Supervising the Behaviour of the Conditioning-State of the Neural Nets by Their Phase-Space-Flow-Behaviour

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Abstract

A new kind of supervising strategy for the neural nets will be presented, which is based on a phase-space-representation of the degrees of freedom of neural nets. It will be shown, that with this phase-space-representation it is possible, to describe the change of the configuration of the neural nets by the trajectories of the degrees of freedom of the net and to detect by the flow of this trajectories local or global changes of the different degrees of freedom, the adaptation of local minima and/or the effects of changes of the network parameters.

Similar to the theory of the quantisation of the neural nets [Reuter 4] it will be shown that out of the phase-space-representation and the potential-oriented formulation of the activity states of neural nets the classification- and the common condition-states of neural nets can be described as different states of a dynamical n-particle-system ‘neural net’.

By dint of this theory in the conditioning state neural nets can be characterised as disturbed equilibrium state of the potential of a so called '**Repräsentant RP$_{_Netz}$**', which can be identified as a vector in the n-dimensioned space of the degrees of freedom of a neural net.

Keywords: phase-space-representation of neural nets, supervising strategies for neural nets.

1. Introduction to the potential oriented description of neural nets

For every neural net a classification potential

$$E_p = E_p\left(\sum_{n=1}^{N}\left(E_p{}^{n}(y_1^n(t),...,y_M^n(t)\right)\right)$$

and a conditioning potential

$$U_p = U_p\left(\sum_{n=1}^{N}\left(U_p{}^{n}(y_1^n(t),...,y_M^n(t)\right)\right)$$

can be defined, where the index n indicates a neuron of the neural net with N neurons and the y_m^n indicates the m[th] degree of freedom of the M degrees of freedom of a neuron n. If the state vector $\vec{y}(t)$ describes all neuronal and synaptic values of the network at time t the neural network reaches steady state when

$$\dot{\vec{y}}(t) = \vec{f}(\dot{\vec{y}}(t)) = \vec{0}$$

holds indefinitely or until new stimuli perturb the system out of the equilibrium described by a the Lagrange function

$$L = L(f(y(t))) = E_p(y(t)) - U_p(y(t)) = 0$$

If and only if the net adapts in the condition state asymptotically to a desired vector called the Repräsentant $RP_{\_Netz} = \vec{y} = (y_1,..., y_n)$ of the net, this Lagrange function can be identified as a Lyapunov function for which holds

$$\dot{L} = \sum_{i=1}^{n} \frac{\partial L}{\partial x_i}\frac{\partial x_i}{\partial t} = \sum_{i=1}^{n} \frac{\partial L}{\partial x_i}\dot{x}_i \le 0$$

where the x_i are the transformed co-ordinates of the state vector $\vec{y}(t)$ of the net in a co-ordination system for which holds

$$\vec{X} = \vec{0}$$

if the net meet it’s equilibrium point.

A possible form of the Lyaponov function will be given by

$$L = \frac{1}{2} \sum_{i=1}^{n} x_i$$

and the dynamics of the degrees of freedom of the net like the inter-neural weights w_{ij} or the parameters of the transfer-function f_{trans} will be given by the form

$$\dot{x}_i = -E_i x_i$$

with

$$x_i = x_i(t=0)\, e^{-E_i t}$$

If we assume that the time derivation of the x_i can be written in the form

$$\dot{x}_i = -grad\, V$$

the change of the x_i can be observed as trajectories that cross the equi-potential levels, described by the values of L, orthogonal.

Depend on this preconditions the conditioning-state of a neural net can be understood as an non-equilibrium-state of the potential-functions E_p and U_p, which forces a change of the degrees of freedom x_i of the net to adapt the co-ordinates of the desired net structure $\vec{X} = \vec{0}$.

If $\Delta\vec{x}(t)$ denotes the flow of the vector $\vec{x}(t)$ and if $F_i = \dot{x}_i$ denotes the conditioning-impulse of the degree of freedom x_i the conditioning of a neural net can be described as the shift of the potential of the Repräsentant RP_Netz as shown in Figure 1 forced by the Lyapunov-Lagrange function $L = E_p - U_P \neq 0$.

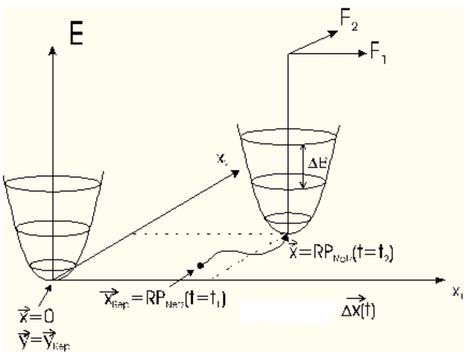

Fig. 1: Translation of the potential of the vector Repräsentant RP_Netz forced by the presents of a Lyapunov-Lagrange function $L = E_p - U_P \neq 0$

2 The neural phase space

With the defined the co-ordinates x_i, y_i and the impulses F_i it is possible to supervise the net's behaviour during the condition state by studying the flow of the degrees of freedom of the trajectories in the phase-space of the net. Moreover the expressions for the flow of the generalised co-ordinates $x_i=q_i$ and the generalised impulses $F_i=p_i$ must follow the Hamilton equations given by

$$\frac{dq_i}{dt} = -\frac{\partial H(q_i, p_i)}{\partial p_i}$$

and

$$\frac{dp_i}{dt} = \frac{\partial H(q_i, p_i)}{\partial q_i}$$

As the divergence of the volume of a phase-space-fluid ρ (describing all degrees of freedoms of the net) will be zero, with the expression

$$\hat{L} = -i \sum_{i=1}^{n} \left(\frac{\partial H(q_i, p_i)}{\partial p_i} \frac{\partial}{\partial q_i} - \frac{\partial H(q_i, p_i)}{\partial q_i} \frac{\partial}{\partial p_i} \right)$$

the change of the phase-space-fluid ρ can be described by the formula

$$-i \frac{\partial \rho}{\partial t} = \hat{L} \rho$$

which leads to the following formula of the time dependence of the phase-space-fluid

$$\rho(t) = e^{-i\hat{L}t} \rho_0(t=0)$$

The spectrum of the operator $\hat{L}$ characterises the flow of ρ [Prigogine] in the way that four different flows of ρ can be expected

- non ergodic flows
- ergodig non-mixing flows
- ergodig mixing flows
- and K-flows

The behaviour of the first three flows is shown in Figure 2. As the K-flows follow the Bäcker-transformation they cannot be visualised.

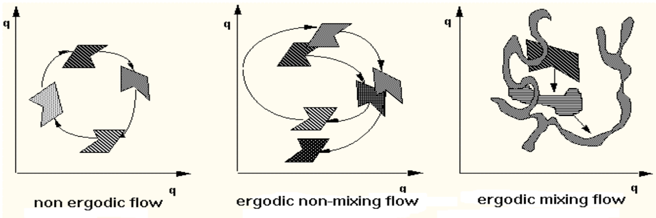

Fig. 2: About the flow of the phase-space-fluid
r

Guided by this classification of the flow of the phase-space-fluid *r* we can learn that the range of the change of the degrees of freedom and the conditions of this change will lead to different classifiers.

So ergodic nets will show after a frequency of conditioning steps the same classification behaviour, as before, a neural 'clock' will be designed.

Ergodig non-mixing nets will represent the common neural nets. After a conditioning-phase all classification criteria learned before will be lost.

Ergodic mixing flows will represent nets which are not loosing all their stored classification knowledge under further conditioning; they can be 'sensitised ' and will show a kind of real-time behaviour [Reuter 1996].

4 About the visualisation of the phase-space-trajectories and experimental results

Based on the theory discussed above an algorithm was implemented in the software-tool zz-2, which enables to visualise the phase-space-flow of the trajectories of the degrees of freedom of a Backpropagation neural net. To supervise the different areas of the net's change and the net's behaviour the visualisation was divided in five windows:

To summarise the change of the net during the conditioning state, two kinds of symmetry visualisation were implemented in the first visualisation window:

- The first visualisation shows the sum of the changes of the weights of the connections from the input-layer to the hidden-layer during two iteration steps and the changes of the weights of the connections form the hidden-layer to the output-layer during two iteration steps.
- The second visualisation shows the global changes of the net in that way that the sum of all changes of the net is deforming a circle and a square.

Following that method, it can be observed at which layer the net structure changes momentary and/or if the net structure is changed at all.

At the next visualisation window the configuration of the different layers was coded in that way that all degrees of freedom form a 2-dimension figure where

- on the x-axis the momentary weight-configurations of the connections of the neurons are shown,
- on the y-axis the gradient of the flow of the changes of this connections are shown.

At the following two windows (block 3 and 4) on the right hand side the flow of individual degrees of freedom are shown. As normally the number of the degrees is very large, the user can select the degrees of freedom to be visualise.

Figure 3 shows the different windows as explained before.

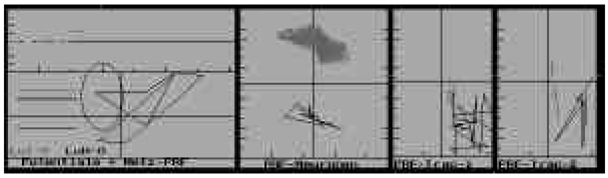

Fig. 3: The phase-space-monitor

The Figures 4a-4c show the phase-space-figures and the flow of the degrees of freedom of a Backpropagation net with two out-put-neurons during a successful conditioning state. It is important to mention that all data have been pre-processed by a DLS and mFD-Operation, so the net has only to learn a system-relevant information-structure.

Figure 4a shows the net structure at the beginning of the conditioning.

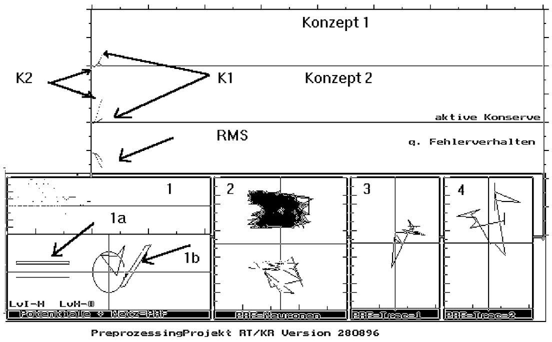

Fig. 4a: Phase-space-behaviour of the degrees of freedom of a backpropagation net at the beginning of the conditioning state

Clearly it can be pointed out of Figure 4a that the classification structure of the net at this time is far away from the desired net-structure, as the classification security indicators K1 and K2 are very low. Also the changes in both layers are very high as mentioned by the stripes 1a. The symmetry figure "square" is extremely deformed, also an indicator that the net is changing it's structure largely.

In Figure 4a four degrees of freedom have been selected to be supervised individually. In block 3 the phase-space-flow of two of them are shown. As this window belongs to the input-hidden-layer, the flows represent the change of the net in this area only. Corresponding block 4 shows the change of two degrees of freedom in the hidden-output-layer.

About 200 iteration steps further on the different net-parameters have changed totally (see Figure 4b). Now the classification security indicators K1 and K2 point out that the classification concept of the net have nearly reached the desired maximum-value ‘1’. On the other hand the phase-space-representation of the net and the flow of degrees of freedom show that the structure of the net are still changing as the figures ‘circle’ and ‘square’ haven't reached a high degree of symmetry yet. Out of the structure of block 2 we can learn, that the major change of the net's structure have been performed in the hidden-output-area, as it's net-structure have changed totally contradict to the input-hidden-structure. This result can be verified also by the changes of block 3 and 4, as a major change of the trajectories can be detected in block 4 only.

Out of the structure of block 2 we can learn that the major change of the net's structure have been performed in the hidden-output-area, as it's net-structure have changed totally on the contrary of the input-hidden-structure.

The result can be verified also by the changes of block 3 and 4, as a major change of the trajectories can be detected in block 4 only.

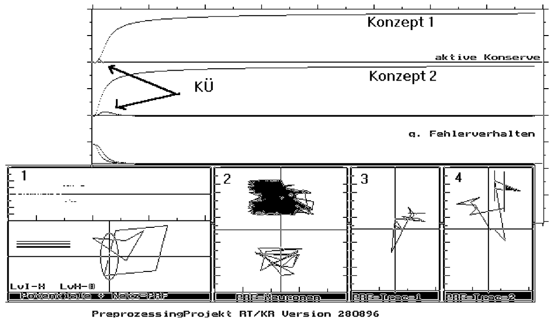

Fig. 4b: Phase-space-behaviour at the middle of the conditioning state

As we know from the theory discussed above, the final structure of the net will be reached if and only if the phase-space-flow is interrupted and the geometrical figures ‘circle’ and ‘square’ have reached a high degree of symmetry.

This state of the net's condition is shown in Figure 4.c

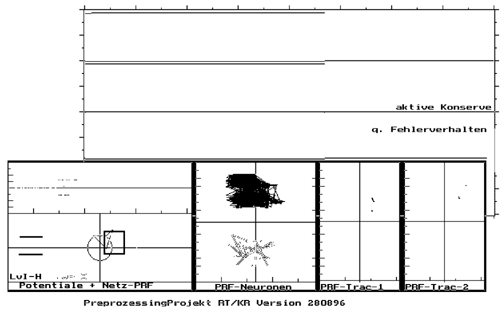

Fig. 4c: Phase-space-behaviour at the end of the conditioning state

Clearly it can be pointed out from Figure 4c that no major changes of the net's structure have been done during the last phase of the conditioning. In block 1 the stripes, representing the changes in the input-hidden-structure and the hidden-output-structure are overlapping and the ‘circle’ and the ‘square’ are highly symmetric. In block 3 and 4 the flow of the different degrees of freedom are interrupted and also the probability of the classification results shown in the upper window are about 99%.

Also it can be pointed out that in this last conditioning phase the net seems to be a ergodig mixing structure (if we assume that the 2-dim projection of all degrees of freedom are distance-invariant), as the net's structure change only locally. So it seems that the net is sensitised in the last phase of the conditioning.

Figure 5 shows an example, how the phase-space-flow-representation can be used to supervise a learning phase more effectively and how this phase space representations enable a user to control or to change the different learning parameters more effectively. In this Figure a typical momentary stoppage of the conditioning phase in the hidden-output-area (block 4) is shown. As the condition state is quite young the weight-configuration is changed largely by every iteration step, but no weight-change is adapted by the Backpropagation algorithm as the structure and the condition for changing the net's structure don't fit. Only the input-hidden-area shows some smoothly changes, announced by the slowly down-ward-shift of the trajectories in block 3.

That the net has not reached it's best configuration can be detected also by the non-symmetry of the figures 'circle' and 'square' in block 1.

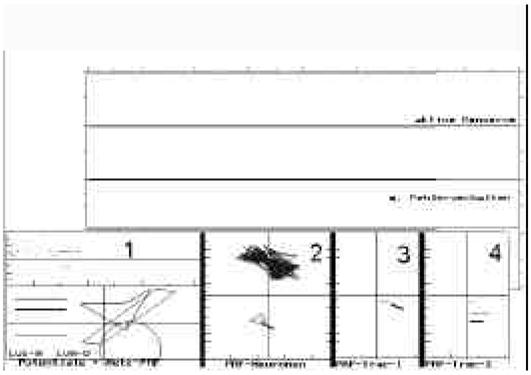

Fig. 5: Phase-space-behaviour announcing a typical local minimum

What we can learn from Figure 5 is that something must be wrong in the hidden-output-area, reps. that the learning parameters for that area have to be changed. This phase space behaviour is a typical indicator a local minimum has been adapted by the learning algorithm.

5 Conclusions

The theory of the phase-space-behaviour of the degrees of freedom of a neural net enables to create and to supervise an individual learning strategy for the system-parameters of neural nets and the supervised design of non ergodic, ergodic non mixing, ergodic mixing or chaotic net structures. As the phase-space-flow of the degrees of freedom indicates the different condition-strategies, special learning effects like the sensitisation of a classifier can be evaluated by the net-designer directly by changing the system-parameters in an adequate way. Also the problem of learning-interrupts by meeting a local minimum can be overcome in a total new way, as now the system-parameters, which have to be changed can be selected directly and individually.

6 Literature

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