

# A Robust Control System for Tentacle Robots in Cooperative Tasks

Mircea Ivanescu\*, Professor, PhD, Viorel Stoian\*, Assoc. Prof., PhD, Nicu Bizdoaca\*, lecturer

Nirvana Alina Popescu\*\*, assistant professor, Decebal Popescu\*\*, assistant professor

\*Automation and Computer Dept, University of Craiova, A. I. Cuza Street, 1100 Craiova, Romania

Phone: +40-51-414398, Fax: +40-51-411688

e-mail: [ivanescu@robotics.ucv.ro](mailto:ivanescu@robotics.ucv.ro); [stoian@robotics.ucv.ro](mailto:stoian@robotics.ucv.ro); [nicu@robotics.ucv.ro](mailto:nicu@robotics.ucv.ro).

\*\* Computer Department, University "Politehnica", Bucharest, Romania

**ABSTRACT:** A robust control system is proposed to solve the local control for a multi-chain robotic system formed by tentacle manipulators grasping a common object with hard point contacts. The two-level hierarchical control is adopted. The upper level coordinator gathers all the necessary information to resolve the distribution force. Then, the lower-level local control problem is treated as an open-chain redundant manipulator control problem. The stability and robustness of a class of the fuzzy logic control (FLC) are investigated and a robust FLC is proposed with uncertainties of the load. The fuzzy rules are established. Simulation results are presented.

**KEYWORDS:** robust control system, multi-chain tentacle robotic system, fuzzy logic control.

## INTRODUCTION

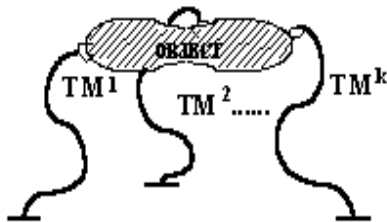


Figure 1: A multiple-chain tentacle robotics system

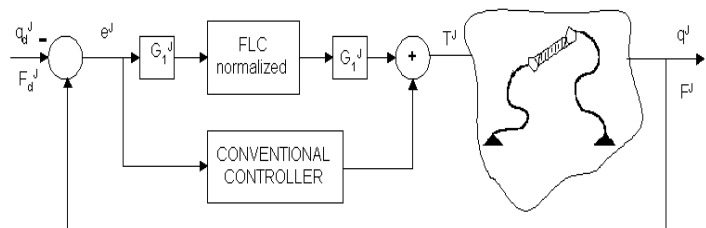


Figure 2: The closed-loop control system

In the past few years, the research in coordinating robotic systems with multiple chains has received considerable attention. The problem of controlling this system in real time is more complicated. A multiple chain tentacle robotic system is more complicated. A tentacle manipulator is a manipulator with a great flexibility, with a distributed mass and torque that can take any arbitrary shape. Technologically, such systems can be obtained by using a cellular structure for each element of the arm. The control can be produced using an electrohydraulic or pneumatic action that determines the contraction or dilatation the peripheral cells. The first problem is the global coordination problem that involves coordination of several tentacles in order to assure a desired trajectory of a load. The second problem is the local control problem, which involves the control of the individual elements of the arm to achieve the desired position. The force distribution is a subproblem in which the motion is completely specified and the internal forces/torques to effect this motion are to be determined. To resolve this large-scale control problem, a two-level hierarchical control scheme (Cheng, 1995) is used. The upper-level system collects all the necessary information and solves the inter-chain coordination problem, the force distribution problem. Then, the problem is decoupled into  $j$  lower-level subsystems, for every arm. The local fuzzy controllers are assigned to solve the local control. In order to obtain the fuzzy control an approximate model of the tentacle arm is used. The stability and robustness of the FLC are investigated and a robust FLC is proposed with respect to the robustness of the load uncertainties. The control strategy is based on the Direct Sliding Mode Control (DSMC) which controls the trajectory towards the switching line and then the motion is forced directly to the origin, on the switching line. A fuzzy controller is proposed and the fuzzy rules are established by using the DSMC procedures. Efficiently considerations of the method are discussed. Numerical simulations for several control problems are presented.

## MODEL FOR COOPERATIVE TENTACLE ROBOTS

A multiple-chain tentacle robotics system is presented in Figure 1. With the chains of the system forming closed-

kinematics loops, the responses of individual chains are tightly coupled with one another through the reference member (object or load). The complexity of the problem is considerably increased by the presence of the tentacle manipulators,  $(TM^j, j=1 \dots k)$ , the systems with, theoretically, a great mobility, which can take any position and orientation in space (Ivanescu, 1984, 1986). The dynamic equations for each chain of the system are (Ivanescu, 1984):

$$TM^j : r_j A^j \int_0^s \left[ \sin(q^j - q'^j) \dot{q}'^j + \cos(q^j - q'^j) \ddot{q}'^j \right] ds' + rAg \int_0^s \cos q^j ds' + t^j = T^j, j=1,2 \quad (1)$$

$$\int_0^{L^j} t^j ds = F_x^j \int_0^{L^j} (-\sin q^j) ds + F_z^j \int_0^{L^j} \cos q^j ds, j=1,2 \quad (2)$$

where we assume that each manipulator  $(TM^j)$  has a uniform distributed mass, with a linear density  $\rho^j$  and a section  $A^j$ . We denote by  $s$  the spatial variable upon the length of the arm,  $s \in [0, L^j]$ . We also use the notations:  $q^j$  - Lagrange generalized coordinate for  $TM^j$  (the absolute angle),  $q^j = q^j(s, t)$ ,  $s \in [0, L^j]$ ,  $t \in [0, t_f]$ ,  $q'^j = q^j(s', t)$ ,  $s' \in [0, s]$ ,  $t \in [0, t_f]$ ,  $T^j = T^j(s, t)$  - the distributed torque over the arm;  $\tau^j = \tau^j(s, t)$  - the distributed moment to give the desired motion specified on the reference member. All these sizes are expressed in the coordinate frame of the arm  $TM^j$ . The  $k$  integral equations are tightly coupled through the terms  $t^j$ ,  $F_x^j$ ,  $F_z^j$  where all of these terms determine the desired motion. We propose a two-level hierarchical control scheme (Cheng, 1995; Zheng, 1998) for this multiple-chain robotic system. The control strategy is to decouple the system into  $k$  lower-level subsystems that are coordinated at the upper level. The function of the upper-level coordinator is to gather all the necessary information so as to formulate the corresponding force distribution problem and then to solve this constrained, optimization problem such that optimal solutions for the contact forces  $F^j$  are generated. These optimal contact forces are then the set-points for the lower-level subsystems. With  $F^0$  - the resultant force vector applied to object expressed in the inertial coordinate frame (0),  ${}^0D_j$  - the partial spatial transform from the coordinate frame for the arm  $TM^j$  to the inertial coordinate frame (0), we consider the hard point contact with friction and the force balance equations on the object may be written as:  $F^0 = \sum {}^0D_j F^j$  (3)

$$\text{The object dynamic equations are obtained by the form } M_0 \ddot{r} = GF^0 \quad (4)$$

$$\text{where } M_0 \text{ is inertial matrix of the object and } r \text{ defines the object coordinate vector } r = (x, z, \phi)^T \quad (5)$$

$$\text{and } r(t) \text{ represents the desired trajectory of the motion. The inequality constraints which include the friction constraints and the maximum force constraints may be associated to (3): } \sum A^j F^j \leq B \quad (6)$$

$$\text{where } A^j \text{ is a coefficient matrix of inequality constraints and } B \text{ is a boundary-value vector of inequality constraints. The problem of the contact forces } F^j \text{ can be treated as an optimal control problem if we associate to the relations (3) - (6) an optimal index } \Psi = \sum C^j F^j \quad (7)$$

This problem is solved in several papers: Cheng (1995), Zheng and Luh (1988), Mason (1981) by the general methods of the optimization or by the specific procedures Cheng (1991). After all of the contact forces  $F^j$  are determinate, the dynamics of each arm  $TM^j$  are decoupled. Now, the equations (1), (2) can be interpreted as same decoupled equations with a given  $\tau^j(s)$ ,  $s \in [0, L^j]$  acting on the tip of the arm.

## APPROXIMATE MODEL

A discrete and simplified model of (1), (2) can be obtained by using a spatial discretization:

$$s_1, s_2, \dots, s_N; \quad s_i - s_{i-1} = \Delta; \quad |q^j(s_i) - q^j(s_k)| < \varepsilon; \quad i, k = 1, 2, \dots, n^j \quad (8)$$

where  $\varepsilon, \Delta$  are constants and  $\varepsilon$  is sufficiently small.

$$\text{We denote } s_i = i\Delta, L^j = n^j\Delta, q_j(s_i) = q_i^j, T^j(s_i) = T_i^j, t^j(s_i) = t_i^j \quad (9)$$

and considering the arm as a lightweight arm, from (1), (2) it results (Ivanescu, 1986):

$$M^j \ddot{q}^j + C^j \dot{q}^j + D^j(q^j) F^j = T^j \quad (10)$$

$$\text{where } M^j, C^j \text{ are } (n^j \times n^j) \text{ contact diagonal matrixes, } D \text{ is } (n^j \times 2) \text{ nonlinear matrix (Ivanescu, 1986; Ivanescu and Stoian, 1995): } F^j = \text{col}(F_x^j, F_z^j); \quad q^j = \text{col}(q_1^j \dots q_{n^j}^j); \quad T = \text{col}(T_1^j \dots T_{n^j}^j) \quad (11)$$

$$\text{In the equation (10), } F^j \text{ assures the load transfer on the trajectory. The uncertainty of the load } m \text{ defines an uncertainty of the force } F^j. F^{MJ} \text{ is an estimation of the force upper bound and we assume that } |F^{MJ} - F^j|_i \leq r_i; i = 1, 2 \quad (12)$$

## FUZZY CONTROL

The control problem asks for determining the manipulatable torques (control variables)  $T_1^J$  such that the trajectory of the overall system (object and manipulators) will correspond as closely as possible to the desired behavior. In order to obtain the control law for a prescribed motion, we shall use the closed-loop control system from Figure 2. Let  $q_d^J$  the desired parameters of the trajectory,  $F_d^J$  the desired force applied to the  $j$ - contact point of the object, and  $q^J, F^J$  the same sizes measured on the real system. For a bounded smooth trajectory, a tracking error is:  $e = q - q_d$  (13)

$\dot{e}^J \setminus e^J$	NBE	NSE	ZRE	PSE	PBE
PBDE	ZRC	NBC	NBC	NBC	NBC
PSDE	PBC	ZRC	NSC	NSC	NBC
ZRDE	PBC	PSC	ZRC	NSC	NBC
NSDE	PBC	PSC	PSC	ZRC	NBC
NBDE	PBC	PBC	PBC	PBC	ZRC

Table 1: The fuzzy rules for the output T

$\dot{e}^J \setminus e^J$	NBE	NSE	ZRE	PSE	PBE
PBDE	B	S	S	S	S
PSDE	S	B	S	S	S
ZRDE	S	S	B	S	S
NSDE	S	S	S	B	S
NBDE	S	S	S	S	B

Table 2: The fuzzy rules for  $k_i$

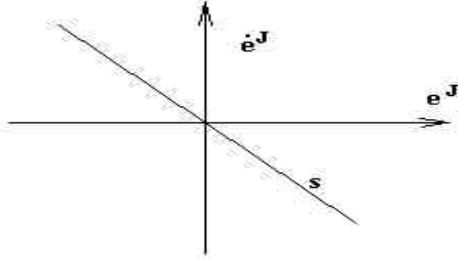


Figure 3: The switching line

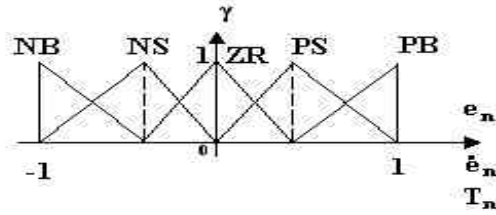


Figure 4: The input/output fuzzy sets

The control system contains two parts: the first component is a conventional controller which implement a classic strategy of the motion control based on the Lyapunov stability and the second is a FLC. A fuzzy control is proposed by using the control law in the neighborhood of the switching line (Table 1 and Figure 3) as a variable structure controller. The physical meaning of the rules is as follows: the output is zero near the switching line ( $s$ ), the output is negative above the switching line, the output is positive below the diagonal line, the magnitude of the output tends to increase in accordance with magnitude of the distance between the switching line and the state  $(e^J, \dot{e}^J)$ .

We consider that all input/output fuzzy sets are assumed to be designed on the normalized space:

$$e_{i,N}^J, \dot{e}_{i,N}^J, T_{i,N}^J \in [-1,1], i=1,2,\dots,n^J \quad (14)$$

and the input/output gains  $G_1^J = [G_e^J, G_{\dot{e}}^J]$  and  $G_c$  serve as scale factors between the normalized space and the

$$\text{corresponding real space of the process } e_{N,i}^J = G_{ei}^J e_i^J; \dot{e}_{N,i}^J = G_{\dot{e}i}^J \dot{e}_i^J; T_{N,i}^J = G_{ci}^J T_i^J \quad (15)$$

If we consider that the switching line  $s$  in the space of normalized values is defined by the diagonal:  $\dot{e}_{N,i}^J + e_{N,i}^J = 0$  (16) it corresponds to the following diagonal line in the real error phase plane (Soo, 1997),

$$s_i^J = \dot{e}_i^J + s_i^J e_i^J = 0, \quad s_i^J = G_{ci}^J / G_{ei}^J > 0 \quad (17)$$

$$\text{or } \dot{e}^J + s e^J = 0 \quad (18)$$

where  $\sigma = \text{diag}(\sigma_1, \sigma_2, \dots, \sigma_{nj})$ . The memberships of the input/output variables are represented in Figure 4, where NB, NS, ZR, PS, PB define the linguistic variables: NEGATIVE BIG, NEGATIVE SMALL, ZERO, POSITIVE SMALL, POSITIVE BIG.

**Theorem 1.** The closed-loop system of Figure 2 is stable if the control is defined by  $T^J = -K^J s + H^J(e, q_d, F^M) + u_F^j$  (19)

where  $K^J$  is a  $(n^J \times n^J)$  symmetric positive definite matrix, which satisfies the condition:

$$K^j - M^j s^j + C^j \text{ is a positive definite matrix} \quad (20)$$

$$\text{and } u_F^j \text{ is the output vector of the fuzzy controller } u_{Fi}^j = -k_{Fi}^j \text{sgn } s_i^j; k_{Fi}^j \geq |H(e, q_d, F) - H^J(e, q_d, F^M)|_i \quad (21)$$

*Proof.* See Appendix 1.

The Theorem 1 determines the conditions which assure the motion control in the neighborhood of the switching line. In order to accelerate the motion on the switching line, we can use the DSMC (Direct Sliding Mode Control). The DSMC was presented in (Ivanescu and Stoian, 1996) and it establishes conditions which force the trajectory along the switching line, directly toward the origin (Figure 5).

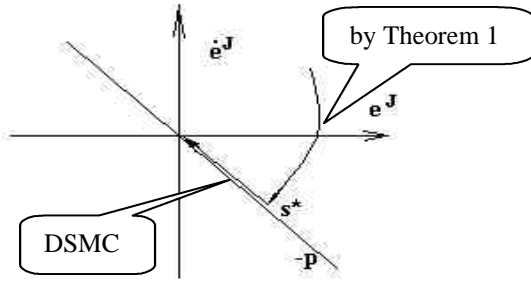


Figure 5: The DSMC procedure.

In the first part of the motion defined by Theorem 1, we considered that the switching line has the slope  $-\sigma_i$ . The DSMC can require a new switching line,  $s_i^* = \dot{e}_i^J - p_i^J e_i^J = 0$  (22)

**Proposition:** The DSMC control is assured if the coefficient  $k_i$  of the controller verifies the condition,

$$(c_i + k_i)^2 \geq 4m_i(s_i(-m_i s_i + c_i + k_i) + h_i) \quad (23)$$

*Proof.* See Appendix 2.

The DSMC control can introduce a new fuzzy output variable, the coefficients  $k_i$ . In the first part of the motion  $k_i$  verifies only the condition (20). When the trajectory penetrates the switching line  $s_i^*$ , the  $k_i$  are increased in order to verify (23). In Figure 6 and Table 2 are presented the memberships functions for  $k_i$  and fuzzy rules, where we defined as  $v_i^1, v_i^2$  the average values which verify the condition (20) and (23), respectively.

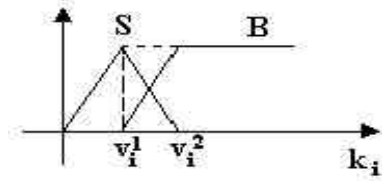


Figure 6: The memberships functions for  $k_i$

## NUMERICAL RESULTS

The purpose of this section is to demonstrate the effectiveness of the method. This is illustrated by solving a fuzzy control problem for a two tentacle manipulator system which operates in XOZ plane (Figure 7).

TM <sup>J</sup>	q <sub>1</sub> <sup>J</sup> (0)	q <sub>2</sub> <sup>J</sup> (0)	q <sub>3</sub> <sup>J</sup> (0)	q <sub>4</sub> <sup>J</sup> (0)	q <sub>5</sub> <sup>J</sup> (0)	q <sub>6</sub> <sup>J</sup> (0)	q <sub>7</sub> <sup>J</sup> (0)
TM <sup>1</sup>	$\pi/6$	$\pi/3$	$7\pi/12$	$2\pi/3$	$\pi/15$	$15\pi/8$	0
TM <sup>2</sup>	$5\pi/6$	$4\pi/5$	$4\pi/5$	$3\pi/4$	$3\pi/4$	$2\pi/3$	$\pi$

Table 3: The initial position of the arms.

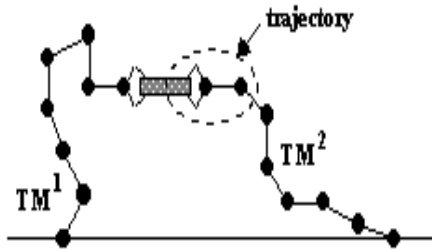


Figure 7: A two tentacle manipulator system.

These two manipulators form a closed-chain robotic system by a common object which is manipulated. An approximate model (10) with  $\Delta = 0.06$  m and  $n^J = 7$  is used ( $J = 1, 2$ ). Also, the length and the mass of the object are 0.2 m and 1 kg, respectively. The initial position of the arms expressed in the inertial coordinate frame are presented in Table 3.

The desired trajectory of the terminal points is defined by:

$$\begin{aligned} x &= x_0 + a \sin \omega t \\ z &= z_0 + b \cos \omega t \end{aligned}$$

With  $x_0 = 0.2$  m,  $z_0 = 0.1$  m,  $a = 0.3$  m,  $b = 0.1$  m,  $\omega = 0.8$  rad/s. The trajectory lies the work envelopes of the both arm and does not go through any workspace singularities. The maximum force constraints are defined by:

$$F_X^J \leq F_{MAX} = 50N \quad F_Z^J \leq F_{MAX} = 50N \quad \text{and the optimal index } \min\left(\sum F_X^{J2}\right) \min\left(\sum F_Z^{J2}\right) \text{ are used.}$$

The uncertainty domain of the mass is defined as  $0.8kg \leq m \leq 1.4kg$ . The solution of the desired trajectory for the elements

of the arms is given by solving the nonlinear differential equation (Ivanescu, 1995):  $\dot{q}_d^J(t) = [J^{JT}(q) \quad J^J(q)]^{-1} J^{JT}(q) \dot{w}(t)$

where  $w = (x, z)^T$  and  $J^J(q)$  is the Jacobian matrix of the arms ( $J = 1, 2$ ). A conventional controller with  $k_i^J = 0.5$

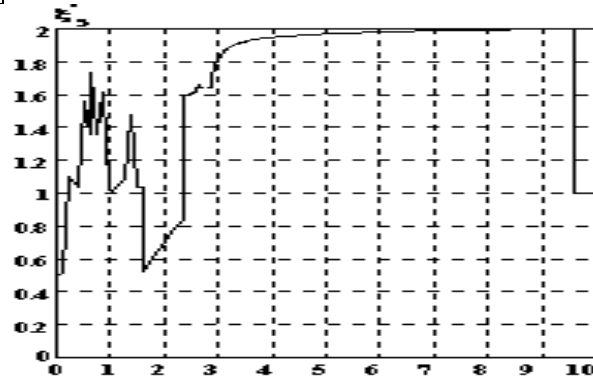


Figure 8: The evolution of  $k_5^1$

( $i=1,..,7, j=1,2$ ) is determined. A FLC is used with the scale factors selected as  $G_{e_i^j} = G_{\dot{e}_i^j} = 10$ ;  $I = 1,..,7; J = 1, 2$ . The conventional and DSMC procedures are used and new switching line is computed. The condition (23) is verified and the new switching line is defined for  $p_i^j = 1.03$ ;  $I = 1,..,7, J = 1, 2$ . In Figure 8 is presented the evolution of  $k_5^1$  for a DSMC procedure and the evolution of the position error  $e_5^1$  and the position error rate  $\dot{e}_5^1$  are presented in Figure 9.

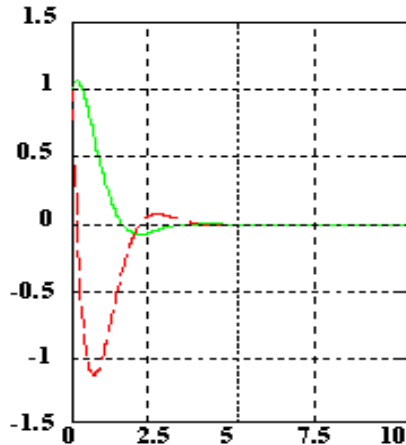


Figure 9: The evolution of the position error  $e_5^1$  and the position error rate  $\dot{e}_5^1$

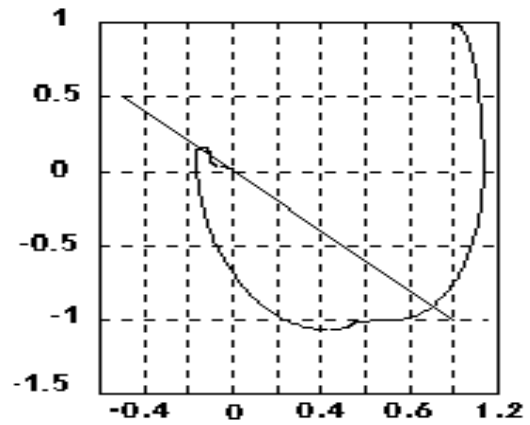


Figure 10: The trajectory in the plane  $(e_5^1, \dot{e}_5^1)$  for conventional procedure

Figure 10 represents the trajectory in the plane  $(e_5^1, \dot{e}_5^1)$  for conventional procedure and Figure 11 the same trajectory for a DSMC procedure for a new switching line. Figure 12 presents the final trajectory. We can remark the error during the 1<sup>th</sup> cycle and the convergence to the desired trajectory during the 2<sup>nd</sup> cycle.

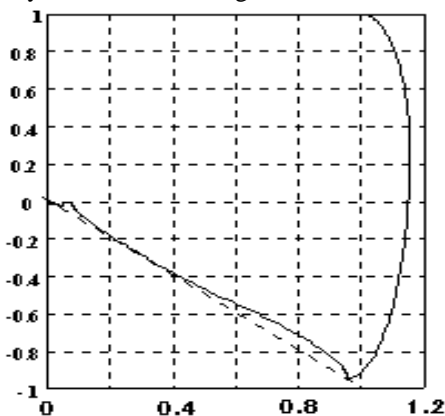


Figure 11: The trajectory in the plane  $(e_5^1, \dot{e}_5^1)$  for DSMC procedure

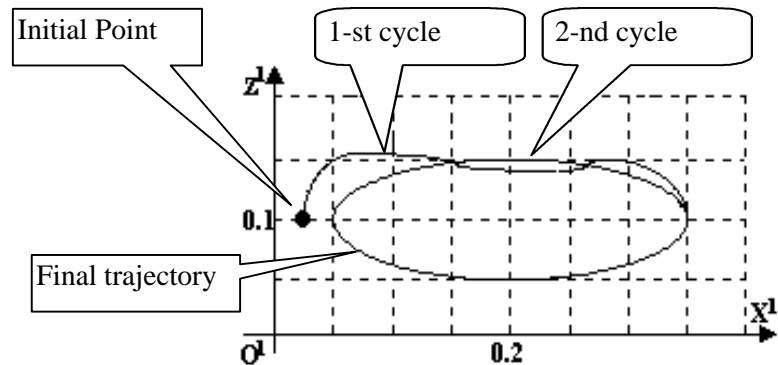


Figure 12: The final trajectory of the object

## CONCLUSION

The two level hierarchical control procedure is constructed in this paper to solve the large-scale control problem of a chain robotic system formed by tentacle manipulators grasping a common object. The upper-level coordinator collects all the necessary information to solve the inter-chain coordination problem, the force distribution. Then, the problem is decoupled into  $j$  lower-level subsystems, for every arm. The local fuzzy controllers are assigned to solve the local control. The stability and robustness of a class of the fuzzy logic control (FLC) are investigated and a robust FLC is proposed in order to cancel the uncertainties of the load. A DSMC procedure is used and the fuzzy rules are established. The simulation problem for two closed-chain tentacle robotic systems has also been studied.

## REFERENCES

- Cheng, Fan-Tien, 1995, "Control and Simulation for a Closed Chain Dual Redundant Manipulator System", Journal of Robotic Systems, pp. 119-133.
- Cheng, Fan-Tien, Orin, David, 1991, "Optimal Force Distribution in Multiple-Chain Robotic Systems", IEEE Trans. On Sys. Man and Cyb.", vol. 21, pp. 13-24.

Cheng, Fan-Tien; Orin, David., 1991, "Efficient Formulation of the Force Distribution Equations for Simple Closed Chain Robotic Mechanisms", IEEE Trans. on Sys. Man and Cyb., Jan. vol. 21, pp. 25-32.

Ivanescu, Mircea, 1984, "Dynamic Control for a Tentacle Manipulator", Proc. of Int. Conf., Charlotte, USA.

Ivanescu, Mircea, 1986, "A New Manipulator Arm: A Tentacle Model", Recent Trends in Robotics, pp. 51-57.

Ivanescu, Mircea; Stoian, Viorel, 1995, "A Variable Structure Controller for a Tentacle Manipulator", Proc. of the 1995 IEEE Int. Conf. on Robotics and Aut., Nagoya, Japan, vol. 3, pp. 3155-3160.

Ivanescu, Mircea; Stoian, Viorel, 1996, "A Sequential Distributed Variable Structure Controller for a Tentacle Arm", Proc. of the 1996 IEEE Intern. Conf. on Robotics and Aut., Minneapolis, USA, vol. 4, pp. 3701-3706.

Khatib, D.E., 1996, "Coordination and Descentralisation of Multiple Cooperation of Multiple Mobile Manipulators, Journal of Robotic Systems, 13 (11), pp. 755-764.

Mason, Matthew, 1981, "Compliance and Force Control", IEEE Trans. Sys. Man Cyb., Nr. 6, pp. 418-432.

Ross, T.J., 1995, "Fuzzy Logic with Engineering Applications", Mc. Grow Hill , Inc.

Schilling, Robert, 1990, "Fundamentals of Robotics", Prenting Hall.

Silverman, L. M., 1969, "Inversion of Multivariable Linear Systems", IEEE Trans. Aut. Contr., Vol AC - 14.

Soo Yeong Yi, 1997, "A robust Fuzzy Logic Controller for Robot Manipulators, IEEE Trans. on Systems, Man and Cybernetics, vol 27, No 4, pp. 706-713.

Zheng, Y.F.; Luh, J.Y.S., 1988, "Optimal Load Distribution for Two Industrial Robots Handling a Single Object", Proc. IEEE Int. Conf. Rob. Autom., pp. 344 - 349.

Wang, Li-Chun , 1996, "Time-Optimal Control of Multiple Cooperating manipulators", Journal of Robot Systems, pp. 229-241.

## APPENDIX 1

We consider the dynamic model of the manipulator defined by (10). The parameter  $s^j$  from (17) represents an error measure of the closed-loop control:  $s = \dot{e} + \sigma e$  A1.1

Where we cancelled the superscript j, for the simplicity.

From (10) and (13) we obtain:  $M(\ddot{e} + \ddot{q}_d) + C(\dot{e} + \dot{q}_d) + D(e + q_d)F = T$  A1.2

$M\dot{s} - M\sigma s + M\sigma^2 e + Cs - C\sigma e + D(e + q_d)F = T$ . We separate the linear part of s, it results,

$M\dot{s} - (M\sigma - C)s + H(e, q_d, F) = T$  A1.3

where H is a ( $n^j \times 1$ ) nonlinear vector defined on the trajectory parameters  $q_d$ , e, F. In order to prove the stability of the closed-loop system, we use a Lyapunov function by the form  $V = \frac{1}{2} s^T M s$  A1.4

Differentiating (A1.4),  $\dot{V} = s^T M \dot{s}$  A1.5

If we substitute (A1.3) we obtain  $\dot{V} = s^T M [M^{-1}(M\sigma - C)s - M^{-1}H(e, q_d, F) + M^{-1}T]$  A1.6

By using the control law of T, (A1.6) becomes  $\dot{V} = s^T (k - M\sigma + C)s + s^T [H(e, q_d, F^M) - H(e, q_d, F)] + s^T u_F$  A1.7

And using (21), (Soo,1997),  $\dot{V} \leq -s^T (k - M\sigma + C)s + \sum_i |s_i| [ |H(e, q_d, F^M) - H(e, q_d, F)|_i - k_{Fi} ]$  A1.8

If we introduce the condition (21) and we denote by  $\lambda_{\min}$  the minimum eigenvalue of  $(K - M\sigma + C)$ , it results,

$V \leq -I_{\min} \|s\|$ , (Q.E.D).

## APPENDIX 2

We consider the dynamic model (A1.3) in the area around switching line ( $u_{Fi}=0$ ),

$M\dot{s} - (M\sigma - C)s + H(e, q_d, F) = -Ks + H(e, q_d, F^M)$  A2.1

From (A1.1) and using the properties of the matrices M, C, K, S (diagonal matrices) we obtain,

$m_i \ddot{e}_i + (c_i + k_i) \dot{e}_i + (-m_i s_i^2 + c_i s_i + k_i s_i) e_i + (H(e, q_d, F) - H(e, q_d, F^M))_i = 0$  A2.2

but  $(H(e, q_d, F) - H(e, q_d, F^M))_i = (\Delta H)_i \cong \frac{\partial H}{\partial F} \frac{\partial F}{\partial q_i} e_i$  A2.3

We denote this term as  $(\Delta H)_i = h_i(q_d, e) e_i$ . From (A2.2), (A2.4) we obtain the switching line:

$\frac{\dot{e}_i}{e_i} = -p_i = \frac{1}{m_i e_i} [ -(c_i + k_i) e_i - s_i (-m_i s_i + c_i + k_i) e_i - h_i(q_d, e) e_i ]$ . This equation determines the slope  $p_i$  if the following

condition is verified (Ivanescu and Stoian, 1996):  $(c_i + k_i)^2 \geq 4(s_i(-m_i s_i + c_i + k_i) + h_i) m_i$ . A2