, k), \\ s_i^2 &= \frac{1}{n} \sum_{j=1}^n (r_{ij} - \bar{r}_i)^2 = \frac{1}{n} \sum_{j=1}^n r_{ij}^2 - \bar{r}_i^2 \quad (i=1, \dots, k), \end{aligned} \quad (12)$$

$$s_i = \sqrt{\frac{1}{n} \sum_{j=1}^n (r_{ij} - \bar{r}_i)^2} \quad (i=1, \dots, k), \quad (13)$$

$$r_{il} = \frac{\frac{1}{n} \sum_{i=1}^n (r_{ij} - \bar{r}_i)(r_{lj} - \bar{r}_l)}{s_i s_l} = \frac{\frac{1}{n} \sum_{i=1}^n r_{ij} r_{lj} - \bar{r}_i \bar{r}_l}{s_i s_l} \quad (i=1, \dots, k; l=1, \dots, k), \quad (14)$$

$$\max_{\mathbf{Q}} H_e(\mathbf{Q}) = H_e(\mathbf{Q}^*) = -\sum_{j=1}^n q_j^* \ln q_j^* = -\sum_{j=1}^n \frac{1}{n} \ln \frac{1}{n} = \ln n$$

Substituting appropriate values (12), (13), (14) in (5) and (6), we'll receive a problem with two criterions (5) and (6) under system of restrictions (8), (10). Thus, the problem of optimization of the portfolio structure of securities, under condition of unknown economic medium probability distribution is reduced to a problem of choice of an optimum structure of a portfolio in conditions of an information situation  $I_1$ . The Pareto vector of a problem (5), (6), (7) under the system of

restrictions (8), (9), (10), (11) looks like  $(\mathbf{x}_1^*, \dots, \mathbf{x}_k^*; \frac{1}{n}, \dots, \frac{1}{n})$ , where the effective portfolio  $(\mathbf{x}_1^*, \dots, \mathbf{x}_k^*)$  is a Pareto

vector of a problem (5), (6) under the system of restrictions (8), (10), if  $m_i, \mathbf{s}_i, \mathbf{r}_{ij}$  are calculated under the formulas (12), (13), (14).

In [8] the application of a game theory concerning shaping a set of effective portfolios is offered. Let's consider game, which has a payment matrix  $R = R_{k \times n} = (r_{ij})$ , where  $r_{ij}$  — possible profit rate of  $i$ -th paper ( $i=1\dots, k, j=1\dots, n$ ). Let in game  $\mathbf{a} = \max_i \mathbf{a}_i = \max_i \min_j r_{ij} < \min_j \max_i r_{ij} = \min_j \mathbf{b}_j = \mathbf{b}$  and it is impossible to simplify it up to a size  $s \times s$ , where  $s < \min\{k; n\}$ . A solution of the given game will be a pair of the optimum mixed strategies

$$\mathbf{P}^* = (p_1^*, \dots, p_k^*), \quad \mathbf{Q}^* = (q_1^*, \dots, q_n^*)$$

And value of a game

$$v = \sum_{i=1}^k \sum_{j=1}^n r_{ij} p_i^* q_j^* \quad (15)$$

**Theorem 1.** Let  $k \geq n$ , the game, which payment matrix  $R = R_{k \times n} = (r_{ij})$ , hasn't a saddle point, exists such optimum mixed strategy of the second player  $\mathbf{Q}^* = (q_1^*, \dots, q_n^*)$ ,  $q_j^* > 0$  ( $j=1\dots, n$ ),  $\sum_{j=1}^n q_j^* = 1$ . Then

$(p_1^*, \dots, p_k^*; \frac{1}{n}, \dots, \frac{1}{n})$  is a Pareto vector of a problem (5), (6), (7) under the system of restrictions (8), (9), (10), (11), where the optimum mixed strategy of the first player  $\mathbf{P}_1^* = (p_1^*, \dots, p_k^*)$  is an effective portfolio.

**Demonstration.** If the second player applies the active strategy, the average prize of the first player will be equaled accordingly to value of a game (15):

$$\sum_{i=1}^k r_{ij} p_i^* = v = \text{const} \quad (j=1\dots, n),$$

that is profit rate  $R_x^*$  of a portfolio  $\mathbf{X}^* = (x_1^*, \dots, x_k^*)$ ,

$$\text{where } x_i^* = p_i^* \quad (i=1\dots, k), \text{ will be } R_x^* = \sum_{i=1}^k R_i x_i^* = \sum_{i=1}^k R_i p_i^* = v = \text{const}.$$

Thus, his risk — the variance of profit rate  $\mathbf{D}(R_x^*) = \mathbf{D}(\text{const}) = 0$  — will be the minimal. For a vector  $(p_1^*, \dots, p_k^*; \frac{1}{n}, \dots, \frac{1}{n})$  the value of Shannon's entropy

$$H_e(\mathbf{Q}) = - \sum_{j=1}^n \frac{1}{n} \ln \frac{1}{n} = \ln n$$

should be the maximal. So, there is no such vector  $(x_1^*, \dots, x_k^*; q_1^*, \dots, q_n^*)$ , which is by more priority, than solution

$$(p_1^*, \dots, p_k^*; \frac{1}{n}, \dots, \frac{1}{n})$$

The theorem is proved.

**Example 1.** Let  $k = 5$ ,  $n = 2$ , functional of an estimation

$$R = R_{5 \times 2} = \begin{pmatrix} 20 & 11 \\ 25 & 18 \\ 11 & 30 \\ 20 & 24 \\ 15 & 20 \end{pmatrix}$$

**Solution.** First of all we shall check up existence of a saddle point in a payment matrix. For this purpose we shall find the lower value of a game

$$\mathbf{a} = \max_i \mathbf{a}_i = \max_i \min_j r_{ij} = \max\{11, 18, 11, 20, 15\} = 20$$

And upper value of a game

$$\mathbf{b} = \min_j \mathbf{b}_j = \min_j \max_i r_{ij} = \min\{25; 30\} = 25 > 20 = \mathbf{a}$$

The saddle point in game is absent.

Let's reduce a matrix  $R = R_{5 \times 2}$  to more simple kind

$$H = H_{2 \times 2} = (h_{ij}) = \begin{pmatrix} 25 & 18 \\ 20 & 24 \end{pmatrix}$$

Also we shall receive

$$p_2^* = \frac{h_{22} - h_{21}}{h_{11} - h_{12} - h_{21} + h_{22}} = \frac{24 - 20}{25 - 18 - 20 + 24} = \frac{4}{11},$$

$$p_4^* = 1 - p_2^* = 1 - \frac{4}{11} = \frac{7}{11}, \quad p_1^* = p_3^* = p_5^* = 0,$$

$$q_1^* = \frac{h_{22} - h_{12}}{h_{11} - h_{12} - h_{21} + h_{22}} = \frac{24 - 18}{25 - 18 - 20 + 24} = \frac{6}{11} > 0,$$

$$q_2^* = 1 - q_1^* = 1 - \frac{6}{11} = \frac{5}{11} > 0,$$

$$v = \frac{h_{11}h_{22} - h_{12}h_{21}}{h_{11} - h_{12} - h_{21} + h_{22}} = \frac{25 \cdot 24 - 18 \cdot 20}{25 - 18 - 20 + 24} = \frac{240}{11}$$

The Pareto vector looks like  $(0, \frac{4}{11}, 0, \frac{7}{11}, 0; \frac{1}{2}, \frac{1}{2})$ , and portfolio

$$X^* = P^* = (0, \frac{4}{11}, 0, \frac{7}{11}, 0)$$

Is effective. Should remark, that  $m_2 = \bar{r}_2 = 21,5$ ,  $m_4 = \bar{r}_4 = 22$ ,  $\mathbf{s}_2 = 3,5$ ,  $\mathbf{s}_4 = 2$ ,  $\mathbf{r}_{12} = -1$ ,

$$\mathbf{M}(R_x^*) = v = \frac{240}{11}, \quad \mathbf{D}(R_x^*) = 0, \quad \text{where} \quad R_x^* = R_2 \cdot \frac{4}{11} + R_4 \cdot \frac{7}{11}.$$

**Theorem 2.** If  $k < n$ , the game, which has a payment matrix  $R = R_{k \times n} = (r_{ij})$ , has no a saddle point, so for an information situation  $I_5$  the Pareto vector of a problem (5), (6), (7) under the system of restrictions (8), (9), (10), (11) looks

like  $(\mathbf{p}_1^*, \dots, \mathbf{p}_k^*; \mathbf{q}_1^*, \dots, \mathbf{q}_n^*)$ , where  $\mathbf{P}^* = (\mathbf{p}_1^*, \dots, \mathbf{p}_k^*)$ ,  $\mathbf{Q}^* = (\mathbf{q}_1^*, \dots, \mathbf{q}_n^*)$  are mixed strategies of

the players for game, which is given by a payment matrix  $R = R_{k \times n} = (r_{ij})$ .

**Demonstration.** The fifth information situation is characterized by availability of antagonistic interests of a medium. The active medium tends to choice of such condition, for which the estimation functional  $R = R^+$  acquires minimum value in a set of the maximum possible values.

The decision process should analyze under the fundamental rules of the theory of antagonistic games. The second player puts the purpose as much as possible to reduce the loss, that is value of a game (15). A solution of the given game will be a pair of the optimum mixed strategies  $\mathbf{P}^* = (\mathbf{p}_1^*, \dots, \mathbf{p}_k^*)$ ,  $\mathbf{Q}^* = (\mathbf{q}_1^*, \dots, \mathbf{q}_n^*)$ , and the average prize of the first player (or average loss of the second player) will be measured by a value of a game (15). If one of the players adheres to

$$\sum_{i=1}^k \sum_{j=1}^n r_{ij} p_i q_j^* \leq v \leq \sum_{i=1}^k \sum_{j=1}^n r_{ij} p_i^* q_j$$

the optimum strategy, it is not favourable to other one to deviate from his own: , that

is  $\max_p \sum_{i=1}^k \sum_{j=1}^n r_{ij} p_i q_j^* = v$ . If  $\mathbf{Q}^* = (\mathbf{q}_1^*, \dots, \mathbf{q}_n^*)$  to substitute in (1), (2), (3), (4), we shall receive

$$m_x = \sum_{i=1}^k m_i X_i = \sum_{i=1}^k \left( \sum_{j=1}^n r_{ij} q_j^* \right) X_i = \sum_{i=1}^k \sum_{j=1}^n r_{ij} p_i q_j^* \leq v$$

That is  $\max_x m_x = v = \sum_{i=1}^k \sum_{j=1}^n r_{ij} p_i^* q_j^*$ .

If the game, which has a payment matrix  $R = R_{k \times n} = (r_{ij})$ , has more than one optimum solution, the optimum mixed strategies  $\mathbf{P}^* = (\mathbf{p}_1^*, \dots, \mathbf{p}_k^*)$  and  $\mathbf{Q}^* = (\mathbf{q}_1^*, \dots, \mathbf{q}_n^*)$  are those, that the risk (6) of portfolio

$\mathbf{x}^* = \mathbf{P}^* = (\mathbf{p}_1^*, \dots, \mathbf{p}_k^*)$ , expressed as a variance  $\mathbf{D}(R_x^*) = \mathbf{D} \left( \sum_{i=1}^k R_i p_i^* \right) = \sum_{i=1}^k \sum_{l=1}^k \mathbf{s}_i \mathbf{s}_l r_{il} p_i^* p_l^*$ , reaches

the minimum in a set of all pairs of the strategies  $\mathbf{P}^* = (\mathbf{p}_1^*, \dots, \mathbf{p}_k^*)$  and  $\mathbf{Q}^* = (\mathbf{q}_1^*, \dots, \mathbf{q}_n^*)$ .

Thus, the economic medium actively counteracts to reaching of the greatest efficiency of solutions, and the optimum strategy  $\mathbf{Q}^* = (\mathbf{q}_1^*, \dots, \mathbf{q}_n^*)$  is by a probability distribution of conditions of an economic medium, and the optimum strategy

$\mathbf{P}^* = (\mathbf{p}_1^*, \dots, \mathbf{p}_k^*)$  ensures the maximum expected value (mathematical expectation) of the profit rate : for a vector

$(\mathbf{p}_1^*, \dots, \mathbf{p}_k^*; \mathbf{q}_1^*, \dots, \mathbf{q}_n^*)$  there is no such vector  $(\mathbf{x}_1^*, \dots, \mathbf{x}_k^*; \mathbf{q}_1^*, \dots, \mathbf{q}_n^*)$ , which is by more priority, than

solution  $(\mathbf{p}_1^*, \dots, \mathbf{p}_k^*; \mathbf{q}_1^*, \dots, \mathbf{q}_n^*)$ .

The theorem is proved.

**Example 2.** Let  $k = 2$ ,  $n = 6$ , estimation functional

$$R = R_{2 \times 6} = \begin{pmatrix} 12 & 18 & 24 & 30 & 24 & 21 \\ 24 & 36 & 24 & 18 & 21 & 18 \end{pmatrix}$$

**Solution.** a) It is easy to notice, that  $\mathbf{a} < \mathbf{b}$ . Really

$$\mathbf{a} = \max_i \mathbf{a}_i = \max_i \min_j r_{ij} = \max\{12, 18\} = 18,$$

$$\mathbf{b} = \min_j \mathbf{b}_j = \min_j \max_i r_{ij} = \min\{24, 36, 24, 30, 24, 21\} = 21$$

Let's reduce a matrix  $R = R_{2 \times 6}$  to more simple kind  $H = H_{2 \times 2} = \begin{pmatrix} 12 & 21 \\ 24 & 18 \end{pmatrix}$  and we shall receive

$$p_1^* = \frac{18 - 24}{12 - 21 - 24 + 18} = \frac{-6}{-15} = \frac{2}{5}, \quad p_2^* = 1 - p_1^* = \frac{3}{5},$$

$$q_1^* = \frac{18 - 21}{12 - 21 - 24 + 18} = \frac{-3}{-15} = \frac{1}{5}, \quad q_6^* = 1 - q_1^* = \frac{4}{5},$$

$$q_2^* = q_3^* = q_4^* = q_5^* = 0, \quad v = \frac{12 \cdot 18 - 21 \cdot 24}{12 - 21 - 24 + 18} = \frac{-288}{-15} = \frac{96}{5}.$$

For the fifth information situation  $I_5$  the Pareto vector looks like  $(\frac{2}{5}, \frac{3}{5}; \frac{1}{5}, 0, 0, 0, 0, \frac{4}{5})$ , and the portfolio

$$X^* = P^* = (\frac{2}{5}, \frac{3}{5}) \text{ is effective.}$$

b) Let  $q_1 = q_2 = q_3 = q_4 = q_5 = q_6 = \frac{1}{6}$ , then

$$m_1 = \bar{r}_1 = \frac{1}{6} \sum_{j=1}^6 r_{1j} = 21,5, \quad m_2 = \bar{r}_2 = \frac{1}{6} \sum_{j=1}^6 r_{2j} = 23,5 > 21,5 = m_1,$$

$$s_1^2 = \frac{1}{6} \sum_{j=1}^6 r_{1j}^2 - \bar{r}_1^2 = 31,25, \quad s_2^2 = \frac{1}{6} \sum_{j=1}^6 r_{2j}^2 - \bar{r}_2^2 = 37,25 > 31,25 = s_1^2,$$

$$r_{12} = \frac{\frac{1}{6} \sum_{j=1}^6 r_{1j} r_{2j} - \bar{r}_1 \cdot \bar{r}_2}{s_1 \cdot s_2} = \frac{-16,25}{\sqrt{31,25} \cdot \sqrt{37,25}} \approx -0,4763$$

Magnitude of a variance of a portfolio

$\mathbf{s}_x^2 = \mathbf{s}_1^2 X_1^2 + \mathbf{s}_2^2 X_2^2 + 2\mathbf{s}_1 \mathbf{s}_2 r_{12} X_1 X_2 = 31,25 X_1^2 + 37,25 X_2^2 - 32,5 X_1 X_2$ , where  $X_1 + X_2 = 1$ ,  $X_1 \geq 0$ ,  $X_2 \geq 0$ , reaches the minimum, when

$$X_1 = X_1^* = \frac{\mathbf{s}_2 (\mathbf{s}_2 - \mathbf{s}_1 r_{12})}{\mathbf{s}_1^2 + \mathbf{s}_2^2 - 2\mathbf{s}_1 \mathbf{s}_2 r_{12}} = \frac{37,25 + 16,25}{31,25 + 27,25 + 32,5} = \frac{107}{202} \geq 0,$$

$$x_2 = x_2^* = 1 - x_1^* = 1 - \frac{107}{202} = \frac{95}{202} \geq 0$$

For the fourth information situation  $I_4$  the Pareto vector looks like  $(x, 1-x; \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6})$ , where  $0 \leq x \leq \frac{107}{202}$ , and any portfolio  $(x, 1-x)$ ,  $0 \leq x \leq \frac{107}{202}$ , is effective.

From a model (5) – (11) it is visible, that the problem of choice of an optimum structure of a portfolio under condition of a unknown probability distribution is a problem with three criterions. On the one hand, the problem (5) – (11) can be reduced to a standard problem of optimization. On the other hand, the coordination (7) maximization of Shannon's

entropy gives dot evaluations of probabilities  $q_j = \frac{1}{n}$  ( $j=1, \dots, n$ ). These evaluations of distribution of a priori probabilities enable to apply a classical economic-mathematical model of shaping of a portfolio for the first information situation  $I_1$  with two criterions.

In the developed form the situation of a decision making is characterized by a matrix  $R^+ = R_{k \times n}^+ = (r_{ij}^+)$ , which elements are  $r_{ij}^+$  — probable profit rates of  $i$ -th paper under condition that the medium is in  $j$ -th condition ( $i=1, \dots, k$ ,  $j=1, \dots, n$ ). If in game of two subjects with the zero sum, which has a payment matrix  $R = R_{k \times n} = (r_{ij})$ ,  $\mathbf{a} < \mathbf{b}$ , where  $\mathbf{a}$  — lower (maxmin) a value of a game,  $\mathbf{b}$  — upper (minmax) the value of a game, the solution of the game is a pair of the optimum mixed strategies  $\mathbf{P}^* = (p_1^*, \dots, p_k^*)$  and  $\mathbf{Q}^* = (q_1^*, \dots, q_n^*)$ . If  $k \geq n$ ,  $q_j^* > 0$  ( $j=1, \dots, n$ ), the portfolio  $\mathbf{x}^* = \mathbf{P}^* = (p_1^*, \dots, p_k^*)$  is effective. If  $k < n$ , the portfolio  $\mathbf{x}^* = \mathbf{P}^* = (p_1^*, \dots, p_k^*)$  is effective in the  $I_5$  situation. If the payment matrix  $R = R_{k \times n} = (r_{ij})$  cannot be reduced to a type  $H = H_{k \times 2}$  or  $H = H_{2 \times n}$ , in many cases it is convenient to solve a problem of the theory of a game as a problem of a linear programming by a simplex method, which allows to use modern COMPUTERS for choice of an optimum structure of a portfolio widely.

Thus it is possible to solve the delivered problem under condition of uncertainty for selected system of hypothesis.

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