

Making Use of Fuzzy Sets of Triangle Membership Function for Locating the Hipocentre of a Group of Tremors in a Mine

Stanislaw Kowalik

Department of Management and Restructuring of Mining
Faculty of Mining and Geology, Silesian Tecnicl University
Akademicka 2, 44-100 Gliwice, Poland
Phone: +048-32-2371842, Fax: +048-32-2371296
email: stan@boss.gorn.polsl.gliwice.pl

ABSTRACT: Tremors in a mine are monitored by means of seismometers. Different calculation methods give different results. Seismometer measurements by means of fuzzy sets have been presented in the paper. The method of locating the hipocentre of group tremors in a mine, using the described fuzzy sets, has been presented. This method consists in determining the argument, for which the membership function has its maximum. The example of locating the hipocentre of group tremors on the basis of the data from coal mine „Szombierki” has been described (for using different measurement methods).

KEYWORDS: tremors, hipocentre, localization, fuzzy sets, triangle membership function.

1. INTRODUCTION

Tremors which occur in a mine are recorded by a set of seismometers. Different mathematical methods (Mendecki (1981)) are used to locate the hipocentre of a group of tremors. These methods can give different results. The aim of this paper is to locate the hipocentre of a group of tremors on the basis of different results given by different methods. Fuzzy sets theory (Czoga³a (1985), Drewniak (1987), Drewniak (1989), Kowalik (1996), Zadeh (1965)) will be used to achieve this purpose. Each method calculates coordinates x , y and z in three dimensional space and gives positional tolerance (measurement error) for each coordinate (Kowalik (1996), Mendecki (1981)). Coordinate x measurement will be considered as fuzzy set X_{fuzzy} with membership function $\mu_{X_{\text{fuzzy}}}(x)$. This function is shown in figure 1, where:
 x_p - is coordinate x measurement,
 t - is tolerance for coordinate x .

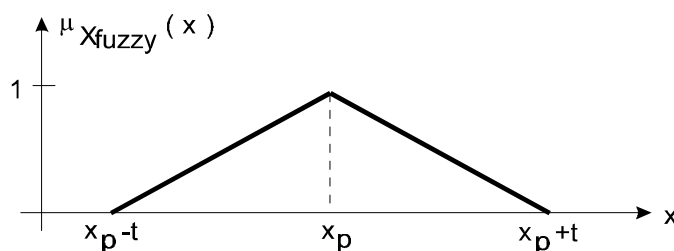


Figure 1: The membership function for one measurement of coordinate x .

The data from the paper (Mendecki (1981)) collected during tremors in coal mine „Szombierki” in the region 2A/509 have been used for calculation.

2. THE EXAMPLE OF THE HIPOCENTRE OF A GROUP OF TREMORS CALCULATION

For example, we will consider one group of tremors which in paper (Mendecki (1981)) have the numbers 124, 126, ...,131. The tremor have been located on the basis of the data from seismometers by five different mathematical methods:

- a) by geophysical mine service using A. Kijki algorithm,
- b) using P method for weighed function of the location error,
- c) using P method for not weighed function of the location error,
- d) using PA method for weighed function of the location error,
- e) using PA method for not weighed function of the location error.

The hipocentre of tremors coordinates are designated by x, y, z. They are measured in meters from a given reference point in the mine. Coordinate z is the depth under the surface.

The following results of the hipocentre location were calculated (Mendecki (1981)). These results are shown in table 1.

Table 1

Locating a group of tremors						
Number of tremor		Method A	Method B	Method c	Method D	Method E
124	x	6800	6794.32	6794	7009	6726
	y	-1650	-1631.06	-1631	-1801	-1694
	z	-600	-922.68	-919	-656	-664
126	x	6770	6732.86	6732	6928	6721
	y	-1680	-1651.38	-1651	-1808	-1717
	z	-620	-397.85	-397	-560	-566
127	x	6810	6780.51	6903	7034	6814
	y	-1600	-1596.20	-1700	-1773	-1670
	z	-570	-499.86	-684	-574	-574
128	x	6830	6819.17	6819	7206	6932
	y	-1630	-1638.27	-1638	-1561	-1730
	z	-540	-575.49	-574	-576	-574
129	x	6770	6767.52	6767	6990	6749
	y	-1640	-1631.23	-1631	-1800	-1699
	z	-520	-603.34	-602	-596	-599
130	x	6880	6945.36	6944	7699	7281
	y	-1640	-1712.61	-1712	-2060	-1864
	z	-540	-852.41	-850	-596	-586
131	x	6820	6765.70	6765	7048	6831
	y	-1600	-1634.36	-1634	-1819	-1714
	z	-570	-369.39	-369	-547	-549

Literature: Mendecki (1981)

The table contains 35 measurement values x, 35 measurement values y and 35 measurement values z. Each number of this table contains the method's error. The following values have been assumed in calculations (Mendecki (1981)) for tolerances:

- 58.33 [m] - for method a),
- 52.01 [m] - for method b), d),
- 99.27 [m] - for method c), e).

Arithmetic averages obtained from the results in methods a), b), c), d), e) have been assumed as the hipocentre of group tremors coordinates by the author of the paper (Mendecki (1981)). He has obtained: $x = 6881.56$, $y = -1689.66$, $z = -586.23$. In this way all the measurements have been dealt with in the same manner. We will pay greater attention to more focused measurements than to measurements that differ considerably from the other ones.

For each measurement on the basis of the above data we create a fuzzy set, which is shown in figure 1. These sets have the following membership functions.

$$\mu_{X_i}(x) = \begin{cases} \frac{x - x_p + t}{t} & \text{dla } x \in (x_p - t, x_p], \\ \frac{x - x_p - t}{-t} & \text{dla } x \in (x_p, x_p + t], \\ 0 & \text{dla } x \notin (x_p - t, x_p + t]. \end{cases} \quad (1)$$

$$\mu_{Y_i}(y) = \begin{cases} \frac{y - y_p + t}{t} & \text{dla } y \in (y_p - t, y_p], \\ \frac{y - y_p - t}{-t} & \text{dla } y \in (y_p, y_p + t], \\ 0 & \text{dla } y \notin (y_p - t, y_p + t]. \end{cases} \quad (2)$$

$$\mu_{Z_i}(z) = \begin{cases} \frac{z - z_p + t}{t} & \text{dla } z \in (z_p - t, z_p], \\ \frac{z - z_p - t}{-t} & \text{dla } z \in (z_p, z_p + t], \\ 0 & \text{dla } z \notin (z_p - t, z_p + t]. \end{cases} \quad (3)$$

Values x_{p-t} , x_p , x_{p+t} , y_{p-t} , y_p , y_{p+t} , z_{p-t} , z_p , z_{p+t} are shown in tables 2, 3, 4.

Table 2

Fuzzy sets X_i characterizing coordinate x

Number of tremor	Method a	Method b	Method c	Method d	Method e
124	x_{p-t} 6741.67 x_p 6800.00 x_{p+t} 6858.33	6742.31 6794.32 6846.33	6694.73 6794.00 6893.27	6956.99 7009.00 7061.01	6626.73 6726.00 6825.27
126	x_{p-t} 6711.67 x_p 6770.00 x_{p+t} 6828.33	6680.85 6732.86 6784.87	6632.73 6732.00 6831.27	6875.99 6928.00 6980.01	6621.73 6721.00 6820.27
127	x_{p-t} 6751.67 x_p 6810.00 x_{p+t} 6868.33	6728.50 6780.51 6832.52	6803.73 6903.00 7002.27	6981.99 7034.00 7086.01	6714.73 6814.00 6913.27
128	x_{p-t} 6771.67 x_p 6830.00 x_{p+t} 6888.33	6767.16 6819.17 6871.18	6719.73 6819.00 6918.27	7153.99 7206.00 7258.01	6832.73 6932.00 7031.27
129	x_{p-t} 6711.67 x_p 6770.00 x_{p+t} 6828.33	6715.51 6767.52 6819.53	6667.73 6767.00 6866.27	6937.99 6990.00 7042.01	6649.73 6749.00 6848.27
130	x_{p-t} 6821.67 x_p 6880.00 x_{p+t} 6938.33	6893.35 6945.36 6997.37	6844.73 6944.00 7043.27	7646.99 7699.00 7751.01	7181.73 7281.00 7380.27
131	x_{p-t} 6761.67 x_p 6820.00 x_{p+t} 6878.33	6713.69 6765.70 6817.71	6665.73 6765.00 6864.27	6995.99 7048.00 7100.01	6731.73 6831.00 6930.27

Table 3

Fuzzy sets Y_i characterizing coordinate y

Number of tremors	Method a	Method b	Method c	Method d	Method e	
124	y_{p-t}	-1708.33	-1683.07	-1730.27	-1853.01	-1793.27
	y_p	-1650.00	-1631.06	-1631.00	-1801.00	-1694.00
	y_{p+t}	-1591.67	-1579.05	-1531.73	-1748.99	-1594.73
126	y_{p-t}	-1738.33	-1703.39	-1750.27	-1860.01	-1816.27
	y_p	-1680.00	-1651.38	-1651.00	-1808.00	-1717.00
	y_{p+t}	-1621.67	-1599.37	-1551.73	-1755.99	-1617.73
127	y_{p-t}	-1658.33	-1648.21	-1799.27	-1825.01	-1769.27
	y_p	-1600.00	-1596.20	-1700.00	-1773.00	-1670.00
	y_{p+t}	-1541.67	-1544.19	-1600.73	-1720.99	-1570.73
128	y_{p-t}	-1688.33	-1690.28	-1737.27	-1613.01	-1829.27
	y_p	-1630.00	-1638.27	-1638.00	-1561.00	-1730.00
	y_{p+t}	-1571.67	-1586.26	-1538.73	-1508.99	-1630.73
129	y_{p-t}	-1698.33	-1683.24	-1730.27	-1852.01	-1798.27
	y_p	-1640.00	-1631.23	-1631.00	-1800.00	-1699.00
	y_{p+t}	-1581.67	-1579.22	-1531.73	-1747.99	-1599.73
130	y_{p-t}	-1698.33	-1764.62	-1811.27	-2112.01	-1963.27
	y_p	-1640.00	-1712.61	-1712.00	-2060.00	-1864.00
	y_{p+t}	-1581.67	-1660.60	-1612.73	-2007.99	-1764.73
131	y_{p-t}	-1658.33	-1686.37	-1733.27	-1871.01	-1813.27
	y_p	-1600.00	-1634.36	-1634.00	-1819.00	-1714.00
	y_{p+t}	-1541.67	-1582.35	-1534.73	-1766.99	-1614.73

Table 4

Fuzzy sets Z_i characterizing coordinate z

Number of tremors	Method a	Method b	Method c	Method d	Method e	
124	z_{p-t}	-658.33	-974.69	-1018.27	-708.01	-763.27
	z_p	-600.00	-922.68	-919.00	-656.00	-664.00
	z_{p+t}	-541.67	-870.67	-819.73	-603.99	-564.73
126	z_{p-t}	-678.33	-449.86	-496.27	-612.01	-665.27
	z_p	-620.00	-397.85	-397.00	-560.00	-566.00
	z_{p+t}	-561.67	-345.84	-297.73	-507.99	-466.73
127	z_{p-t}	-628.33	-551.87	-783.27	-626.01	-673.27
	z_p	-570.00	-499.86	-684.00	-574.00	-574.00
	z_{p+t}	-511.67	-447.85	-584.73	-521.99	-474.73
128	z_{p-t}	-598.33	-627.50	-673.27	-628.01	-673.27
	z_p	-540.00	-575.49	-574.00	-576.00	-574.00
	z_{p+t}	-481.67	-523.48	-474.73	-523.99	-474.73
129	z_{p-t}	-578.33	-655.35	-701.27	-648.01	-698.27
	z_p	-520.00	-603.34	-602.00	-596.00	-599.00
	z_{p+t}	-461.67	-551.33	-502.73	-543.99	-499.73
130	z_{p-t}	-598.33	-904.42	-949.27	-648.01	-685.27
	z_p	-540.00	-852.41	-850.00	-596.00	-586.00
	z_{p+t}	-481.67	-800.40	-750.73	-543.99	-486.73
131	z_{p-t}	-628.33	-421.40	-468.27	-599.01	-648.27
	z_p	-570.00	-369.39	-369.00	-547.00	-549.00
	z_{p+t}	-511.67	-317.38	-269.73	-494.99	-449.73

These functions are shown in figure 2, 3, 4.

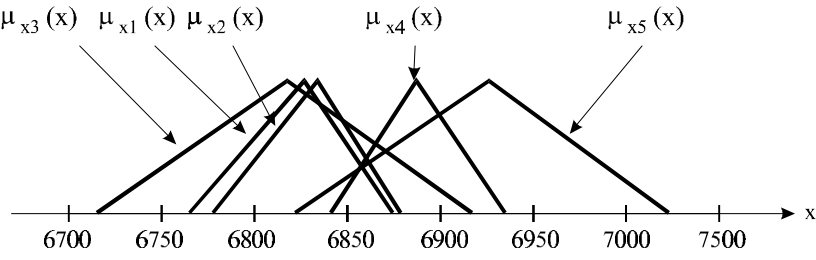


Figure 2. The membership functions for fuzzy sets for x variable.

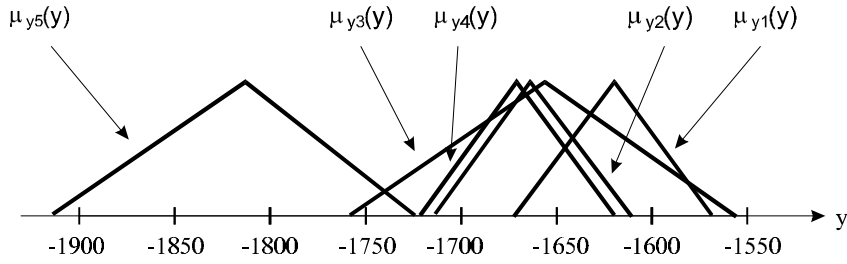


Figure 3. The membership functions for fuzzy sets for y variable.

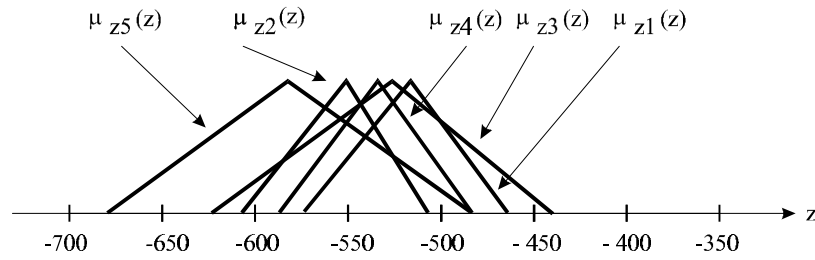


Figure 4. The membership functions for fuzzy sets for z variable.

Next each fuzzy set from tables 2, 3, 4 has been multiplied by scale coefficient $\alpha=1/35$, because for coordinates x, y, z there are 35 measurements. The multiplication of a fuzzy set by coefficient α is executed according to the following definition.

Definition 1.

The product of fuzzy set A by real number $\alpha \in [0, 1]$ is set αA defined by the membership function:

$$\forall_{x \in X} \mu_{\alpha A}(x) = \alpha \mu_A(x). \quad (4)$$

Fuzzy sets from table 2, 3, 4 multiplied by scale coefficient $\alpha=1/35$ are added using the definition of limited sum.

Definition 2

Limited sum of fuzzy sets A and B is set $A \oplus B$ defined by the membership function:

$$\forall_{x \in X} \mu_{A \oplus B}(x) = \min(\mu_A(x) + \mu_B(x), 1). \quad (5)$$

After calculating the sum of fuzzy sets from table 2 on the ground of definition 2 we obtain fuzzy set

$$S_X = \alpha X_1 \oplus \alpha X_2 \oplus \dots \oplus \alpha X_{35} = \alpha (X_1 \oplus \dots \oplus X_{35}) = (1/35) (X_1 \oplus \dots \oplus X_{35}) \quad (6)$$

After calculating the sum of fuzzy sets from table 3 on the ground of definition 2 we obtain fuzzy set

$$S_Y = \alpha Y_1 \oplus \alpha Y_2 \oplus \dots \oplus \alpha Y_{35} = \alpha (Y_1 \oplus \dots \oplus Y_{35}) = (1/35) (Y_1 \oplus \dots \oplus Y_{35}) \quad (7)$$

After calculating the sum of fuzzy sets from table 4 on the ground of definition 2 we obtain fuzzy set

$$S_Z = \alpha Z_1 \oplus \alpha Z_2 \oplus \dots \oplus \alpha Z_{35} = \alpha (Z_1 \oplus \dots \oplus Z_{35}) = (1/35) (Z_1 \oplus \dots \oplus Z_{35}) \quad (8)$$

For fuzzy set S_X we calculate point x_0 , in which membership function $\mu_{S_X}(x)$ reaches maximum

$$\max_x \mu_{S_X}(x) = \mu_{S_X}(x_0). \quad (9)$$

On the ground of table 2 we obtain

$$\max_x \mu_{S_X}(x) = \mu_{S_X}(6780.51) = 0.3771. \quad (10)$$

For fuzzy set S_Y we calculate point y_0 , in which membership function $\mu_{S_Y}(y)$ reaches maximum

$$\max_y \mu_{S_Y}(y) = \mu_{S_Y}(y_0). \quad (11)$$

On the ground of table 3 we obtain

$$\max_y \mu_{S_Y}(y) = \mu_{S_Y}(-1638.27) = 0.4715. \quad (12)$$

For fuzzy set S_Z we calculate point z_0 , in which membership function $\mu_{S_Z}(z)$ reaches maximum

$$\max_z \mu_{S_Z}(z) = \mu_{S_Z}(z_0). \quad (13)$$

On the ground of table 4 we obtain

$$\max_z \mu_{S_Z}(z) = \mu_{S_Z}(-574.00) = 0.4679. \quad (14)$$

Therefore the hypocentre of tremors in the mine has the following coordinates:

$$x_0 = 6780.51 \text{ [m]}, \quad (15)$$

$$y_0 = -1638.27 \text{ [m]}, \quad (16)$$

$$z_0 = -574.00 \text{ [m]}. \quad (17)$$

Comparing coordinates x_0 , y_0 , z_0 with the arithmetic averages of the results calculated in methods a), b), c), d), e) [5] shows that there are differences in the location of the hypocentre of tremors. For x coordinate there is difference of about 101 [m], for y 51[m] and for z 12 [m]. Such small differences were obtained, because in the method presented in the paper, the measurements which were too far away from the focused group were rejected and had a smaller influence on the calculated results.

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