A SUBSPACE METHOD FOR SEPARATING COCHANNEL TDMA SIGNALS

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ABSTRACT

In this paper, we describe an adaptive algorithm that uses a subspace method for separating cochannel time-division multiple-access (TDMA) signals impinging on an antenna array. A frame synchronization algorithm is initially employed to identify the cochannel scenario. The adaptive algorithm uses a subspace decomposition of the array signals to constrain the beamformer weights for the signal of interest to be in a specific subspace. This subspace corresponds to the orthogonal complement of the subspace spanned by the direction vectors of the interferers. A follow-on linear equalizer removes the intersymbol interference (ISI) introduced by the transmit filter.

1. INTRODUCTION

In recent years, several methods have been proposed to separate and demodulate cochannel signals. For example, the algorithm described in [1] employs blind equalization based on second-order statistics. The approach proposed in [2] recovers the transmitted symbols by oversampling the received signals, and it exploits the finite alphabet property of digital signals, as well as the inherent structure of the matrices in the signal model. The subspace approach described in [3] recovers the transmitted signals by exploiting the orthogonality of certain matrices constructed from the second-order statistics of the received signals.

However, a fundamental property of TDMA systems, such as those specified in IS-136 (Interim Standard 136) [4] and GSM (Global System for Mobile) [5], is that the bursts are relatively short, containing only a few hundred symbols. Previous subspace methods, such as those mentioned above, typically require more than a few hundred symbols of data to achieve satisfactory performance. On the other hand, the beamforming algorithm described in [6] employs the synchronization sequence and the guard symbols provided in each burst to accurately separate cochannel TDMA signals. We extended this work in [7] by employing a constrained least-squares (LS) formulation that utilizes information from adjacent user bursts. In this paper, we improve the performance of the constrained beamforming algorithm by replacing the LS formulation with an adaptive algorithm that uses a subspace approach.

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2. PROBLEM FORMULATION

In a scenario with L cochannel users, each with M_i propagation paths, the signals received at an N-element antenna array can be modeled as follows:

$$\mathbf{x}(t) = \sum_{i=1}^{L} \sum_{j=1}^{M_i} \alpha_{i,j}(t)\mathbf{a}(\theta_{i,j})s_i(t-\tau_{i,j}) + \mathbf{n}(t) \qquad (1)$$

where

$$s_i(t) = \sum_{n=-\infty}^{\infty} d_i(n)g(t-nT), \qquad (2)$$

$\{d_i(n)\}$ are the transmitted data symbols, and $g(\cdot)$ is the transmit filter with a square-root raised-cosine (SRRC) frequency response [4]. The direction vector $\mathbf{a}(\theta_{i,j})$ depends on the angle of arrival (AOA) $\theta_{i,j}$, $\tau_{i,j}$ is a propagation delay, $\alpha_{i,j}(t)$ is an attenuation factor (which is time-varying and includes fading), and $\mathbf{n}(t)$ is additive white Gaussian noise (AWGN).

If the received signal vector is sampled at the symbol rate, (1) can be written as

$$\mathbf{x}(kT) = \sum_{i=1}^{L} \sum_{n=-\infty}^{\infty} \mathbf{h}_i(n;k)d_i(k-n) + \mathbf{n}(kT) \qquad (3)$$

where

$$\mathbf{h}_i(n;k) = \sum_{j=1}^{M_i} \alpha_{i,j}(kT)\mathbf{a}(\theta_{i,j})g(nT-\tau_{i,j}) \qquad (4)$$

describes the effective channel (i.e., it includes the transmit filter, the direction vectors, and the multipath pa-

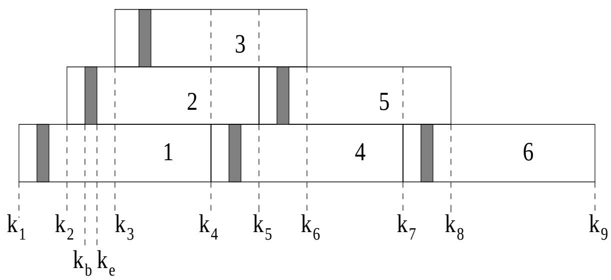

Figure 1: Example cochannel TDMA scenario.

rameters). In practice, this channel would have a finite impulse response such that the limits of the second summation in (3) are finite.

The goal of the signal separation algorithm is to recover the transmitted symbols $\{d_i(k)\}$ from $\mathbf{x}(kT)$. The algorithm proposed in this paper recovers the data by processing the received signals with a beamformer (to separate the cochannel signals) and a linear equalizer (to mitigate the ISI introduced by the transmit filter).

3. BEAMFORMING ALGORITHM

For convenience, assume that the oversampling factor is unity and that the fading is negligible. We proceed to describe the signal separation algorithm by way of an example. Consider the cochannel scenario illustrated in Figure 1. Note that the algorithm can be used for signals with either the IS-136 or GSM signal format. The training sequences (shaded regions) shown in the figure are located near the beginning of the times slots (corresponding to IS-136 bursts).

The received data are partitioned into blocks of a predetermined length. Since the frame and time-slot structures in a TDMA system are fixed and known, the start times of the training sequences for the bursts in a block of data also provide us with information about the cochannel scenario for that entire block. The frame synchronization algorithm in [6] and [8] can be used to locate the training sequences. Once the cochannel scenario is identified, as labeled in Figure 1, the beamforming algorithm processes the data in two steps.

3.1. Step 1

Each slot has a training sequence which may overlap only a subset of the interferers for that slot. As a result, the beamformer weights computed from such a training sequence are appropriate only for those time points involving interferers that overlap the training sequence. For the example in Figure 1, the weights computed for slot 2 are appropriate for the region $k \in [k_2, k_3]$; the training sequence of slot 2 overlaps slot 1, but does not overlap slot 3.

Let the number of users during the time period of a given training sequence be L, and let k_b and k_e be the beginning and ending times of that training sequence (see slot 2 in Figure 1). Observe that $L = 1$ for slot 1, $L = 2$ for slots 2 and 6, and $L = 3$ for slots 3, 4, and 5. The received data vectors for the region $k \in [k_b, k_e]$ are collated together to form the matrix

$$\mathbf{X}_1 \triangleq [\mathbf{x}(k_b), \cdots, \mathbf{x}(k_e)]. \tag{5}$$

In the absence of noise, the column rank of $\mathbf{X}_1$ is L. Therefore, performing a singular value decomposition (SVD), $\mathbf{X}_1$ can be approximated by the components associated with the L significant eigenvalues $\{\lambda_i\}$ as follows [9]:

$$\mathbf{X}_1 \approx \sum_{i=1}^{L} \lambda_i \mathbf{c}_i \mathbf{r}_i^H \triangleq \widehat{\mathbf{X}}_1 \tag{6}$$

where $\{\mathbf{c}_i\}$ and $\{\mathbf{r}_i\}$ are the left and right eigenvectors, respectively. The optimal beamformer weight vector is thus given by

$$\mathbf{w}_1^H = \mathbf{t}^H \widehat{\mathbf{X}}_1^+ = \sum_{i=1}^{L} (1/\lambda_i) \mathbf{t}^H \mathbf{r}_i \mathbf{c}_i^H \tag{7}$$

where $\widehat{\mathbf{X}}_1^+$ is the pseudo-inverse of $\widehat{\mathbf{X}}_1$, and $\mathbf{t}^H = [t^*(1), t^*(2), \cdots, t^*(k_e - k_b + 1)]$ is the training sequence. Note that $\mathbf{w}_1$ lies entirely in the column space of $\widehat{\mathbf{X}}_1$ spanned by $\{\mathbf{c}_1, \mathbf{c}_2, \cdots, \mathbf{c}_L\}$.

3.2. Step 2

In Step 2, the beamformer weights for those regions of each slot that were not beamformed in Step 1 are obtained by projecting the beamformer weights obtained in Step 1 onto a subspace that is determined by the interferers.

The regions of each slot that were not beamformed in Step 1 are divided into subslots such that the cochannel scenario remains unchanged in a subslot. For the example in Figure 1, slot 2 is divided into three subslots, defined by the regions $k \in [k_2, k_3), [k_3, k_4)$, and $[k_4, k_5)$. A different set of beamformer weights must be computed for each subslot. In order to obtain the weights for a given subslot, we first identify the time points where the interferers do not overlap with the current signal of interest. Let us enumerate the union of these data points as $\{k^{(1)}, k^{(2)}, \cdots, k^{(n)}\}$. For the subslot in slot 2 specified by $k \in [k_3, k_4)$, for example,

User	AOA	Bit Delay
1	0°	0
2	45°	108
3	75°	216
4	15°	324
5	−45°	432
6	30°	648

Table 1: Cochannel Signal Parameters

these data points are $[k_1, k_2) \cup [k_5, k_6)$. The corresponding received data vectors are then collated to form the matrix

$$\mathbf{X}_2 \triangleq [\mathbf{x}(k^{(1)}), \cdots, \mathbf{x}(k^{(n)})]. \tag{8}$$

Each column of $\mathbf{X}_2$ may have contributions from several users. If we denote the number of different users that contribute to the columns of $\mathbf{X}_2$ by P, then the column rank of $\mathbf{X}_2$ (in the absence of noise) is P. In our example, $P = 4$ for slot 2, corresponding to users 3, 4, and 5 in region $k \in [k_5, k_6)$, and user 1 in region $k \in [k_1, k_2)$. The SVD of $\mathbf{X}_2$ is given by

$$\mathbf{X}_2 = \sum_{i=1}^{N} \beta_i \mathbf{u}_i \mathbf{v}_i^H \tag{9}$$

where $\beta_1, \cdots, \beta_P >> \beta_{P+1}, \cdots, \beta_N$. As a result, we can approximate this expression by

$$\mathbf{X}_2 \approx \sum_{i=1}^{P} \beta_i \mathbf{u}_i \mathbf{v}_i^H \triangleq \widehat{\mathbf{X}}_2. \tag{10}$$

The beamformer weights for a given subslot are obtained by projecting $\mathbf{w}_1$ obtained for the slot in Step 1 (via 7) onto the subspace spanned by $\{\mathbf{u}_{P+1}, \mathbf{u}_{P+2}, \cdots, \mathbf{u}_N\}$ (which is the left null space of $\widehat{\mathbf{X}}_2$), yielding

$$\mathbf{w}_2 = \sum_{i=P+1}^{N} (\mathbf{u}_i^H \mathbf{w}_1) \mathbf{u}_i. \tag{11}$$

Hence, by using this procedure, the entire slot can be beamformed. The resulting beamformed signal is then processed by a linear equalizer as described in [8].

4. COMPUTER SIMULATIONS

The performance of the adaptive beamforming algorithm is illustrated by applying it to the cochannel scenario in Figure 1 with the parameters specified in Table 1. The bit delay refers to the number of bits that a user is delayed relative to the first bit of user 1. (Recall that there are two bits per symbol because differential QPSK is employed in IS-136 [4].) There are $N = 8$ antenna elements, the attenuation factors $\{\alpha_{i,j}\}$ are all unity, and the noise variance is $\sigma_n^2 = 0.01$. Eight IS-136 frames of data were processed by the algorithm; the resulting signal constellations are shown in Figure 2 (the center and radial points are caused by the guard and ramp-up symbols). It is clear from these plots that the signal separation algorithm has successfully recovered all six cochannel sources.

5. REFERENCES

[1] L. Tong, G. Xu, and T. Kailath, "Blind identification and equalization based on second-order statistics: A time domain approach," *IEEE Trans. Information Theory*, vol. 40, pp. 340–349, Mar. 1994.

[2] A. J. van der Veen, S. Talwar, and A. Paulraj, "A subspace approach to blind space-time signal processing for wireless communication systems," *IEEE Trans. Signal Processing*, vol. 45, pp. 173–190, Jan. 1997.

[3] K. Abed-Meraim, J.-F. Cardoso, A. Y. Gorokhov, P. Loubaton, and E. Moulines, "On subspace methods for blind identification of single-input multiple-output FIR systems," *IEEE Trans. Signal Processing*, vol. 45, pp. 42–55, Jan. 1997.

[4] T. S. Rappaport, *Wireless Communications: Principles and Practice*. Upper Saddle River, NJ: Prentice Hall PTR, 1996.

[5] S. M. Redl, M. K. Weber, and M. W. Oliphant, *An Introduction to GSM*. Norwood, MA: Artech House, 1995.

[6] J. Leary and R. P. Gooch, "Adaptive beamforming for TDMA signals," in *Proc. Twenty-Ninth Asilomar Conf. Signals, Systems, and Computers*, (Pacific Grove, CA), pp. 1378–1382, Nov. 1995.

[7] R. Chandrasekaran, A. V. Keerthi, and J. J. Shynk, "A modified sequential beamforming algorithm for cochannel IS-136 signals," in *Proc. Thirty-First Asilomar Conf. Signals, Systems, and Computers*, (Pacific Grove, CA), pp. 1564–1569, Nov. 1997.

[8] A. V. Keerthi and J. J. Shynk, "Separation of cochannel signals in TDMA mobile radio," *IEEE Trans. Signal Processing*, vol. 46, Oct. 1998.

[9] G. H. Golub and C. F. Van Loan, *Matrix Computations*. Baltimore, MD: The Johns Hopkins Univ. Press, 1984.

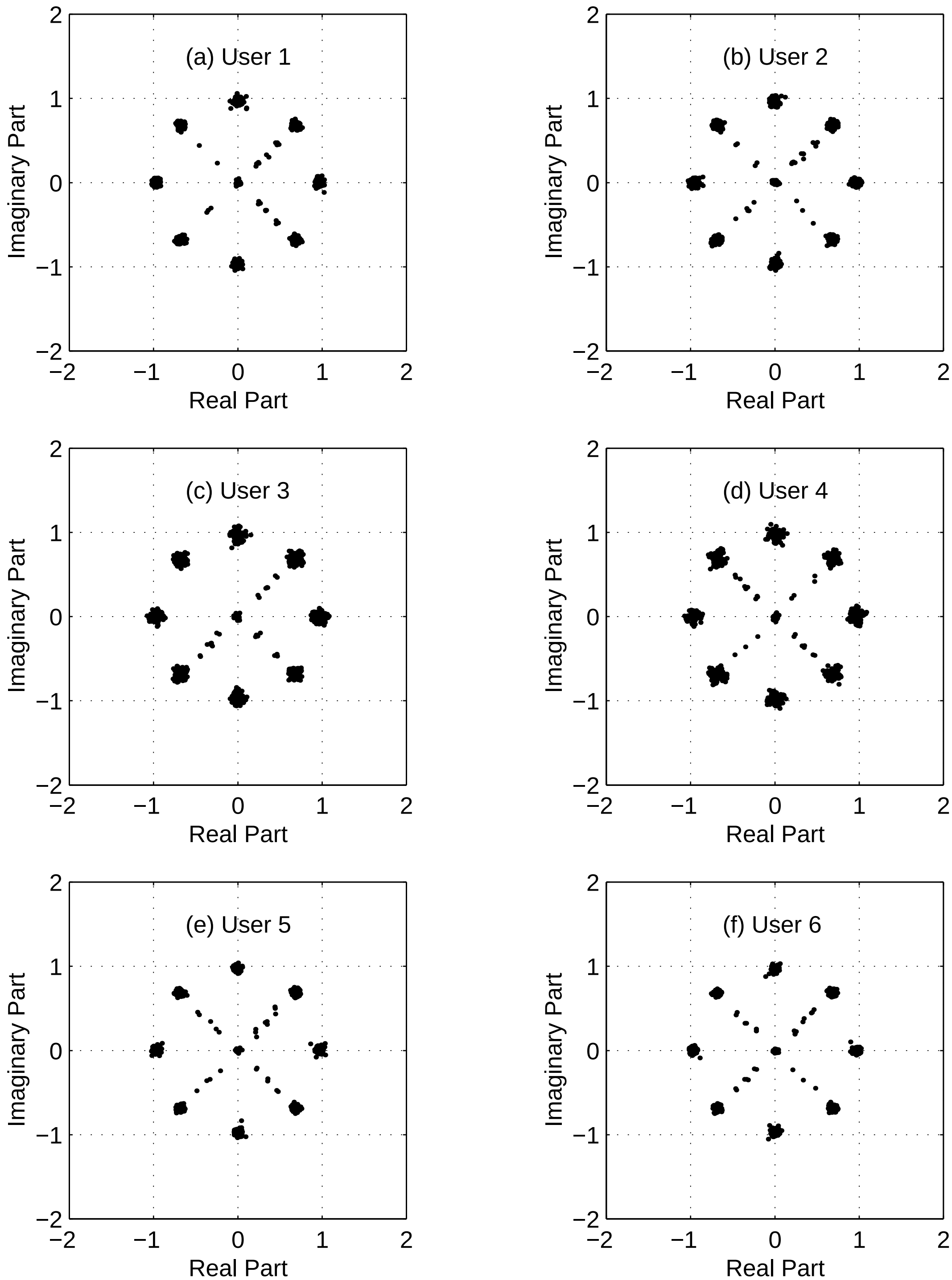

Figure 2: Signal constellations for the cochannel scenario in Figure 1 and Table 1. (a) User 1. (b) User 2. (c) User 3. (d) User 4. (e) User 5. (f) User 6.