A BROADBAND APPLICATION OF MEMORYLESS NARROWBAND GSC/NLMS ADAPTIVE BEAMFORMERS

D. Peters

Nortel Technologies, 16 Place du Commerce, Montréal, Québec, CANADA H3E 1H6

ABSTRACT

In this article, the adaptive performance of the normalized least mean-squares algorithm in the context of the generalized sidelobe canceller beamformer is considered. The implications of both the convergence behaviour and the misadjustment on various beamforming applications are discussed. In particular, an important case is identified for which there is near-instantaneous convergence. A misadjustment limit for which coherent post-processing is viable is also derived. Finally, a novel approach to coherent broadband beamforming is introduced and then tested via simulation.

INTRODUCTION

Adaptive beamformers can provide significant gains under conditions of anisotropic noise and interference. The Griffiths-Jim Generalized Sidelobe Canceller (GSC) is a useful structure for the implementation of this processing [1]. The GSC abstracts the constraints of the adaption problem so as to permit the use of a standard unconstrained adaption algorithm such as Least Mean-Squares (LMS) or Recursive Least-Squares (RLS) (see, for example, [2]).

The LMS algorithm and its variants suffer the stigma of having excessively long convergence times. Further, this difficulty is exacerbated by correlation between successive input vectors [2]. Recently, however, Slock reports "much faster convergence for some ill-conditioned input covariance matrices than for the white noise case [3]." It is of interest, in consequence, to identify applications in which this "much faster convergence" can be exploited. In this article, the limiting case will be examined. It will be shown that a particular interference scenario of interest is ideally suited for the convergence of LMS-type algorithms.

After introductory preliminaries on the GSC and NLMS, the performance of the memoryless GSC/NLMS combination is considered. The near-instantaneous limiting convergence is then derived, and implications with regard to tracking performance and output SNR are then discussed. Observations with respect to the detection performance of adaptive beamformers and their support of post-processing are made. Finally, a novel approach to coherent broadband beamforming is introduced, and its performance is compared to existing beamformers via simulation.

THE MEMORYLESS GSC/NLMS BEAMFORMER

Let us examine the "memoryless" case in which the GSC reduces to that shown in Figure 1. To form the input, $\mathbf{z}$, to the GSC of Figure 1, the time-series at the output of an M-sensor array is segmented, and Fourier Transforms are used to bring the data into the frequency domain. The resulting complex values are then pre-steered so that a vector of complex sensor outputs, $\mathbf{z}_{k,f,\theta}$, result at each snapshot, k, and at each frequency bin, f, and look direction, θ. Let us consider the processing at some arbitrary frequency and look direction so that the corresponding subscripts may be dropped. For basic Minimum Variance Distortionless Response (MVDR) processing, the so-called quiescent response for the pre-steered case is simply a vector of ones (i.e., $\mathbf{w}_q = \mathbf{1}_M$). The input to the unconstrained adaptive filter is the product of a "signal blocking matrix" and the sensor outputs: $\mathbf{x} = \bar{\mathbf{C}}\mathbf{z}$, where $\bar{\mathbf{C}}\mathbf{w}_q = \mathbf{0}$. With respect to the standard adaptive filtering terminology, the so-called "desired response" is simply the conventional beamformer output, $d = \mathbf{w}_q^H\mathbf{z}$, where the superscript H denotes conjugate transpose. An adaptive filter is driven to reflect the correlation between the energy at the output of the conventional beamformer (d) and that which arrives from other directions ($\mathbf{x}$). The adaptive filter output, which at convergence will represent any interferences present, is then subtracted from the conventional beamformer output to provide the output of the adaptive beamformer. This difference quantity is simply the adaptive "error": $e = d - \hat{\mathbf{u}}^H\mathbf{x}$. In the case of Normalized LMS (NLMS) processing, this adaptation can be represented by its weight update equation,

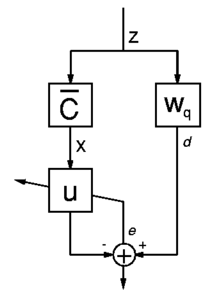

Figure 1. Generalized Sidelobe Canceller.

$$\hat{\mathbf{u}}_{k+1} = \hat{\mathbf{u}}_k + \frac{\bar{\mu}e_k^\dagger}{\alpha + \mathbf{x}_k^H\mathbf{x}_k}\mathbf{x}_k,$$

where α is some small positive number designed to maintain stability (assumed to be zero here), and $\bar{\mu}$ is the so-called stepsize or convergence-controlling parameter for the NLMS filter.

GSC/NLMS LIMITING CONVERGENCE

Consider the case in which there are two planewave signals in otherwise isotropic noise. The assumption that one of these signals is in the precise steering direction is also made. The pre-steered covariance matrix is now

$$E\left(\mathbf{z}\mathbf{z}^H\right) = \sigma_n^2\left(\mathbf{I}_M + h_i\mathbf{q}_i\mathbf{q}_i^H + h_s\mathbf{q}_s\mathbf{q}_s^H\right),$$

where σ_n^2 is the variance of the noise at each sensor, and the vectors $\mathbf{q}_i$ and $\mathbf{q}_s$ represent the beam-weighted sensor outputs due to the "interferer" (signal arriving off the look direction) and "signal" (that arriving in the look direction), respectively. We set $\mathbf{q}_i^H\mathbf{q}_i = \mathbf{q}_s^H\mathbf{q}_s = 1$ so that the quantities h_i and h_s represent the in-beam Interference-to-Noise and Signal-to-Noise Ratios (INR and SNR) respectively. Note that due to the pre-steering, these conditions imply that $\mathbf{q}_s = M^{-\frac{1}{2}}\mathbf{1}_M$. The covariance matrix of the adaptive inputs is now

$$E\left(\mathbf{x}\mathbf{x}^H\right) = \sigma_n^2\left(\bar{\mathbf{C}}\bar{\mathbf{C}}^H + h_i\bar{\mathbf{C}}\mathbf{q}_i\mathbf{q}_i^H\bar{\mathbf{C}}^H\right).$$

In order to facilitate the subsequent analysis, we also consider an orthonormal signal blocking matrix, i.e., $\bar{\mathbf{C}}\bar{\mathbf{C}}^H = \mathbf{I}_{M-1}$. The results to follow will represent a performance limit in the case of more computationally motivated signal blocking matrices. Given an orthonormal $\bar{\mathbf{C}}$, we have a $M-1 \times M-1$ adaptive filter input covariance matrix with $M-2$ eigenvalues of $\lambda_n = \sigma_n^2$ and one eigenvalue of $\lambda_i = \sigma_n^2(1 + h_i\mathbf{q}_i^H\bar{\mathbf{C}}^H\bar{\mathbf{C}}\mathbf{q}_i)$. Clearly, the dominant eigenvector is in the direction of $\mathbf{r}_1 = \bar{\mathbf{C}}\mathbf{q}_i$.

It will now be shown that the Wiener filter is to be found in the same direction as this dominant direction. As a result, the adaptive filter inputs will tend to drive its adjustable weights directly toward the target of the adaptation process, and near-instantaneous convergence will result. For the problem at hand, the Wiener filter is given by

$$\begin{aligned}\mathbf{u}^{\text{opt}} &= E\left(\mathbf{x}\mathbf{x}^H\right)^{-1}E\left(\mathbf{x}d^\dagger\right)\\ &= \left[\bar{\mathbf{C}}E\left(\mathbf{z}\mathbf{z}^H\right)\bar{\mathbf{C}}^H\right]^{-1}\bar{\mathbf{C}}E\left(\mathbf{z}\mathbf{z}^H\right)\mathbf{w}_q.\end{aligned}$$

where the superscript $\dagger$ denotes complex conjugate. Multiplying the equation above by $E\left(\mathbf{x}\mathbf{x}^H\right)$ and substituting for d and $\mathbf{x}$ yields

$$E\left(\mathbf{x}\mathbf{x}^H\right)\mathbf{u}^{\text{opt}} = E\left(\mathbf{x}d^\dagger\right) = \sigma_n^2 h_i(\mathbf{q}_i^H\mathbf{w}_q)\mathbf{r}_1,$$

implying that the Wiener weights coincide with the dominant input direction, i.e.,

$$\mathbf{u}^{\text{opt}} = \frac{h_i\mathbf{q}_i^H\mathbf{w}_q}{1 + h_i\mathbf{r}_1^H\mathbf{r}_1}\mathbf{r}_1.$$

This is, of course, intuitive: the function of the GSC is to steer a null in the direction of the interferer.

If the adaptive filter weights are initialized to zero, as is typical, then the adaptive weight error only exists in the direction of the dominant eigenvector of the input covariance matrix. Consequently, the decoupled modal convergence analysis found in [3] can be applied. Since only one of the modal weight errors exists initially, the global NLMS convergence reduces to the convergence of that mode. In this case, the time constant of the NLMS learning curve is given by [3]

$$\tau_{\text{NLMS}} = \frac{\lambda_i + (M-2)\lambda_n}{\lambda_i\bar{\mu}(2-\bar{\mu})}.$$

For large interference to noise ratio (INR), that is, $\lambda_i > (M-2)\lambda_n$, this time constant is less than two for $\bar{\mu} = 1$. This represents a considerable gain in convergence over sample matrix inversion methods, which require at least M snapshots before their output becomes meaningful. In a richer environment, the convergence for the narrowband GSC/NLMS adaptive beamformer will, of course, be slower.

The RLS algorithm can also demonstrate this remarkable convergence behaviour. With an initial inverse covariance estimate of $\mathbf{I}_{M-1}$, for example, the first RLS iteration is equivalent to that of an NLMS filter with α equal to the RLS forgetting factor. Unfortunately, the efficient (so-called fast) implementations of the RLS algorithm are not available in an adaptive beamforming context, necessitating $O(M^2)$ multiplications per snapshot per beam. As a result, the NLMS adaptation algorithm, with $O(M)$ multiplications per snapshot per beam, is much more attractive in terms of its complexity. Hence, the focus of this paper is the GSC/NLMS beamformer.

In principle, anisotropies resulting in large suboptimality for the conventional beamformer will be nulled quickly whereas convergence toward more subtle, and hence less deleterious, anisotropies will tend to be slower. An exception to this rule is the case in which two closely-spaced interferers exist.

TRACKING PERFORMANCE

In some applications, fast convergence translates directly into enhanced tracking ability. Unfortunately, this is not necessarily the case for narrowband GSC/NLMS adaptive beamforming. Our tracking performance will be related to the excitation of the direction that the target filter is drifting. Unfortunately, this relationship is difficult to quantify and may change dramatically as an interferer moves relative to the array. In some conditions, the target filter drift due to an interferer's motion is almost parallel to the excitation, in which circumstances NLMS would be expected to track well. The opposite situation, in which the excitation is orthogonal to the target drift, is no less likely to arise, however. In this event we might expect a very poor tracking behaviour on the part of the NLMS algorithm.

This difficulty with the tracking performance of the NLMS adaptive filter does not necessarily suggest that the RLS filter will track better. The NLMS/RLS tracking comparisons of [3], for example, simply do not apply since the input nonstationarity under the conditions being considered impacts the tracking performance of the RLS algorithm. In any event, the tracking performance of the NLMS/GSC adaptive beamformer may be considerably poorer than desired.

MISADJUSTMENT AND OUTPUT SNR

The term "misadjustment" refers to the steady-state noise performance of the processing, and in particular the noise injected into the output due to the variability of the adaptive weights. In all adaptive systems, a trade-off exists between convergence times and the misadjustment. If a system is tuned to provide low misadjustment, its convergence behaviour must necessarily suffer. Let us consider, therefore, the implications of significant misadjustment in the adaptive beamforming context.

The minimum mean squared error (MSE) is given by

$$\xi^{\text{opt}} = E\left[\left(d - \mathbf{u}^{\text{opt}H}\mathbf{x}\right)^2\right]$$

$$= \left(\mathbf{w}_q - \bar{\mathbf{C}}^H \mathbf{u}^{\text{opt}}\right)^H E\left(\mathbf{z}\mathbf{z}^H\right)\left(\mathbf{w}_q - \bar{\mathbf{C}}^H \mathbf{u}^{\text{opt}}\right).$$

For the model environment under consideration, this expression reduces to

$$\xi^{\text{opt}} = \sigma_n^2 \mathbf{w}_q^H \left(\mathbf{I}_M + \frac{h_i}{1 + h_i \mathbf{r}_1^H \mathbf{r}_1} \mathbf{q}_i \mathbf{q}_i^H + h_s \mathbf{q}_s \mathbf{q}_s^H\right) \mathbf{w}_q.$$

As a result, the potential SNR at the output of a MVDR beamformer is given by

$$SNR_{\text{MVDR}}^{\text{opt}} = \frac{h_s}{1 + \frac{h_i \zeta_i}{1 + h_i(1-\zeta_i)}}. \qquad (1)$$

where $\zeta_i = \mathbf{w}_q^H \mathbf{q}_i \mathbf{q}_i^H \mathbf{w}_q / \mathbf{w}_q^H \mathbf{w}_q$ ($\zeta_s = 1$), and $\mathbf{r}_1^H \mathbf{r}_1 = 1 - \zeta_i$ due to the orthogonality between $\bar{\mathbf{C}}$ and $\mathbf{w}_q$.

Since the conventional output SNR is given by

$$SNR_{\text{CB}} = \frac{h_s}{1 + h_i \zeta_i},$$

the potential SNR gain is given by

$$\frac{SNR_{\text{MVDR}}^{\text{opt}}}{SNR_{\text{CB}}} = \frac{1 + h_i \zeta_i}{1 + \frac{h_i \zeta_i}{1 + h_i(1-\zeta_i)}} = 1 + \frac{h_i^2 \zeta_i (1 - \zeta_i)}{1 + h_i} \qquad (2)$$

in agreement with, for example, [4].

Expression (2) represents an unachievable gain in true SNR in the case of zero misadjustment. The practical MSE, ξ, of an adaptive filter is given by

$$\xi \triangleq E\left(|e|^2\right) = \xi^{\text{opt}}(1 + \mathcal{M})$$

where $\mathcal{M}$ is the misadjustment [2]. When the adaptive misadjustment is taken into consideration, the SNR at the output of the adaptive beamformer becomes

$$SNR_{\text{MVDR}} = \frac{h_s}{\left[1 + \frac{h_i \zeta_i}{1 + h_i(1-\zeta_i)}\right](1 + \mathcal{M}) + h_s \mathcal{M}}. \qquad (3)$$

In effect, there are now four components present in the adaptive beamformer output power:

1. h_s;
2. $1 + \frac{h_i \zeta_i}{1 + h_i(1-\zeta_i)}$;
3. $h_s \mathcal{M}$;
4. $\left[1 + \frac{h_i \zeta_i}{1 + h_i(1-\zeta_i)}\right] \mathcal{M}$.

Only the first component represents coherent signal power while the last two components are due to the misadjustment. Note that the signal in the look direction contributes significantly to the noise in the adaptive beamformer output via component 3. For $h_i = 0$ and $h_s \to \infty$, we observe that $SNR_{\text{MVDR}} \to \mathcal{M}^{-1}$. This suggests an interesting interpretation for the misadjustment as the inverse of a ceiling on the true output SNR. By comparison, the SNR at the output of the conventional beamformer in the same circumstances is unbounded, equaling h_s.

This is of particular importance when the beamforming takes place over a number of bins, f, and a number of snapshots, k. The effect of misadjustment is that the fluctuations in the adaptive weights decorrelate the beamformer outputs, resulting in a loss of coherence between adjacent bins and adjacent snapshots. When any post-processing is performed after beamforming, this inter-bin and inter-snapshot coherence can be vital. For example, spectral zoom processing assumes coherence between adjacent snapshots. The application of a matched filter, on the other hand would also require inter-bin coherence. In order for adaptive beamforming to be a suitable for these situations, the true adaptive SNR must be greater than that obtained due to conventional processing. In other words, the misadjustment must be lower than

$$\mathcal{M}_{\text{crit}} = \frac{h_i^2 \zeta_i (1 - \zeta_i)}{(1 + h_i)(1 + h_s) - h_s h_i \zeta_i},$$

at which value the misadjustment losses equal the adaptive gains. As a result of this limit, a very low value for misadjustment is often required.

In some applications, however, the information of interest is simply the power in a given look direction at a given frequency. In these cases, all of the output power resulting directly from the signal of interest may be considered to be "signal". That is, the incoherent output power component $h_s \mathcal{M}$ may actually be considered to be part of the output signal power since it is an indication of the signal present in the look direction. The apparent SNR now becomes the optimal SNR as given by (1). Here, the coherence of the adaptive beamformer output is not an issue. The only consideration is the detection of the signal in the look direction. Since the adaptive misadjustment has the effect of amplifying the signal power with the same gain as it amplifies the noise power, the detection problem becomes independent of misadjustment.

BROADBAND PROCESSING

A simple extension of the narrowband beamformer under consideration to the broadband beamforming problem will now be introduced. For narrowband processing, an independent set of adaptive weights is maintained for each frequency bin and each beam, meaning that only one input vector is available to drive each adaptive filter at each snapshot. In constrast, the proposed coherent (with respect to the look direction) broadband processing method maintains only one set of adaptive weights per beam. These weights are then updated using all the N_{bb} independent pre-steered frequency bins which the broadband signals are expected to span.

The manner in which any given set of bin-vectors is applied to the update of a given adaptive filter is important, however. In a field consisting of one broadband interferer, the Wiener filter is different at each frequency due to the dependence of the quantity $\mathbf{q}_i$ on frequency. As a result, it is important to process frequency bins sequentially, so that the Wiener filter changes as little as possible between input vectors. In order to maintain this policy from snapshot to snapshot, it is necessary to alternate processing directions, applying the bin-vectors from low to high frequency in one snapshot and from high to low in the next.

In principle, the application of N_{bb} independent input vectors to each adaptive filter per snapshot should result in convergence that is faster by a factor of N_{bb}. Consequently, lower values of $\bar{\mu}$ may be used to provide lower misadjustment. With sufficiently low misadjustment, little signal coherence is lost so that further processing, such as matched filters, can be successfully applied while still maintaining a near-instantaneous convergence (in terms of snapshots rather than input vectors). Unfortunately, the coherent performance of the coherent broadband GSC/NLMS beamformer depends more on the tracking ability of the NLMS algorithm than its convergence performance. For broadband detection, on the other hand, which is simply a power detection exercise, there is no need to choose a low value of $\bar{\mu}$ or, for that matter, to adopt a coherent rather than an incoherent processing policy, as has already been discussed. Perhaps counter-intuitive, this result stems from the fact that the misadjustment of the adaptive process equally boosts the power of the signal and the (cancelled) interferers. In consequence, the stability of the adaptive weight vector is not an issue in the context of broadband detection.

EXPERIMENTAL RESULTS

This section presents the results of a simulation that tests the performance of broadband beamforming method described above. At the same time, a number of the analytic observations above are also demonstrated.

A broadband planewave source (interferer) with strength $h_i = 30$ was simulated to arrive at 7° away from the broadside of a equi-spaced linear array with $M = 16$ sensors. The source frequency characteristics can be taken to span 80% of the array's design frequency being centered at 60% of that frequency. The processing resolution was chosen so that 201 independent input bin-vectors were available per beam per snapshot. Finally, σ_n^2 was chosen to be M^{-1} so that the conventional beamformer (CBF) would expect 0dB output apart from the interferer.

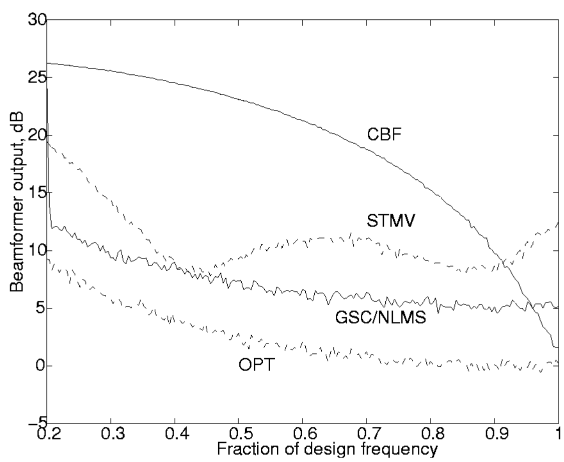

Figure 2. NB Response of Coherent BB Beamformers.

An ensemble average of 200 independent trials of the first snapshot was constructed for a number of different processing methods. Figure 2 shows the resulting bin-outputs. Of the four curves shown on this figure, the uppermost is the conventional beamformer response. The effects of the interferer are clear at low frequencies. The lowermost line represents the optimal beamformer, having $\mathbf{u} = \mathbf{u}^{\text{opt}}$ for all bins. While this processing is impractical, it is included for reference. The remaining two curves are the bin-outputs for GSC/NLMS and the Steered Minimum Variance (STMV) technique of [5]. For the NLMS algorithm, $\bar{\mu} = 1$ was used. As a result, near-instantaneous convergence is evident at the lowest frequency, where the processing began. Moreover, the effects of adaptive misadjustment are evident toward the design frequency, where the conventional beamformer outperforms both of the adaptive methods. The broadband outputs (sum over all bins) of the four beamformers are summarized below.

Table 1. Beamformer output powers, dB

Method	Output
CBF	45.2
OPT	26.3
GSC/NLMS	32.0
STMV	35.0

The experiment in question could, of course, be interpreted quite differently. A brief discussion of its duality may be of interest. Instead of one snapshot of a stationary broadband scenario with N bins, the experiment can equally be considered to be a nonstationary narrowband experiment over N snapshots. With the processing frequency equal to the design frequency, the interferer travels from 1.4° to 7° away from broadside. In this event, the STMV technique corresponds to a batch MVDR method. Using the narrowband interpretation, it is easy to picture the interferer traversing the main lobe of the conventional beamformer from the appropriate curve of Figure 2. Only the labels for the abscissa of Figure 2 would change under this interpretation.

CONCLUSIONS

In this paper, the performance of the narrowband memoryless GSC beamformer is considered, and its limiting convergence with the NLMS adaptive filter has been derived. The implications of the misadjustment of these adaptive beamformers have also been discussed. By recursive application of narrowband beam-outputs to a common adaptive filter, a coherent broadband beamforming method results. This approach has also been shown to be able to outperform existing coherent broadband methods in a scenario of interest via simulation.

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