

# HIGHLY EFFICIENT SIGNAL PROCESSING FOR FREQUENCY AGILE POWER LINE COMMUNICATIONS

Maja Sliskovic

Faculty of Electrical Engineering and Computing  
Unska 3, Zagreb, HR10000, Croatia

## ABSTRACT

A modified frequency hopping signaling scheme has recently received a considerable attention by the designers of power line communication systems because of its insensitivity to frequency-selective and time-variant attenuation and high level of interference. In frequency hopping system, the increase of the data rate without increase of the hop rate is possible only if the receiver can detect all information-bearing tones simultaneously. This paper presents the structure of the noncoherent optimum receiver that minimizes number of required multiplications and memory locations. The receiver structure is based on computation of equidistant DFT coefficients. The proposed demodulator can be used in systems with variable bit rate. With slightly modified algorithm, rough synchronization can also be achieved. Comparison of computational complexity confirms superiority of presented algorithm over direct DFT calculation and FFT algorithm.

## 1. INTRODUCTION

Wide spread low voltage electrical power distribution networks represent an attractive medium for digital communications. The general opening of telecommunications market and deregulation of energy market in Europe have enabled the electric utilities to offer the users some services till recently reserved only for communication companies, and thereby raised the importance of reliable and cost effective power line communication system. The electric utility applications include, e.g., remote meter reading, distribution automation and demand-side management.

Power lines are, however, heavily stressed with interference from various sources and attenuation exhibits unpredictable variations. Such a communication hostile environment calls for a frequency redundant modulation scheme. It has been shown in [1] that frequency hopping (FH) enables reliable power line communications, even when a rather simple synchronization scheme based on zero-crossings of the mains voltage is used.

Since oscillators that are used at the transmitter and the receiver are generally not synchronous and the phase characteristic of the transmission medium is changed rapidly and in random manner [1], noncoherent detection has to be employed. Noncoherent detector computes energy at expected information-bearing frequencies. Although FH demodulator requires only two matched-filter-type detectors [1], the possibility to detect all transmitted frequencies would be advantageous. The data rate could be increased by applying symbol processing [2], where

symbols are encoded as permutations of hop tones, and more than one channel can be realized by using orthogonal frequency division multiplexing (OFDM) [3]. However, this increases computational complexity. Therefore, the demodulator structure which reduces number of computational operations would be useful.

The purpose of this paper is to present optimum-receiver-equivalent demodulator with reduced number of multiplications. The principle of the optimum receiver is discussed in section 2. In section 3, the new receiver structure based on DFT computation is presented. The computational gain is discussed in section 4.

## 2. OPTIMUM RECEIVER

The optimum receiver for M-ary frequency modulated signal corrupted only by additive white Gaussian noise (AWGN) is illustrated in Figure 1. The demodulation of the received signal  $r(t)$  is accomplished by using two correlators for each possible transmitted frequency. The detector computes  $M$  envelopes [3]

$$r_m = \sqrt{r_{mc}^2 + r_{ms}^2}, \quad m = 1, \dots, M \quad (1)$$

(or squared envelopes  $|r_m|^2$ ), and selects signal with the largest envelope or squared envelope<sup>1</sup>. Outputs of the correlators are sampled at the end of each chip interval. The clock used to sample correlator outputs must be synchronous with the chip clock in the transmitter. A synchronization based on zero crossings of the mains voltage is the simplest solution [1].

In discrete-time system, integrators on Figure 1 are realized as accumulators. The two reference signals for the correlators are cosine and sine waveforms with corresponding frequency  $f_m$ . Therefore, the outputs from correlators after  $N$  input samples are given by:

$$r_{mc} = \sum_{n=0}^{N-1} r(nT_s) \cdot \cos(2\pi f_m nT_s), \quad (2)$$

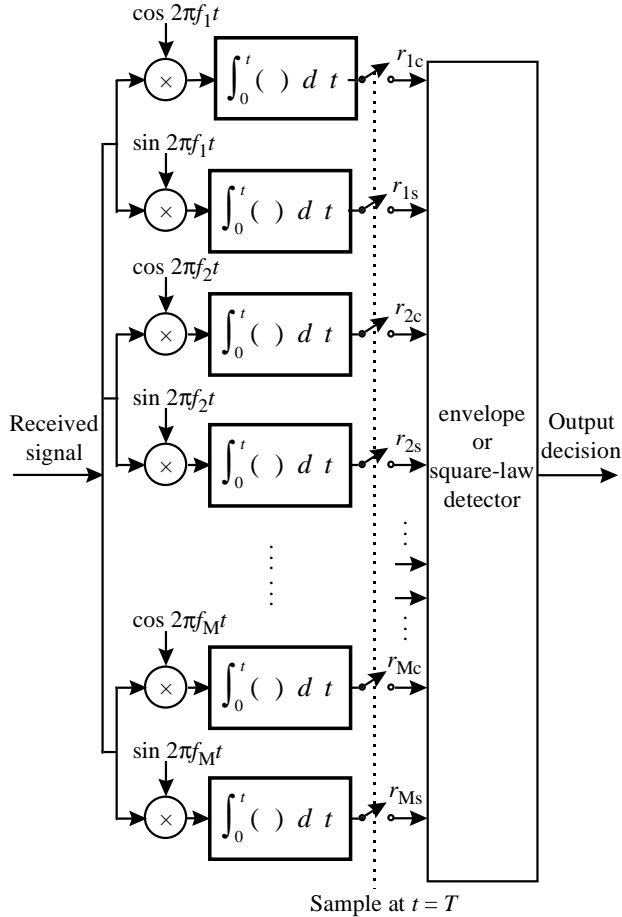
$$r_{ms} = \sum_{n=0}^{N-1} r(nT_s) \cdot \sin(2\pi f_m nT_s) \quad (3)$$

where  $T_s=1/f_s$  is the sampling period.

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<sup>1</sup> Optimum demodulator for the M-ary frequency shift keying (M-FSK) signals has the same structure. Therefore, the results presented in this paper can be applied to M-FSK receiver too.

It is obvious that cosine referenced correlator delivers real part of one DFT component and sine referenced correlator imaginary part. The envelope demodulator output represents the absolute value of DFT components. For computation of DFT components, computationally more efficient methods can be used.



**Figure 1.** Optimum receiver for  $M$ -ary frequency hopping, orthogonal information-bearing tones

### 3. DFT BASED RECEIVER

At this point, we shall suppose that information-bearing frequencies  $f_m$  are equidistant:

$$f_{m+1} - f_m = f_m - f_{m-1} = \Delta f, \quad m=1, \dots, M \quad (4)$$

and thus uniformly distributed across the whole available frequency range. This condition is usually inherently fulfilled in communication systems employing orthogonal information-bearing tones. Orthogonal frequencies are equidistant, and, from the point of reducing the error probability, it is advantageous to use tones distributed across the whole available frequency range [1], [3].

The second condition is that information-bearing tones are shifted to the baseband. This can be accomplished by multiplication with the carrier and low pass filtration or by bandpass sampling [4].

Frequencies of the mutually orthogonal tones on the chip interval  $T$  are

$$f_i = i \cdot \frac{1}{T}, \quad i \in \mathbf{N}. \quad (5)$$

Equidistant information-bearing tones, fulfilling the above conditions (4) and (5), have frequencies

$$f_m = (mL + k) \cdot \frac{1}{T}, \quad m = 0, \dots, M - 1, \quad (6)$$

where  $L$  determines distance between tones and  $k$  is the offset from the null frequency. The period  $L$  is determined by available signal bandwidth  $B$

$$B = M \cdot \Delta f = ML \frac{1}{T}. \quad (7)$$

By applying the Nyquist sampling frequency

$$f_s = 2B = 2M \cdot \Delta f \quad (8)$$

the components with frequencies  $f_m$  (6) can be computed as DFT coefficients:  $k, L+k, 2L+k, \dots, (M-1)L+k$ . Thus, all possible orthogonal frequencies are not used.

Generally, the number of required signal samples is the same as the number of needed Fourier coefficients. However, in order to reduce the impact of the synchronization error and impulse noise by averaging, the DFT coefficients are calculated using all samples in one chip period.

There are methods of operation savings when reduced number of DFT components is required [5], [6], [7]. None of them is, however, designed for the case when only equidistant coefficients have to be computed. Such a method will be developed in this paper. The only restriction is that the number of samples  $N$  in one chip period is the integer multiple of the coefficient distance  $L$ , i.e.

$$N = L \cdot P, \quad P \in \mathbf{N}. \quad (9)$$

This condition is fulfilled for frequencies satisfying equations (6) and (8). First, we shall develop the algorithm for the simplest case, which is the most common case in the practice.

#### A. First frequency is null frequency

This condition means that  $k=0$ , i.e. the tone frequencies are

$$f_m = mL \frac{1}{T}, \quad m = 0, \dots, M - 1. \quad (10)$$

These frequencies correspond to DFT coefficients with index  $mL$ :

$$R(mL) = \sum_{n=0}^{N-1} r(n) \cdot e^{-j \frac{mLn}{N} 2\pi} \quad (11)$$

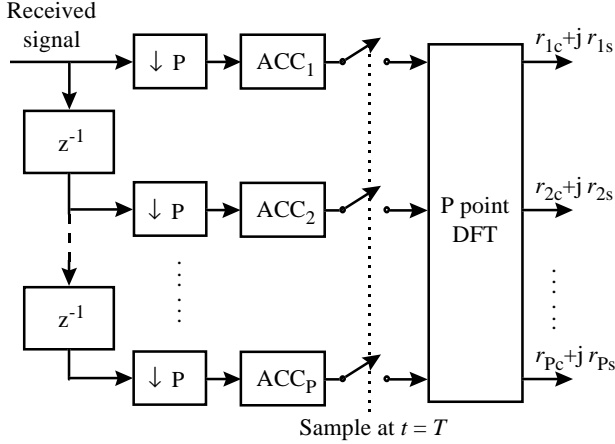
By applying relation (9), the equation (11) becomes

$$R(mL) = \sum_{n=0}^{N-1} r(n) \cdot e^{-j \frac{mm}{P} 2\pi} \quad (12)$$

The exponential function  $e^{-j\frac{mn}{P}2\pi}$  is now periodic with period  $P$ , and summation (12) can be broken to two summations:

$$R(mL) = \sum_{p=0}^{P-1} \left[ \left( \sum_{l=0}^{L-1} r(Pl+p) \right) \cdot e^{-j\frac{mp}{P}2\pi} \right] \quad (13)$$

The realization is illustrated in Figure 2. The inner summation is realized in the form of accumulators whose content is sampled and cleared after  $N$  input samples (chip period  $T$ ).



**Figure 2.** Realization of the system that computes equidistant DFT coefficients when first frequency equals zero ( $\downarrow P$  stands for decimator)

Let us examine the properties of the structure shown in Figure 2. For a larger chip period  $T_1 > T$ :

$$T_1 = \frac{L+q}{L} T, \quad (14)$$

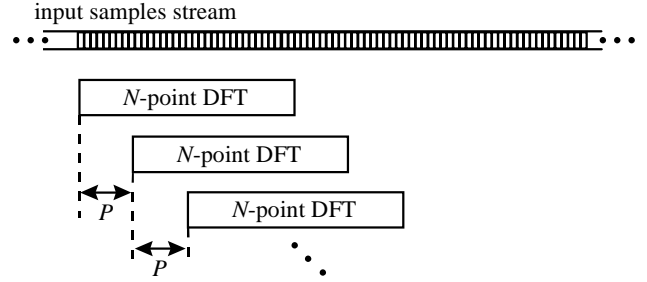
the structure in Figure 2 computes DFT coefficients with index  $m(L+q)$ ,  $m=0, \dots, P-1$ . The frequencies corresponding to those coefficients are

$$\begin{aligned} f_{m(L+q), T_1} &= m(L+q) \frac{1}{T_1} = \frac{m(L+q)f_s}{P(L+q)} = \\ &= m \frac{f_s}{P} = mL \frac{f_s}{N} = mL \frac{1}{T} = f_{mL, T} \end{aligned} \quad (15)$$

This result means that computed DFT components always correspond to the same set of physical frequencies, regardless of the period  $T$ . This fact enables the use of this algorithm in systems with variable chip period, for example in communication systems with adaptive data rate.

If the accumulators in Figure 2 are substituted with moving-average-filters of length  $L$ , and if their outputs are sampled after  $P$  input samples, the DFT on  $N=L \cdot P$  input samples will be computed after each block of  $P$  input samples (Figure 3). By locating the local maximum in the output streams  $r_m$  (1), the rough synchronization can be achieved when the mains voltage is

off. The maximal synchronization error is  $|\varepsilon_{sync}| = \frac{T}{2L}$ . This advantage has to be paid by increased number of registers in moving average circuits.



**Figure 3.** Possibility to calculate DFT coefficients on overlapped blocks of input samples

### B. General case ( $k \neq 0$ )

Generally, by selection of tone frequencies, the condition in the case A ( $k=0$ ) may not be fulfilled. The required DFT coefficients are

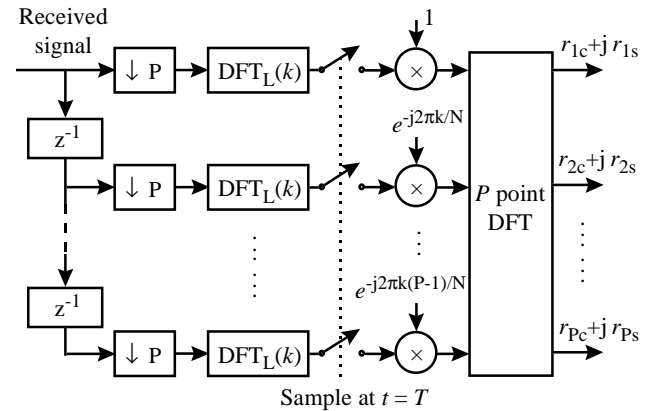
$$R(mL+k) = \sum_{n=0}^{N-1} r(n) e^{-j\frac{(mL+k)n}{N}2\pi} \quad (16)$$

Let us break summation (16) to two summations ( $n=P \cdot l+p$ ):

$$\begin{aligned} R(mL+k) &= \\ &= \sum_{p=0}^{P-1} \sum_{l=0}^{L-1} r(Pl+p) \cdot e^{-j\frac{(mL+k)(Pl+p)}{N}2\pi} \end{aligned} \quad (17)$$

After simple transformations, equation (17) reduces to

$$R(mL+k) = \sum_{p=0}^{P-1} \left[ \left( e^{-j\frac{kp}{N}2\pi} \sum_{l=0}^{L-1} r(Pl+p) e^{-j\frac{kl}{L}2\pi} \right) \cdot e^{-j\frac{mp}{P}2\pi} \right] \quad (18)$$



**Figure 4.** Realization of the system that computes equidistant DFT coefficients, general case

The realization is illustrated in Figure 4. The inner summation is recognized as being  $k$ -th DFT coefficient calculated on  $L$  samples ( $\text{DFT}_L(k)$  on Figure 4), and the outer summation is  $P$ -point DFT. The general case does not preserve properties of the case A, i.e. work with adaptive data rate and synchronization are not possible.

#### 4. NUMBER OF MULTIPLICATIONS

In this section, the number of multiplications in the presented algorithm is calculated and compared to the number of multiplications in the direct DFT method and in the Cooley-Tukey FFT algorithm.

We will assume that all tone frequencies in the available bandwidth that satisfy condition (6) are used for data transmission, i.e.  $P=2M$ , where  $M$  is the number of information-bearing tones and consequently the number of DFT coefficients to be calculated. The number of samples in one chip period is  $N$ .

By using direct computation of DFT coefficients, the number of required complex multiplications is

$$n_{dir} = M \cdot N \quad (19)$$

By using the above algorithm to compute equidistant DFT coefficients, case A, the total number of multiplications equals number of multiplications needed to calculate DFT in  $P$  points. If the FFT algorithm is used, the total number of complex multiplications is

$$n_A = 2M \cdot \log_2 2M \quad (20)$$

In the case B, one can see from Figure 4 that  $L \cdot P = N$  multiplications are necessary for calculation of  $P$  Fourier components because for each component only  $L$  multiplications are required. Furthermore,  $P$  multiplications with complex exponentials are required and, if the FFT algorithm is used,  $P \cdot \log_2 P$  multiplications for  $P$ -point DFT computation. If  $P=2M$ , the total number of complex multiplications for the case B is

$$n_B = N + 2M + 2M \cdot \log_2 2M \quad (21)$$

Table I shows the comparison of the number of multiplications in few algorithms.  $N$  is the number of samples in one chip period and  $M$  is the number of tone frequencies. The advantage of using the proposed algorithm to compute equidistant DFT coefficients is obvious because  $N$  is always larger than  $2M$  ( $N=2ML > 2M$ ).

**Table I.** Comparison of the number of multiplications

Algorithm	No. of multiplications
Direct method	$M \cdot N$
FFT	$N \cdot \log_2 N$
DFT in equidistant points (case A)	$2M \cdot \log_2(2M)$
DFT in equidistant points (case B)	$N + 2M + 2M \cdot \log_2(2M)$

#### 5. CONCLUSIONS

An algorithm that calculates equidistant DFT components is proposed. The computational savings are calculated and it is shown that this algorithm is computationally more efficient than FFT or direct method.

The application of presented algorithm in communication systems employing M-ary frequency modulation schemes is outlined. The most usual case in practice is the case A, when the tone frequencies are the integer multiples of the difference between them. In such a case, the algorithm enables calculation of DFT coefficients in variable time interval and can be used in systems with adaptive data rate. Furthermore, the possibility of symbol synchronization with slightly modified algorithm is demonstrated, and therefore the system can operate when the mains voltage is off.

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