TIME DELAY ESTIMATION IN UNKNOWN SPATIALLY UNCORRELATED GAUSSIAN NOISES USING HIGHER-ORDER STATISTICS

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ABSTRACT

Bispectrum methods have been proposed for non-Gaussian signal Time Delay Estimation(TDE) problem. When the signal is non-Gaussian and additive noises are spatially uncorrelated Gaussian, the bispectrum methods are outperformed by Generalized Cross-Correlation (GCC) methods. This problem is addressed in this paper and new methods are proposed to improve the TDE performance. The new methods exploit the Higher-Order Statistics characteristics of the signals and formulate weighting functions to improve the time delay estimation. Computer simulation results show that the new methods outperform both the GCC and the bispectrum methods.

1. INTRODUCTION

Time delay estimation(TDE) between received signals at two sensor locations is an important problem in many fields such as sonar, radar, biomedicine, geophysics, etc. Generalized Cross-Correlation (GCC) approaches(SCOT, ROTH, PHAT, etc.) have been the conventional methods for TDE [1], [2]. Conventional bispectrum method(CBM) and parametric bispectrum method(PBM) were proposed in [3] for non-Gaussian signal TDE problem. When the signal is non-Gaussian and additive noises are spatially correlated Gaussian, the bispectrum methods(CBM and PBM) outperform GCC approaches. But in the case where the signal is non-Gaussian and additive noises are spatially uncorrelated Gaussian, GCC approaches exhibit better performance than the bispectrum methods as indicated in [3].

The above problem is addressed in this paper and new methods are developed to improve the TDE of non-Gaussian signal buried in spatially uncorrelated Gaussian noise. The new methods are based on Higher-Order Statistics(HOS) characteristics of the signals and weighting functions are proposed to improve the time delay estimation. Our computer simulation results show that the new methods outperform both the GCC and the bispectrum methods.

2. BACKGROUND

The fundamental physical problem can be described as follows. There are two receiving sensors, and it is assumed that $\{x(n)\}$ and $\{y(n)\}$ are two sensor measurements. The TDE problem can be modeled as

$$\begin{aligned} x(n) &= s(n) + w_1(n) \\ y(n) &= s(n-D) + w_2(n) \end{aligned} \quad (1)$$

It is assumed that $s(n)$ is the non-Gaussian source signal , $w_1(n)$ and $w_2(n)$ are the additive spatially uncorrelated Gaussian noises at the respective sensors, independent of the signal, and D is the time delay between the signals at the two sensors.

Bispectrum methods have been proposed to estimate the time delay in the HOS domain. But the simulation results in [3] show that SCOT method (one type of GCC methods) exhibits better performance than the CBM and PBM for TDE of non-Gaussian signal buried in spatially uncorrelated Gaussian noises.

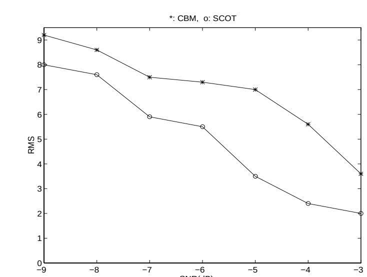

Figure 1: RMS error of TDE, *: CBM, o: SCOT

Fig.1 shows the Root Mean-Square (RMS) error of TDE result for SCOT method and CBM. From Fig.1, we can see that SCOT method is better than CBM in the sense that its RMS error is smaller. This indicates that the SCOT approach can effectively suppress the noise through the specially chosen SCOT weighting function.

Even though bispectrum methods can suppress Gaussian noise in theory, it actually does not suppress the noises completely in the estimated cumulants due to the estimator variance, which degrades the performance of CBM and PBM.

Since $w_1(n)$ and $w_2(n)$ are assumed to be spatially uncorrelated Gaussian noise and the third-order cumulant of a zero-mean Gaussian process is zero (in theory)[4], [5], the Gaussian noises can be suppressed by computing the third-order cumulant sequence

$$\begin{aligned} R_{xxx}(\tau,\rho) &\stackrel{\triangle}{=} E\{x(n)x(n+\tau)x(n+\rho)\} \\ &= E\{s(n)s(n+\tau)s(n+\rho)\} + \\ &\quad E\{w_1(n)w_1(n+\tau)w_1(n+\rho)\} \\ &= R_{sss}(\tau,\rho) + R_{w_1w_1w_1}(\tau,\rho) \\ &= R_{sss}(\tau,\rho) \end{aligned} \quad (2)$$

$$\begin{aligned} R_{xyx}(\tau,\rho) &\stackrel{\triangle}{=} E\{x(n)y(n+\tau)x(n+\rho)\} \\ &= E\{s(n)s(n-D+\tau)s(n+\rho)\} + \\ &\quad E\{w_1(n)w_2(n+\tau)w_1(n+\rho)\} \\ &= R_{sss}(\tau-D,\rho) + R_{w_1w_2w_1}(\tau,\rho) \\ &= R_{sss}(\tau-D,\rho) \end{aligned} \quad (3)$$

where we define the following third-order statistics as

$$R_{sss}(\tau,\rho) \stackrel{\triangle}{=} E\{s(n)s(n+\tau)s(n+\rho)\} \quad (4)$$

$$R_{w_1w_1w_1}(\tau,\rho) \stackrel{\triangle}{=} E\{w_1(n)w_1(n+\tau)w_1(n+\rho)\} = 0 \quad (5)$$

$$R_{w_1w_2w_1}(\tau,\rho) \stackrel{\triangle}{=} E\{w_1(n)w_2(n+\tau)w_1(n+\rho)\} = 0 \quad (6)$$

from Eq.(2) and (3), we have the following third-order cumulant relationship

$$R_{xyx}(\tau,\rho) = R_{xxx}(\tau-D,\rho) \quad (7)$$

It is clear that the cross-cumulant $R_{xyx}(\tau,\rho)$ is a delayed version of $R_{xxx}(\tau,\rho)$ and the time delay is D. Therefore it is possible to estimate the time delay in cumulant domain using an approach similar to the traditional GCC methods.

The advantage of estimating the time delay through the above model is that Gaussian noise is completely suppressed in the HOS domain(in theory). Therefore we can expect a better time delay estimation at low SNR compared with the conventional GCC approaches, which are based on the traditional TDE model in time domain. GCC approaches fail to get accurate time delay estimation at low SNR due to the corrupting Gaussian noise effect. We can improve the TDE performance through transforming the traditional TDE model of the covariance-based domain to that of the third-order statistics based domain.

3. NEW METHODS

3.1. HOS-SCOT method

Since we have Eq.(7), define

$$R_{12}(k,\rho) \stackrel{\triangle}{=} E\{R_{xxx}(\tau,\rho)R_{xyx}(\tau+k,\rho)\} \quad (8)$$

$$R_{11}(k,\rho) \stackrel{\triangle}{=} E\{R_{xxx}(\tau,\rho)R_{xxx}(\tau+k,\rho)\} \quad (9)$$

$$R_{22}(k,\rho) \stackrel{\triangle}{=} E\{R_{xyx}(\tau,\rho)R_{xyx}(\tau+k,\rho)\} \quad (10)$$

and

$$G_{12}(\omega_1,\omega_2) \stackrel{\triangle}{=} FT\left[R_{12}(k,\rho)\right] \quad (11)$$

$$G_{11}(\omega_1,\omega_2) \stackrel{\triangle}{=} FT\left[R_{11}(k,\rho)\right] \quad (12)$$

$$G_{22}(\omega_1,\omega_2) \stackrel{\triangle}{=} FT\left[R_{22}(k,\rho)\right] \quad (13)$$

where $FT[\cdot]$ denotes double Fourier Transform.

then we have

$$R(m,n) = \int_{-\pi}^{\pi}\int_{-\pi}^{\pi} \frac{G_{12}(\omega_1,\omega_2)}{\sqrt{G_{11}(\omega_1,\omega_2)G_{22}(\omega_1,\omega_2)}} e^{j\omega_1 m}e^{j\omega_2 n}d\omega_1 d\omega_2 \quad (14)$$

and

$$J(m) = \sum_{n=-\infty}^{\infty} R(m,n) \quad (15)$$

peaks at $m = D$.

$J(m)$ is the proposed criterion for TDE. In the criterion, the frequency weighting function

$$W(\omega_1,\omega_2) = \frac{1}{\sqrt{G_{11}(\omega_1,\omega_2)G_{22}(\omega_1,\omega_2)}} \quad (16)$$

is used to enhance the time delay estimation. Since the weighting function is similar to SCOT method, we refer to it as HOS-SCOT method.

By applying the weighting function, the remaining Gaussian noise contribution in the estimated cumulants due to the estimator variance is further suppressed. Thus the time delay estimation is improved by suppressing the noise effect .

3.2. HOS-ROTH method

The weighting function can also be selected as

$$W(\omega_1,\omega_2) = \frac{1}{G_{11}(\omega_1,\omega_2)} \tag{17}$$

therefore, we have

$$R(m,n) = \int_{-\pi}^{\pi}\int_{-\pi}^{\pi} \frac{G_{12}(\omega_1,\omega_2)}{G_{11}(\omega_1,\omega_2)} e^{j\omega_1 m} e^{j\omega_2 n} d\omega_1 d\omega_2 \tag{18}$$

and the TDE criterion is

$$J(m) = \sum_{n=-\infty}^{\infty} R(m,n) \tag{19}$$

which peaks at $m = D$. Since the weighting function is similar to that one used in the ROTH method, this method is called HOS-ROTH method. The weighting function helps to get the correct time delay by suppressing the noise effect.

3.3. HOS-PHAT method

The weighting function can also be selected as

$$W(\omega_1,\omega_2) = \frac{1}{\mid G_{12}(\omega_1,\omega_2) \mid} \tag{20}$$

therefore, we have

$$R(m,n) = \int_{-\pi}^{\pi}\int_{-\pi}^{\pi} \frac{G_{12}(\omega_1,\omega_2)}{\mid G_{12}(\omega_1,\omega_2) \mid} e^{j\omega_1 m} e^{j\omega_2 n} d\omega_1 d\omega_2 \tag{21}$$

and the TDE criterion is

$$J(m) = \sum_{n=-\infty}^{\infty} R(m,n) \tag{22}$$

which peaks at $m = D$. This weighting function is similar to that one used in the PHAT method, therefore it is called HOS-PHAT method.

The ROTH, PHAT and SCOT processors have been proposed for time delay estimation in second-order statistics domain and they are also widely used in practical problems to suppress the noise effect and improve the TDE performance. Similar weighting functions are proposed and used in the 3rd-order statistics domain to get better time delay estimation. The HOS property of non-Gaussian signal and Gaussian noise is exploited to suppress the corrupting noises. And weighting functions are also used in the proposed criterion to further improve the performance of time delay estimation. In the next section, the performance of the proposed methods is compared to that of GCC(SCOT, ROTH, PHAT) and CBM. The computer simulation results indicate that the proposed methods are superior to GCC and CBM.

4. SIMULATION RESULTS

Computer simulations are carried out to investigate the performance of the proposed methods. The signal $s(n)$ is a zero-mean non-Gaussian sequence(one-sided exponentially distributed). The two additive noise sources are zero-mean Gaussian sequence, uncorrelated with each other and with the signal. 512 samples of the signal and noise are used in the 50 Monte Carlo runs with SNR from $-3dB$ to $-9dB$. The true time delay is chosen as $D = 16$. The CBM, SCOT, ROTH and PHAT methods are also employed in the simulations for comparison purposes.

Fig.3 illustrates the RMS error of the time delay estimate. From Fig.3, we see that the RMS of CBM is the highest at all SNR values and the RMS of SCOT is smaller than that of CBM, whereas the RMS of the proposed HOS-SCOT method is smaller than that of CBM and SCOT. The TDE criteria versus time lag of 50 Monte Carlo runs at $SNR = -3dB$ are given in Fig.2, Fig.4 and Fig.5 to show the performance of HOS-SCOT, HOS-ROTH and HOS-PHAT respectively. All of the cost functions derived in Monte Carlo simulations for TDE are plotted together in one figure to illustrate the performance of time delay estimation. From Fig.2, we can see that the peak generated by the CBM and SCOT is not very good for time delay estimation. In contrast, a sharp peak is generated by HOS-SCOT in multiple simulations and the peak position corresponds to the time delay. It is clear that the proposed criterion HOS-SCOT is superior to the CBM and SCOT method for time delay estimation.

Similarly, Fig.4 shows that the proposed HOS-ROTH is better than CBM and ROTH, and Fig.5 shows that the HOS-PHAT is better than the CBM and PHAT, as clearly indicated by the simulation results.

5. CONCLUSION

A well-known problem with the bispectrum-based TDE methods (CBM and PBM) is that they are outperformed by GCC methods in time delay estimation of non-Gaussian signal buried in spatially uncorrelated Gaussian noises. New approaches are proposed in this paper to solve this problem. A time delay model is formulated in the third-order cumulant domain and is used to estimate the time delay. Based on the third-order statistics of the signals, three different kind of weighting functions are formulated to suppress the Gaussian noise effect. These new TDE methods are called HOS-SCOT, HOS-ROTH and HOS-PHAT respectively. Computer simulation results verified the performance improvement of the new methods.

6. REFERENCES

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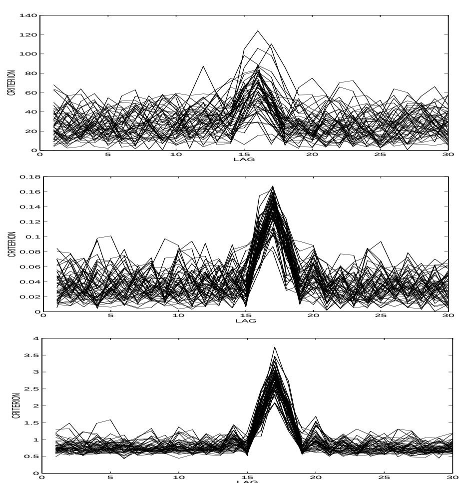

Figure 2: Criterion versus Time Lag, top: CBM, middle: SCOT, bottom: HOS-SCOT method

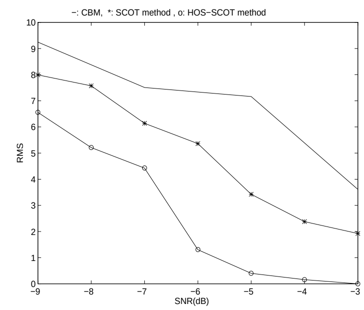

Figure 3: RMS error of TDE, -: CBM, *: SCOT, o: HOS-SCOT method

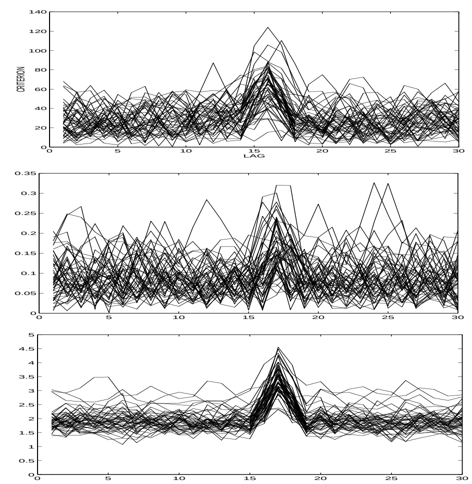

Figure 4: Criterion versus Time Lag, top: CBM, middle: ROTH , bottom: HOS-ROTH method

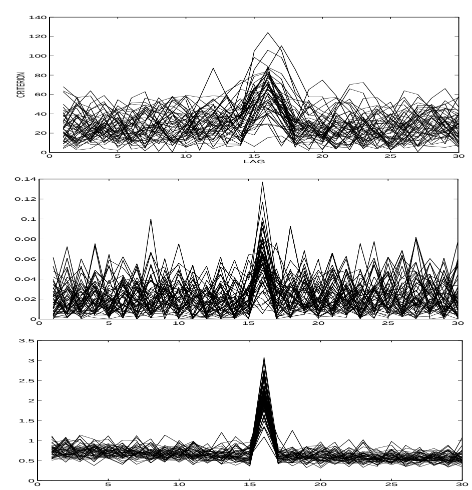

Figure 5: Criterion versus Time Lag, top: CBM, middle: PHAT, bottom: HOS-PHAT method