PERFORMANCE ANALYSIS OF MULTICARRIER MODULATION SYSTEMS USING COSINE MODULATED FILTER BANKS

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ABSTRACT

In this paper we compare the performance of biorthogonal cosine-modulated transmultiplexer filter banks with today's multicarrier modulation systems whose transceivers are based on DFT. In contrast to early works on transmultiplexer filter banks that concentrated on the derivation of perfect reconstruction constraints of the filter bank or prototype design, this study takes into consideration a typical twisted pair copper line transmission channel into consideration and examines the influence of different system parameters as filter length, number of channels, and the overall system delay on the distortion at the receiver. Biorthogonal filter banks have the advantage that filter length and overall system delay can be chosen independently. Restricting the equalizer at the receiver to a single scalar tap per subchannel, we show that cosine-modulated filter banks outperform DFT based multicarrier systems without a guard interval and obtain a similar performance to DFT based systems with a guard inverval and time domain equalization but at a lower computational cost and a higher throughput data rate.

1. INTRODUCTION

Multicarrier Modulation (MCM) is an efficient method for transmitting information over a linearly distorted channel. Its basic idea is to divide the channel spectrum into parallel, narrowband subchannels. These ideas were proposed more than 30 years ago [1], but were not pursued due to limitations of processing technology. However, advances in the area of digital signal processors have made this concept practically feasible and competitive with traditional single-carrier systems [2]. Multicarrier signalling offers the advantages of simpler equalization, immunity to impulse noise and flexibility in allocating subchannels.

The main motivation for research in MCM over the last decade has been high-speed (of the order of Mbit/s) broadband data transmission over the twisted pair channel (High bit-rate Digital Subscriber Loop - HDSL and Asymmetrical Digital Subscriber Loop - ADSL), which causes significant impairments to the signal due to great variations in gain across its bandwidth. In [3] an overview of the systems that were proposed for these services and the associated problems can be found.

The working standard for ADSL signalling is called Discrete Multitone (DMT) and uses the Inverse Discrete Fourier Transform (IDFT) to modulate the carriers at the transmitter, and a DFT to separate the symbols of each subchannel out of the signal at the receiver. This method results in a simple and computationally efficient implementation of the system, using the FFT. However, in a dispersive channel, the significant overlap between signalling filters (sidelobes of -13 dB) results in performance degradation. There is significant crosstalk between subchannels because of the distortion introduced by the channel.

An alternative scheme, employing the synthesis/analysis bank of a multirate filter bank in the modulator/demodulator was proposed in [4, 5]. These filter banks use filters of greater length than the rectangular filters of DMT and typically result in much lower sidelobe levels. Better stopband attenuation and subchannelization results in both lower levels of interchannel interference (ICI) and greater robustness to narrowband interference. In particular, cosine-modulated filter banks (CMFB) are used because they can be implemented in a computationally efficient manner using polyphase representation of the filter bank and fast DCT, and the design is simpler than for general filter banks because all the filters are derived from a single prototype [6]. The comparative performance of DMT and paraunitary CMFB based MCM was already studied in [7, 8] and it was demonstrated that CMFB systems are worthy candidates for MCM. Experimental results have demonstrated the power and flexibility of the CMFB approach for combatting radio-frequency interference in the broadband environment [9].

In this study we extend previous research to the case of CMFBs where the overall system delay (a crucial parameter in online applications such as video conferencing) can be chosen independently of the filter length. Thus, we can obtain filters with a better stopband attenuation than the DFT even when keeping the delay as a constant. In applications where the overall system delay is not so crucial the stopband attenuation can be further improved at the price of an increase in the system delay and a higher computational cost. As a criterion for the system performance we use the signal-to-interference-ratio (SIR) based on a theoretical estimation of the interference at the output of the receiver derived in the next section. We then present experimental results and compare them with the theoretical predictions.

2. SYSTEM DESCRIPTION AND THEORETICAL ANALYSIS

2.1. Block diagram of the system

The block diagram of a basic multicarrier modulation system, using multirate filter banks is shown in Figure 1. In particular, the modulator and the demodulator can be implemented efficiently using polyphase filters and fast transforms. The exact form of the modulation obtaining the filters from the prototype will be given in the next section.

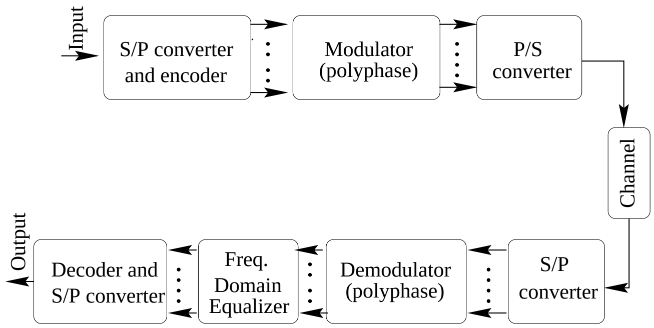

Figure 1: Multicarrier modulation transceiver using multirate filter banks.

The coder and the decoder map the incoming bits into blocks of M symbols, with each subchannel using a bit allocation based on channel measurements made during an initialization period. The coding is a simple Pulse Amplitude Modulation (PAM) coding. The frequency domain equalizer is a single tap equalizer that scales each subchannel output with the inverse of the channel frequency response at the subcarrier frequency. However, the channel may be non-linear phase, and because the data in the CMFB are real numbers, phase rotation cannot be compensated with the above scheme [7]. One would thus resort to optimal combiner methods, which use the outputs of the neighbouring filters in addition to that of the k-th filter to decide which symbol was transmitted in the k-th carrier (see [10, 11] and references therein).

2.2. Theoretical estimate of interference

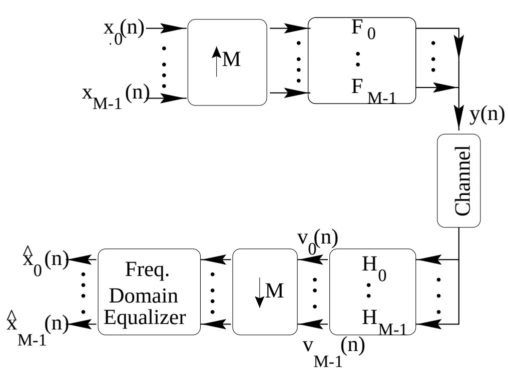

Figure 2: Various signals in a MCM system.

Let x_i denote the transmitted sequence in the i-th subchannel and F_i; $i = 0, 1, ..., M-1$, the synthesis (modulator) filters, as shown in Figure 2. Then the transmitted signal, y is such that:

$$Y(e^{j\omega}) = \sum_{i=0}^{M-1} X_i(e^{j\omega M}) F_i(e^{j\omega}). \tag{1}$$

Let the analysis filters be H_i; $i = 0, 1, ..., M-1$ and the channel frequency response be $H_c(e^{j\omega})$. The output signal of the k-th analysis filter is :

$$V_k(e^{j\omega}) = \sum_{i=0}^{M-1} X_i(e^{j\omega M}) F_i(e^{j\omega}) H_k(e^{j\omega}) H_c(e^{j\omega}) \tag{2}$$

which after downsampling and frequency domain equalization becomes :

$$\hat{X}_k(e^{j\omega}) = \frac{1}{M H_c(\frac{\pi(2k+1)}{2M})} \left\{ X_k(e^{j\omega}) \sum_{l=0}^{M-1} F_k(e^{j\omega_l}) H_c(e^{j\omega_l}) H_k(e^{j\omega_l}) + \sum_{i=0, i\neq k}^{M-1} X_i(e^{j\omega}) \sum_{l=0}^{M-1} F_i(e^{j\omega_l}) H_c(e^{j\omega_l}) H_k(e^{j\omega_l}) \right\} \tag{3}$$

where $\omega_l = (\omega - 2\pi l)/M$. The first term represents the output due to the symbols transmitted in the $k-$th channel and represents the sum of the required output and the intersymbol interference (ISI), from the other symbols in the same subchannel. It is due to the fact that the channel frequency response is not exactly constant within each subchannel, and to the non-ideal nature of the filters within their passbands. The second term represents interchannel interference (ICI), i.e. the interference from other subchannels resulting from the finite attenuation of the filters in their stopbands. Eq. (3) can be written as :

$$\hat{\mathbf{X}}(e^{j\omega}) = \mathbf{S}(e^{j\omega}) \mathbf{X}(e^{j\omega}) \tag{4}$$

where $\hat{X} = [\hat{X}_0 \hat{X}_1 ... \hat{X}_{M-1}]^T$, $X = [X_0 X_1 ... X_{M-1}]^T$ and $S_{k,i} X_i$ represents the output of the k-th subchannel at the receiver due to the input of the i-th subchannel. Therefore we have :

$$S_{k,i}(e^{j\omega}) = \frac{\sum_{l=0}^{M-1} F_i(e^{j\omega_l}) H_c(e^{j\omega_l}) H_k(e^{j\omega_l})}{M \; H_c(\frac{\pi(2k+1)}{2M})} \tag{5}$$

Ideally, we would like $S_{k,i}(e^{j\omega})$ to be zero for $k \neq i$ and one for $k = i$. The interference at the output is the power in the unwanted terms in Eq. (3):

$$P_k = \int_{-\pi}^{\pi} \left| S_{k,k}(e^{j\omega_l}) - 1 \right|^2 d\omega + \sum_{i=0, i\neq k}^{M-1} \int_{-\pi}^{\pi} \left| S_{k,i}(e^{j\omega_l}) \right|^2 d\omega \tag{6}$$

Here, the first term represents ISI and the second term the ICI. In the ideal case where the filters have brick-wall characteristics, the ICI is zero and interference is caused only by the fact that the channel is not flat within each subchannel. This important observation implies that for a given number of subchannels, there exists an upper bound on the performance in terms of signal-to-interference-ratio (SIR) at the receiver. This bound is essentially imposed by the non-ideal channel characteristics within each subchannel. The only way to combat this would be to increase the number of subchannels, thus approaching the ideal case of flat frequency response within each subchannel.

Modelling the channel as a causal FIR filter and assuming, for reasons of simplicity, that all frequency responses in the transmission system are real, we can calculate a lower bound on the interference power. Though this is an unrealistic assumption, it gives us some insight into how the interference varies with the filter characteristics. Using a CMFB where the synthesis and analysis filters have stopband attenuation δ_s and passband error δ_p, and assuming that the channel frequency response varies linearly within each subchannel, it can be easily shown by evaluating Eq. (6) that (neglecting second order error terms):

$$P_k \leq \frac{\pi}{6}\left[\frac{H_c(\frac{\pi}{M}k) - H_c(\frac{\pi}{M}(k+1))}{H_c(\frac{\pi}{2M}(2k+1))}\right]^2$$

$$+8\pi\delta_p^2\left[\frac{H_c(\frac{\pi}{M}k)}{H_c(\frac{\pi}{2M}(2k+1))}\right]^2 + \sum_{i=0, i\neq k}^{M-1} 2\pi\delta_s^2$$

$$\left[\left(\frac{H_c(\frac{\pi}{M}k)}{H_c(\frac{\pi}{2M}(2k+1))}\right)^2 + \left(\frac{H_c(\frac{\pi}{M}i)}{H_c(\frac{\pi}{2M}(2k+1))}\right)^2\right] \quad (7)$$

The first two terms constitute the ISI and the third term is due to the ICI. The above expression indicates that, on improving the filter attenuation characteristics beyond a point, there would not be any significant improvement in SIR at the receiver since the ISI (especially the first term in Eq. (7)) eventually dominates. Thus there is an optimum modulator and demodulator filter length. A further increase in the filter length is not accompanied by an improvement in the SIR that would justify the increase in computational complexity of the filters.

It is known that for filters of a fixed length, the system delay may be reduced from the delay of a linear phase CMFB system, at the cost of lower stopband attenuation of the filters. The system delay can be an important consideration in real-time applications like video conferencing or cable telephony. One can reduce the system delay by trading it off against the error-rate performance. Conversely, one might obtain better filter characteristics by increasing the filter length while maintaining a constant system delay, via the use of biorthogonal CMFB's.

3. SIMULATION RESULTS

The MCM system was simulated using CMFB, for number of subchannels equalling $M = 8, 16$ and 32. The channel used was a linear-phase channel describing a typical subcriber loop. Its magnitude response is shown in Figure 3. We assumed perfect knowledge of the channel characteristics, for the frequency domain equalization. In practical ADSL systems, where the channel is not known beforehand but is fairly stable, there is an initial training period to estimate the channel frequency response.

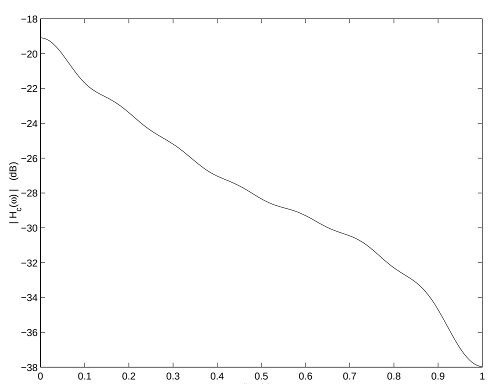

Figure 3: Magnitude response of the channel

For each value of M, simulations were performed for varying filter lengths LM and varying system delays $D = 2(s+1)M - 1$ where s is an integer (the delay parameter). The filters were obtained from linear phase prototype filters, originally designed using the Quadratic Constrained Least Squares method [12], by increasing the filter length using the lifting schemes proposed in [13].

The analysis and the synthesis filters, $h_k(n)$ and $f_k(n)$, respectively, were calculated from the prototype filter $p(n)$ according to:

$$h_k(n) = 2p(n)\cos\left[(2k+1)\frac{\pi}{2M}(n - \frac{D}{2}) + (-1)^k\frac{\pi}{4}\right] \quad (8)$$

$$f_k(n) = 2p(n)\cos\left[(2k+1)\frac{\pi}{2M}(n - \frac{D}{2}) - (-1)^k\frac{\pi}{4}\right] \quad (9)$$

The stopband energy of the filters obtained are given in Table 1. In general, stopband energy A_s decreases with an increase in the system delay for a fixed filter length and number of subchannels. A_s also decreases with an increase of the filter length while keeping the delay constant.

		M=8	M=16	M=32
L=2	s=0	1.4009e+00	7.5990e-01	4.3035e-01
L=4	s=0	2.3755e-01	1.3616e-01	8.3122e-02
	s=1	1.4845e-01	8.4657e-02	5.4007e-02
L=8	s=0	1.1683e-01	6.8658e-02	4.4645e-02
	s=1	2.5327e-02	2.1141e-02	2.0595e-02
	s=3	6.0716e-03	3.7574e-03	2.6148e-03
L=12	s=0	9.2305e-02	5.3810e-02	3.8630e-02
	s=1	1.6998e-02	1.4867e-02	1.3109e-02
	s=3	2.4359e-03	2.8456e-03	2.1094e-03
	s=5	1.5408e-03	1.9188e-03	1.8102e-03

Table 1: Prototype stopband energy for various M, L, s; the prototypes have been normalized such that $P(\omega = 0) = 1$

Table 2 gives the SIR for the cases of $M = 8, 16, 32$ for various choices of filter length LM and delay parameter s. The results obtained using DFT filter banks are also shown, for comparison purposes. In the DFT case, the use of a time domain equalizer (TDE) along with a cyclic prefix [7] results in significant improvement in performance, which is obtained at the cost of increased complexity and reduced throughput. The improvement obtained using a TDE, in case of CMFB was shown to be minimal [7] and thus not taken into consideration.

Let us first interpret the results for the CMFB from Table 2. As expected from the theoretical derivations in the last section, the SIR improves as the number of subchannels increases and also with an increase in the filter length. Also, as discussed above, beyond a certain point, the increased filter length does not result in large improvements in the SIR. This is because the ISI (especially the first term in Eq. (7)) eventually becomes the predominant interference component. Furthermore, the performance improves with an increase in the system delay. However for large M and L, the improvement is not very significant beyond a certain point. This is a very important observation, since in time critical applications, the system delay becomes a significant performance measure and one needs to optimize it against the SIR.

Let us now compare the SIR for CMFB with those of the DFT-bases system. Note that all simulations where the delay parameter of the CMFB has been chosen to $s = 0$ provide the same system delay as the DFT-based system where a $2M$-point DFT has

		M=8	M=16	M=32
DFT		13.47	16.86	19.32
DFT (with cyclic prefix, TDE)		23.53	29.60	35.58
L=2	s=0	18.32	22.25	26.01
L=4	s=0	20.00	25.04	29.58
	s=1	20.44	26.03	31.13
L=8	s=0	20.62	26.17	31.46
	s=1	21.42	27.20	31.71
	s=3	21.83	27.74	34.14
L=12	s=0	20.59	26.15	31.45
	s=1	21.66	27.51	33.08
	s=3	21.97	27.75	34.15
	s=5	22.00	27.83	34.25

Table 2: SIR (dB) for various choices of M, L, s

to be calculated. For $L = 2$ the raw DFT-based system and the CMFB are comparable in terms of computational complexity but the CMFB clearly outperforms the DFT for all values of M.

When applying DFT with cyclic prefix and TDE we obtain a somewhat better performance than with any of the CMFBs employing one-tap frequency-domain equalization . However, the introduction of a cyclic prefix of length N reduces the data throughput rate by a factor of $\frac{M}{M+N}$, as well as increasing the system's computational complexity [10]. The use of a TDE not only increases complexity, but also has been shown in [7] to be sensitive to the presence of noise. Since the insertion of a cyclic prefix and a TDE is not appropriate for CMFBs other means to increase their performance need to be investigated, such as the use of longer frequency domain equalizers (up to the length of the TDE) [10] or the use of oversampled filter banks [11].

4. CONCLUSIONS

CMFB's hold great promise in maximizing SNR over the difficult twisted-pair copper channel. We have described a new multicarrier modulation scheme based on biorthogonal CMFB's, which present greater flexibility in system latency/performance tradeoffs than linear-phase CMFB's. A theoretical analysis of the received signal and interference in the new multicarrier system was developed, with simplifying assumptions on the equalizers and channel responses. A family of prototype filters for CMFB's was designed with varying lengths, delays, and numbers of subchannels to illustrate the tradeoff space. We then evaluated these CMFB systems using simulations, and compared them with traditional DFT-based multicarrier systems. The CMFB approach was superior to a raw DFT-based scheme of similar latency. Furthermore, a biorthogonal low-latency CMFB attains most of the performance improvement of a linear-phase CMFB, while keeping system latency near that of a DFT modem. Future research will focus on more sophisticated equalization techniques to surpass the performance of a DFT system with cyclic prefixes and a time-domain equalizer.

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