Transmitter Optimization for Single Carrier and Multicarrier Transceivers on Crosstalk-Impaired ISI Channels

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Abstract

Transmitter optimization techniques for maximizing the throughput of linear ISI channels impaired by additive–Gaussian noise and crosstalk are presented. Transmitter ends of both single carrier and multicarrier transceiver structures are optimized subject to a fixed average input energy constraint. The effect of transmitter optimization on channel throughput is quantified by comparison with scenarios where both the desired user and the crosstalker use a flat energy distribution across the transmission bandwidth.

1 Introduction

Limited bandwidth resources in many spectrally–efficient digital communications systems often result in having multiple users, with the same transmission power spectral density characteristics, share the same frequency band and thus interfere with each other. Among the scenarios where this interference is performance limiting are *Co-channel Interference* (CCI) in digital cellular radio systems [1] and *crosstalk* (both near–end and far–end) in the emerging high-speed digital subscriber line (DSL) systems [2].

Effective signal processing techniques are implemented at the *receiver* to mitigate crosstalk such as ***decision-feedback equalization*** (DFE) in single–carrier modulation (SCM) systems [1] or *FFT processing* in multicarrier modulation (MCM) systems [2]. Full optimization of a communication system entails optimizing both the receiver ***and*** transmitter ends where the second task requires optimizing the transmission bandwidth and the power spectral density shape of the input signal. While transmitter optimization for multicarrier systems on noisy ISI channels with crosstalk has received considerable attention recently [3, 4, 5], this has not been the case for single–carrier systems where published studies either assume no crosstalk (see [3, 6] and the references therein) or an infinite–length transmit filter as in [7]. In addition, previous transmitter optimization studies for multicarrier systems are for the Discrete Multitone (DMT) implementation where channel spectrum partitioning is effected by using the IFFT/FFT modulating vectors and adding a cyclic prefix to the input block. The present author is not aware of any multicarrier transmitter optimization studies in the presence of crosstalk for Vector Coding (VC) multicarrier systems [8] where zero stuffing and optimum eigenvector–based modulating/demodulating vectors are used.

In this paper, we present a unified framework for optimizing the transmitter of ***finite-complexity*** single carrier and multicarrier modulation systems on linear ISI channels impaired by additive–Gaussian noise and crosstalk. The *performance metric* assumed for transmitter optimization is channel throughput (in bits/symbol) at a given symbol rate.

2 Single-Carrier Transmitter Optimization

2.1 Input-Output Model

We adopt the following discrete–time representation of an additive–noise dispersive channel impaired by crosstalk

$$\mathbf{y}_k = \sum_{m=0}^{\nu} \mathbf{h}_m x_{k-m} + \mathbf{n}_k + \sum_{i=0}^{\nu_x} \mathbf{g}_i \tilde{x}_{k-i}\,, \qquad (1)$$

where $\mathbf{h}_m \stackrel{def}{=} \begin{bmatrix} h_{l-1,m} & \cdots & h_{0,m} \end{bmatrix}^t$ and $\mathbf{g}_i \stackrel{def}{=} \begin{bmatrix} g_{l-1,i} & \cdots & g_{0,i} \end{bmatrix}^t$ are the m^{th} main channel and the i^{th} crosstalk channel (vector) impulse response coefficients having memories of ν and ν_x, respectively, and oversampled by a factor of l. We assume a continuous transmission bandwidth and perfect knowledge of the desired and crosstalker channel and the noise characteristics at the transmitter and receiver ends. The input sequence, $\{x_k\}$, the crosstalker sequence $\{\tilde{x}_k\}$, and the noise sequence, $\{\mathbf{n}_k\}$, are assumed to be stationary, zero-mean, independent of each other, and have non-singular auto-correlation matrices denoted by $\mathbf{R}_{xx}$, $\mathbf{R}_{\tilde{x}\tilde{x}}$, and $\mathbf{R}_{nn}$, respectively.

The input and crosstalker sequences are generated by the same FIR transmit filter according to

$$x_k = \sum_{n=0}^{\nu_t} p_n \epsilon_{k-n} \quad \text{and} \quad \tilde{x}_k = \sum_{n=0}^{\nu_t} p_n \eta_{k-n}, \qquad (2)$$

where $\{p_i\}_{i=0}^{\nu_t}$ are the transmit filter coefficients and $\{\epsilon_k\}$ and $\{\eta_k\}$ are white unit-energy sequences.

Over any block of N output symbols, Equation (1) can be expressed as follows

$$\begin{bmatrix} \mathbf{y}_{k+N-1} \\ \mathbf{y}_{k+N-2} \\ \vdots \\ \mathbf{y}_k \end{bmatrix} = \begin{bmatrix} \mathbf{n}_{k+N-1} \\ \mathbf{n}_{k+N-2} \\ \vdots \\ \mathbf{n}_k \end{bmatrix}$$

$$+ \begin{bmatrix} \mathbf{h}_0 & \mathbf{h}_1 & \cdots & \mathbf{h}_\nu & 0 & \cdots & 0 \\ 0 & \mathbf{h}_0 & \mathbf{h}_1 & \cdots & \mathbf{h}_\nu & 0 & \cdots \\ \vdots & & & & & & \vdots \\ 0 & \cdots & 0 & \mathbf{h}_0 & \mathbf{h}_1 & \cdots & \mathbf{h}_\nu \end{bmatrix} \begin{bmatrix} x_{k+N-1} \\ x_{k+N-2} \\ \vdots \\ x_{k-\nu} \end{bmatrix}$$

$$+ \begin{bmatrix} \mathbf{g}_0 & \mathbf{g}_1 & \cdots & \mathbf{g}_\nu & 0 & \cdots & 0 \\ 0 & \mathbf{g}_0 & \mathbf{g}_1 & \cdots & \mathbf{g}_\nu & 0 & \cdots \\ \vdots & & & & & & \vdots \\ 0 & \cdots & 0 & \mathbf{g}_0 & \mathbf{g}_1 & \cdots & \mathbf{g}_\nu \end{bmatrix} \begin{bmatrix} \tilde{x}_{k+N-1} \\ \tilde{x}_{k+N-2} \\ \vdots \\ \tilde{x}_{k-\nu_x} \end{bmatrix}$$

or more compactly :

$$\begin{aligned} \mathbf{y}_{k+N-1\,k} &= \mathbf{H}_{SCM}\mathbf{x}_{k+N-1\,k-\nu} + \mathbf{n}_{k+N-1\,k} & (3) \\ &+ \mathbf{G}_{SCM}\tilde{\mathbf{x}}_{k+N-1\,k-\nu_x} . & (4) \end{aligned}$$

Similarly, the vector representations of (2) are

$$\mathbf{x}_{k+N-1\,k-\nu} = \mathbf{P}\epsilon_{k+N-1\,k-\nu-\nu_t} \quad \text{and}$$

$$\tilde{\mathbf{x}}_{k+N-1\,k-\nu_x} = \tilde{\mathbf{P}}\eta_{k+N-1\,k-\nu_x-\nu_t} ,$$

where $\mathbf{P} = \tilde{\mathbf{P}}$ when $\nu = \nu_x$, otherwise, one will be a submatrix of the other.

It follows from (2.1) that $\mathbf{R}_{xx} = \mathbf{P}\mathbf{P}^*$ and $\mathbf{R}_{\tilde{x}\tilde{x}} = \tilde{\mathbf{P}}\tilde{\mathbf{P}}^*$ which guarantees that the input and crosstalker auto-correlation matrices are Hermitian positive semi-definite matrices whose size is *independent* of the transmit filter length $(\nu_t + 1)$. Perhaps less obvious is the fact that since $\mathbf{P}$ and $\tilde{\mathbf{P}}$ are *fully-windowed* Toeplitz matrices, $\mathbf{R}_{xx}$ and $\mathbf{R}_{\tilde{x}\tilde{x}}$ will also be Toeplitz with an (i,j) element equal to $\sum_{k=0}^{\nu_t} p_k p^*_{k+|j-i|}$. Therefore, $\mathbf{R}_{xx}$ will be a *submatrix* of $\mathbf{R}_{\tilde{x}\tilde{x}}$ for $\nu \leq \nu_x$ and vice versa.

2.2 Channel Throughput (Noise-Plus-Crosstalk Case)

In the presence of crosstalk, the total noise-plus-crosstalk auto-correlation matrix is given by

$$\mathbf{R}_{nn,tot} = \mathbf{R}_{nn} + \mathbf{G}_{SCM}\mathbf{R}_{\tilde{x}\tilde{x}}\mathbf{G}^*_{SCM} .$$

Assuming the crosstalker signal is Gaussian [1], it can be shown that the channel throughput is given by

$$\bar{\mathbf{I}}_{SCM} = \frac{\log_2 |\mathbf{I}_{N+\nu} + \mathbf{H}^*_{SCM}\mathbf{R}^{-1}_{nn,tot}\mathbf{H}_{SCM}\mathbf{R}_{xx}|}{(N+\nu)} . \qquad (5)$$

Then, the channel throughput optimization problem in the presence of crosstalk can be stated as follows

$$\max_{\{p_0, p_1, \ldots, p_{\nu_t}\}} \bar{\mathbf{I}}_{SCM} \quad \text{subject to} \quad \sum_{i=0}^{\nu_t} |p_i|^2 = 1 . \qquad (6)$$

The unit-energy transmit filter constraint is equivalent to $\frac{trace(\mathbf{R}_{xx})}{(N+\nu)} = 1$ or $\frac{trace(\mathbf{R}_{\tilde{x}\tilde{x}})}{(N+\nu_x)} = 1$. As a performance baseline, the channel throughput for the case of a flat transmit filter is given by

$$\bar{\mathbf{I}}_{flat} = \frac{\log_2 |\mathbf{I}_{N+\nu} + \mathbf{H}^*_{SCM}(\mathbf{R}_{nn} + \mathbf{G}_{SCM}\mathbf{G}^*_{SCM})^{-1}\mathbf{H}_{SCM}|}{(N+\nu)} . \qquad (7)$$

This is a constrained and nonlinear optimization problem that we solved numerically using the *constr* function of the *MATLAB Optimization Toolbox* [9] which implements the *Sequential Quadratic Programming* (SQP) algorithm. This algorithm solves the Kuhn-Tucker equations using an approximation of the Hessian of (5), computed using a quasi-Newton updating procedure to solve a quadratic programming subproblem and establish a search direction for a line search procedure.

3 Multicarrier Transmitter Optimization

3.1 Vector Coding Case

In vector coding, each input block of size N is padded with ν zeros to eliminate interblock interference, thus isolating successive transmitted blocks [8]. Therefore, the input-output relationship can be expressed in matrix form as follows

[1] It was shown in [7] that this assumption results in a lower bound on channel throughput when the crosstalker is not Gaussian.

$$\begin{bmatrix} \mathbf{y}_{k+N-1} \\ \mathbf{y}_{k+N-2} \\ \vdots \\ \mathbf{y}_k \\ \mathbf{y}_{k-1} \\ \vdots \\ \mathbf{y}_{k-\nu} \end{bmatrix} = \begin{bmatrix} \mathbf{h}_0 & \mathbf{0} & \cdots & \cdots & \mathbf{0} \\ \vdots & \ddots & \ddots & & \vdots \\ \mathbf{h}_\nu & \cdots & \mathbf{h}_0 & 0 & \vdots \\ 0 & \mathbf{h}_\nu & \cdots & \mathbf{h}_0 & \vdots \\ 0 & \cdots & \mathbf{h}_\nu & \cdots & \mathbf{h}_0 \\ \vdots & \cdots & & \ddots & \vdots \\ 0 & \cdots & \cdots & 0 & \mathbf{h}_\nu \end{bmatrix} \begin{bmatrix} \mathbf{x}_{k+N-1} \\ \mathbf{x}_{k+N-2} \\ \vdots \\ \mathbf{x}_k \end{bmatrix}$$

$$+ \begin{bmatrix} \mathbf{n}_{k+N-1} \\ \mathbf{n}_{k+N-2} \\ \vdots \\ \mathbf{n}_k \\ \mathbf{n}_{k-1} \\ \vdots \\ \mathbf{n}_{k-\nu} \end{bmatrix} + \begin{bmatrix} \mathbf{g}_0 & \mathbf{0} & \cdots & \cdots & \mathbf{0} \\ \vdots & \ddots & \ddots & & \vdots \\ \mathbf{g}_\nu & \cdots & \mathbf{g}_0 & 0 & \vdots \\ 0 & \mathbf{g}_\nu & \cdots & \mathbf{g}_0 & \vdots \\ 0 & \cdots & \mathbf{g}_\nu & \cdots & \mathbf{g}_0 \\ \vdots & \cdots & & \ddots & \vdots \\ 0 & \cdots & \cdots & 0 & \mathbf{g}_\nu \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{x}}_{k+N-1} \\ \tilde{\mathbf{x}}_{k+N-2} \\ \vdots \\ \tilde{\mathbf{x}}_k \end{bmatrix},$$

or more compactly

$$\mathbf{y}_{k+N-1\ k-\nu} = \mathbf{H}_{VC}\mathbf{x}_{k+N-1\ k} + \mathbf{n}_{k+N-1\ k-\nu} + \mathbf{G}_{VC}\tilde{\mathbf{x}}_{k+N-1\ k}\ .$$

The achievable channel throughput in this case is

$$\bar{\mathbf{I}}_{VC} = \frac{\log_2 |\mathbf{I}_N + \mathbf{H}^*_{VC}(\mathbf{R}_{nn} + \mathbf{G}_{VC}\mathbf{R}_{xx}\mathbf{G}^*_{VC})^{-1}\mathbf{H}_{VC}\mathbf{R}_{xx}|}{(N+\nu)}, \quad (8)$$

where the auto-correlation matrix of the input and crosstalk sequences are identical and both denoted by $\mathbf{R}_{xx}$. For VC-based MCM systems, $\mathbf{R}_{xx}$ is only constrained to be a Hermitian positive semi-definite matrix, hence, it admits the Cholesky factorization $\mathbf{R}_{xx} = \mathbf{L}_x\mathbf{L}_x^*$ where $\mathbf{L}_x$ is a lower-triangular matrix. The maximization of $\bar{\mathbf{I}}_{VC}$ is performed over the $\frac{N(N+1)}{2}$ elements of $\mathbf{L}_x$, subject to the constraint $trace(\mathbf{R}_{xx}) = (N+\nu)$. Once the optimum $\mathbf{R}_{xx}$ is determined, its eigen-decomposition $\mathbf{R}_{xx} = \mathbf{V}\mathbf{\Theta}\mathbf{V}^*$ can be computed. The non-zero elements of the diagonal matrix $\mathbf{\Theta}$ determine the subchannels that should be used for transmission and the corresponding columns of $\mathbf{V}$ are the optimum transmit filters for those subchannels [8].

3.2 Discrete Multitone Case

In DMT-based MCM systems, each length-N input block $\begin{bmatrix} x_{k+N-1} & \cdots & x_k \end{bmatrix}^t$ is cyclically extended to the block $\begin{bmatrix} x_{k+\nu-1} & \cdots & x_k & x_{k+N-1} & \cdots & x_k \end{bmatrix}^t$. Therefore, the input-output model can be cast in matrix form as follows

$$\begin{bmatrix} \mathbf{y}_{k+N-1} \\ \mathbf{y}_{k+N-2} \\ \vdots \\ \mathbf{y}_k \end{bmatrix} = \begin{bmatrix} \mathbf{h}_0 & \ddots & \mathbf{h}_\nu & \cdots & \mathbf{h}_1 \\ \ddots & \ddots & \ddots & \ddots & \cdots \\ \cdots & \mathbf{h}_1 & \mathbf{h}_0 & \mathbf{0} & \mathbf{h}_\nu \\ \mathbf{h}_\nu & \cdots & \mathbf{h}_1 & \mathbf{h}_0 & \mathbf{0} \\ \cdots & \mathbf{h}_\nu & \cdots & \mathbf{h}_1 & \mathbf{h}_0 \end{bmatrix} \begin{bmatrix} x_{k+N-1} \\ x_{k+N-2} \\ \vdots \\ x_k \end{bmatrix}$$

$$+ \begin{bmatrix} \mathbf{n}_{k+N-1} \\ \mathbf{n}_{k+N-2} \\ \vdots \\ \mathbf{n}_k \end{bmatrix} + \begin{bmatrix} \mathbf{g}_0 & \ddots & \mathbf{g}_\nu & \cdots & \mathbf{g}_1 \\ \ddots & \ddots & \ddots & \ddots & \cdots \\ \cdots & \mathbf{g}_1 & \mathbf{g}_0 & \mathbf{0} & \mathbf{g}_\nu \\ \mathbf{g}_\nu & \cdots & \mathbf{g}_1 & \mathbf{g}_0 & \mathbf{0} \\ \cdots & \mathbf{g}_\nu & \cdots & \mathbf{g}_1 & \mathbf{g}_0 \end{bmatrix} \begin{bmatrix} \tilde{x}_{k+N-1} \\ \tilde{x}_{k+N-2} \\ \vdots \\ \tilde{x}_k \end{bmatrix},$$

or more compactly

$$\mathbf{y}_{k+N-1\ k} = \mathbf{H}_{DMT}\mathbf{x}_{k+N-1\ k} + \mathbf{n}_{k+N-1\ k} + \mathbf{G}_{DMT}\tilde{\mathbf{x}}_{k+N-1:k}\ .$$

The achievable channel throughput of DMT is

$$\bar{\mathbf{I}}_{DMT} = \frac{\log_2 |\mathbf{I}_N + \mathbf{H}^*_{DMT}(\mathbf{R}_{nn} + \mathbf{G}_{DMT}\mathbf{R}_{xx}\mathbf{G}^*_{DMT})^{-1}\mathbf{H}_{DMT}\mathbf{R}_{xx}|}{(N+\nu)}, \quad (9)$$

where the insertion of the cylic prefix makes the $N \times N$ matrices $\mathbf{H}_{DMT}$, $\mathbf{G}_{DMT}$, and $\mathbf{R}_{xx}$ circulant. Therefore, they admit the following eigen-decompositions

$$\mathbf{H} = \mathbf{Q}\mathbf{\Sigma}\mathbf{Q}^*\ ;\ \mathbf{G} = \mathbf{Q}\mathbf{\Gamma}\mathbf{Q}^*\ ;\ \mathbf{R}_{xx} = \mathbf{Q}\mathbf{\Delta}\mathbf{Q}^*\ , \quad (10)$$

where $\mathbf{Q}$ is the orthogonal DFT matrix with (k,m) element equal to $\mathbf{Q}(k,m) = \frac{1}{\sqrt{N}}e^{-j\frac{2\pi(k-1)(m-1)}{N}}$ $1 \le k,m \le N$. In general, $\mathbf{R}_{nn}$ is not circulant, however, we consider here the case of white noise, i.e., $\mathbf{R}_{nn} = \sigma_n^2\mathbf{I}_N$. Substituting (10) in (9) and using the facts that $|\mathbf{I}+\mathbf{AB}| = |\mathbf{I}+\mathbf{BA}|$, $\mathbf{Q}^*\mathbf{Q} = \mathbf{Q}\mathbf{Q}^* = \mathbf{I}_N$, and that diagonal matrices commute, we get

$$\begin{aligned} \bar{\mathbf{I}}_{DMT} &= \frac{1}{(N+\nu)}\log_2 |\mathbf{I}_N + \mathbf{\Sigma}^2\mathbf{\Delta}(\sigma_n^2\mathbf{I}_N + \mathbf{\Gamma}^2\mathbf{\Delta})^{-1}| \\ &= \frac{1}{N+\nu}\sum_{i=1}^{N}\log_2(1 + \frac{\sigma_i^2\delta_i}{(\sigma_n^2 + \gamma_i^2\delta_i)})\ . \end{aligned}$$

Using the Lagrange multipliers optimization technique, it can be shown that the optimum *input energy distribution* is given by

$$\delta_i = \max(\frac{-B_i + \sqrt{B_i^2 - 4A_iC_i}}{2A_i}, 0)$$

$$A_i = \gamma_i^2(\gamma_i^2 + \sigma_i^2)\ ;\ B_i = \sigma_n^2(2\gamma_i^2 + \sigma_i^2)\ ;\ C_i = \sigma_n^2(\sigma_n^2 - \lambda\sigma_i^2)\ ,$$

and λ satisfies $trace(\mathbf{R}_{xx}) = \sum_{i=1}^{N}\delta_i = N$. [2]

4 Simulation Results

In Figures 1 and 2, we consider the main channel $h(D) = \frac{1+\alpha D}{\sqrt{1+|\alpha|^2}}$ and the crosstalk channel $g(D) = \frac{1+\beta D}{\sqrt{SIR(1+|\beta|^2)}}$ and examine the variation of channel throughput gain with α (for $-1 \le \alpha \le 1$), β (for $-1 \le \beta \le 1$), and input SNR level. The highest throughput gain is achieved when

[2] Due to the extra energy required to transmit the cyclic prefix, the input energy constraint for DMT is $\frac{(N+\nu)}{N}trace(\mathbf{R}_{xx}) = N+\nu$ or equivalently $trace(\mathbf{R}_{xx}) = N$.

$\alpha = 1$ and $\beta = -1$ in which case the main channel has *low-pass* characteristics and the crosstalk channel has *high-pass* characteristics (or vice versa). Therefore, by having a *low-pass* characteristic, the optimum transmit filter can simultaneously enhance the desired signal and attenuate the crosstalker. This throughput gain becomes even higher as input SNR is increased from 10 to 20 dB at a fixed SIR level of 10 dB.

Next, we examined the effect of transmitter optimization on the throughput of MCM systems. A representative result is depicted in Figure 3. As expected, the throughput gain of VC over DMT decreases as the blocklength increases since the cyclic prefix overhead becomes negligible and the DMT subchannels approach the ideal memoryless characteristics.

References

[1] N. Lo, D.D.Falconer, and A. Sheikh. "Adaptive Equalization for Co-Channel Interference in a Multipath Fading Environment". *IEEE Transactions on Communications*, pages 1441–1453, Feb./March/April 1995.

[2] J. Chow, J. Tu, and J.M.Cioffi. "A Discrete Multitone Transceiver System for HDSL Applications". *IEEE Journal on Selected Areas in Communications*, 9(6):895–908, August 1991.

[3] J.M.Cioffi and G. Dudevoir and M.V. Eyuboglu and G.D. Forney, Jr. "Minimum Mean-Square-Error Decision Feedback Equalization and Coding - Parts I and II". *IEEE Transactions on Communications*, October 1995.

[4] J. Aslanis and J.M.Cioffi. "Achievable Information Rates on Digital Subscriber Loops: Limiting Information Rates with Crosstalk Noise". *IEEE Transactions on Communications*, pages 361–372, February 1992.

[5] P. Chow, J.M.Cioffi, and J. Bingham. "A Practical Discrete Multitone Transceiver Loading Algorithm for Data Transmission over Spectrally Shaped Channels". *IEEE Transactions on Communications*, pages 773–775, Feb/Mar/April 1995.

[6] N. Al-Dhahir and J.M.Cioffi. "Block Transmission over Dispersive Channels : Transmit Filter Optimization and Realization, and MMSE-DFE Receiver Performance". *IEEE Trans. on Information Theory*, pages 137–160, January 1996.

[7] I. Kalet and S. Shamai (Shitz). "On the Capacity of a Twisted-Wire Pair : Gaussian Model". *IEEE Transactions on Communications*, COM-38(3):379–383, March 1990.

[8] S. Kasturia, J. Aslanis, and J.M. Cioffi. "Vector Coding for Partial-Response Channels". *IEEE Transactions on Information Theory*, 36(4):741–762, July 1990.

[9] Optimization Toolbox Manual, MATLAB Software Package Version 5.0. The MathWorks, Inc., 1996.

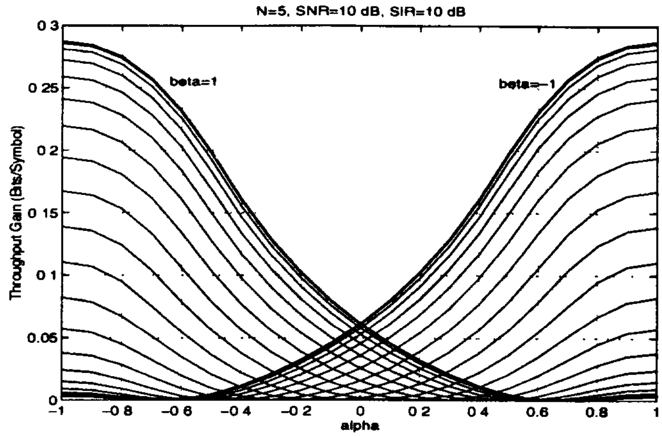

Figure 1: Channel throughput variation versus main and crosstalk channel characteristics with optimized FIR transmit filter

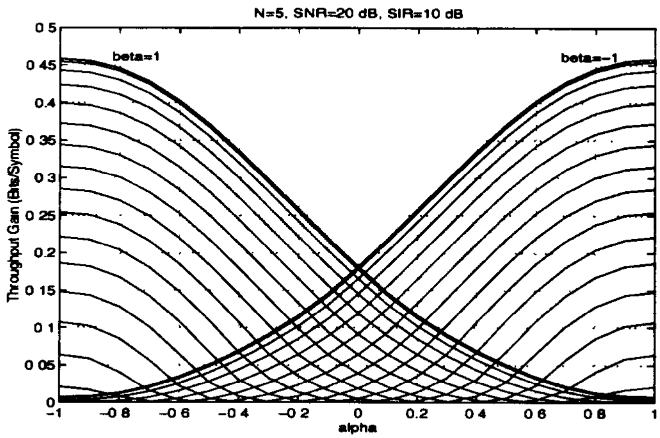

Figure 2: Channel throughput variation versus main and crosstalk channel characteristics with optimized FIR transmit filter

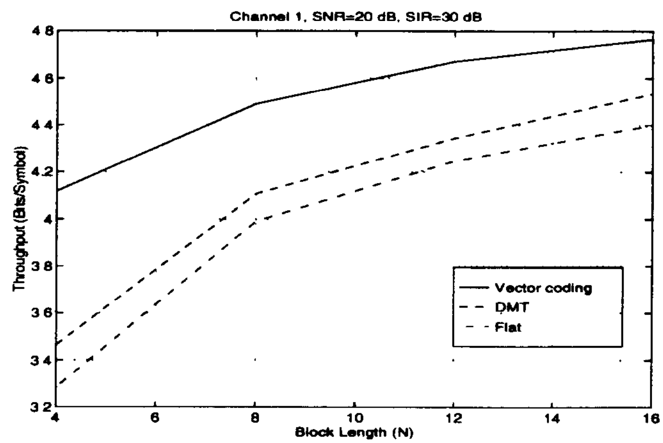

Figure 3: Channel throughput variation of VC, DMT, and flat multicarrier transmission schemes with blocklength