CONSTRAINED MINIMUM-BER MULTIUSER DETECTION

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Abstract

A new linear multiuser detector for binary signaling in code-division multiple-access communication systems is described. The new detector directly minimizes the bit-error rate (BER) subject to a set of reasonable constraints. It is shown that any local minimum of the constrained BER cost function is a global minimum; hence a robust constrained minimization algorithm always leads to a detector with good performance. Although the proposed detector cannot be shown to be optimal among linear multiuser detectors because of the constraints imposed, our analysis and simulations indicate that it always outperforms the decorrelating detector and is optimal for most realistic systems.

I. Introduction

The capacity of direct-sequence code-division multiple-access (DS-CDMA) systems is primarily limited by the near-far problem. This has motivated considerable effort to develop near-far resistant multiuser detectors. Linear multiuser detectors such as the decorrelating detector [1-2] and the minimum mean-squared error (MMSE) detector [3] are among the most popular due to a number of advantages. As indicated by their names, the decorrelating detector and the MMSE detector minimize multiple-access interference (MAI) and mean-squared error, respectively. Since these criteria are not directly related to the BER, it is of great interest to develop a new linear multiuser detector that minimizes BER. In fact, an *approximate* minimum BER criterion has been recently proposed for combating intersymbol interference in single-user communication systems [4]. It was shown that even an approximate minimum BER criterion can yield significant performance gain over the conventional zero-forcing and MMSE criteria. A similar improvement should also be possible in the case of multiuser communication systems.

In this paper, we study the minimum BER criterion as applied to linear multiuser detection for binary signaling and its biorthogonal extensions in DS-CDMA communication systems. Our attention is focused on base stations where information about the signature signals, timing, and received amplitudes of all active users is available or can be accurately estimated. Hence the linear multiuser detector which minimizes BER can be designed prior to its application. It is important to stress that the BER function is highly nonlinear and, as a result, convergence to the global minimum cannot be guaranteed in general. This might be the main reason why the minimum BER criterion did not receive significant attention in the existing literature. In order to avoid local minima, we propose a *constrained* minimum-BER (CMBER) multiuser detector that minimizes the BER cost function subject to a set of convex constraints. Global convergence of such a constrained optimization problem is guaranteed. The resulting detector always achieves better performance than the decorrelating detector. Even though the proposed detector cannot be shown to be optimal among linear detectors because of the constraints imposed, our simulations indicates that it is optimal for most realistic systems.

II. Preliminaries

We consider binary-phase-shift-keying (BPSK) transmission through an additive white Gaussian noise (AWGN) channel in a DS-CDMA system. For synchronous systems, user detection can be performed symbol by symbol. For asynchronous systems, a window approach can be adopted that results in a symbol-by-symbol detection [5][6]. In such a case, each symbol within the observation window, which usually spans an odd number of symbol intervals [5], is deemed to originate from an individual imaginary user and only the symbols in the central symbol interval of the window are detected during the processing of one observation window. Consequently, if there exist m asynchronous users in a system and the observation window spans p symbol intervals, then there are $(p+1)m-1$ imagi-

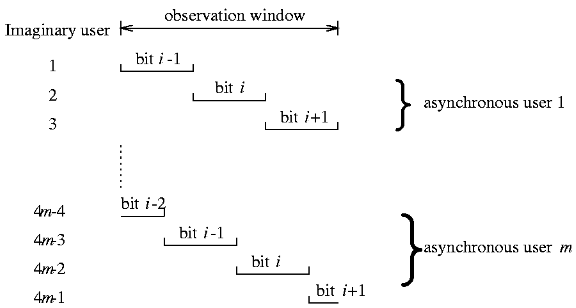

Figure 1: Equivalent synchronous transmission of m asynchronous users seen by user 1 when the observation window spans 3 symbol intervals.

nary synchronous users, as illustrated in Fig. 1. Assume that there are K (imaginary) synchronous users and denote the information bit of the ith user and its amplitude as b_i and A_i, respectively. Within the observation window, the critically sampled version of the received baseband signal $\mathbf{r}$ can be expressed as

$$\mathbf{r} = \mathbf{S}\mathbf{b} + \mathbf{n} \tag{1}$$

where

$$\begin{aligned} \mathbf{r} &= [r_1 \ r_2 \ \cdots \ r_N]^T \\ \mathbf{b} &= [b_1 \ b_2 \ \cdots \ b_K]^T \\ \mathbf{S} &= [A_1\mathbf{s}_1 \ A_2\mathbf{s}_2 \ \cdots \ A_K\mathbf{s}_K] \end{aligned}$$

In (1), $\mathbf{n}$ is an AWGN signal with zero mean and variance σ^2, and $\mathbf{s}_i$ is the signature signal of the ith (imaginary) synchronous user. For asynchronous systems, if the original signature signal of the real user, who transmitted the ith information bit, is $\hat{\mathbf{s}}$, then from Fig. 1 $\mathbf{s}_i$ will be of the form $[0 \ \cdots \ 0 \ \hat{\mathbf{s}} \ 0 \ \cdots \ 0]$.

A linear multiuser detector can be viewed as a linear filter followed by a sampler which samples the output of the filter at $t = nT$, where T is the duration of the symbol interval. The decorrelating detector seeks to completely eliminate MAI regardless of the presence of background noise. This so-called zero-forcing (ZF) solution can be achieved by employing a receiving filter with the coefficient vector

$$\mathbf{c}_d = \mathbf{S}\mathbf{R}^{-1}\mathbf{e}_k \tag{2}$$

where $\mathbf{R} = \mathbf{S}^T\mathbf{S}$ is the cross-correlation matrix among signature signals, $\mathbf{e}_k$ is the kth coordinate vector, and k is the index of the desired user. Note that we have assumed that $\mathbf{R}$ is positive definite, as is usually the case in practice [1]; otherwise, a nearly ZF solution can be achieved by replacing the $\mathbf{S}\mathbf{R}^{-1}$ in (2) by the Moore-Penrose psuedoinverse of $\mathbf{S}^T$.

In contrast to the decorrelating detector, the MMSE detector seeks to minimize the mean-squared error or, equivalently, maximize the signal-to-interference-plus-noise ratio. The coefficient vector of the MMSE detector is given by

$$\mathbf{c}_m = \mathbf{S}(\mathbf{R} + \sigma^2)^{-1}\mathbf{e}_k \tag{3}$$

The above two linear detectors both achieve the optimal near-far resistance and both provide significant performance gain as compared with the conventional matched-filter receiver [1-3]. However, since their decision criteria are not directly related to the BER, the possibility to develop an improved linear detector that directly minimizes the BER exists, as will be discussed in the rest of the paper.

III. Constrained Minimum-BER Multiuser Detection

Consider the BER performance of a linear multiuser receiver with coefficient vector $\mathbf{c}$ for a multiuser channel and assume that the two binary values of the signal, $\{\pm 1\}$, are equally likely. The BER of the kth user can be readily found as

$$P(\mathbf{c}) = \frac{1}{2^{K-1}} \sum_{i=1}^{2^{K-1}} Q\left(\frac{\mathbf{c}^T\mathbf{v}_i}{||\mathbf{c}||\sigma}\right) \tag{4}$$

where $\mathbf{v}_i = \mathbf{S}\hat{\mathbf{b}}_i$, $\hat{\mathbf{b}}_i$ for $1 \le i \le 2^{K-1}$ is a possible information vector given that its kth entry is $b_k = 1$, and

$$Q(x) = \int_x^\infty e^{-v^2/2} dv$$

Since the BER cost function (4) of $\mathbf{c}$ depends only on the direction, the existence of a global minimum of $P(\mathbf{c})$ is obvious. The detector whose coefficient vector $\mathbf{c}^*$ minimizes (4) is optimal among linear detectors and will be referred to as the optimal linear detector. However, unlike the case of $\mathbf{c}_d$ and $\mathbf{c}_m$, there is no closed-form expression for $\mathbf{c}^*$. Furthermore, since the BER function is highly nonlinear and there may exist more than one local minimum, convergence to $\mathbf{c}^*$ cannot generally be guaranteed for most optimization algorithms. A detailed interpretation of the minima of the BER cost function can be found in [4].

The following proposition will be useful in the subsequent analysis.

Propostion 1: Any local minimum point of the BER cost function in (4) subject to

$$\mathbf{c}^T\mathbf{v}_i \ge 0 \quad \text{for } 1 \le i \le 2^{K-1} \tag{5}$$

is a global minimum point.

Proof: Since the BER cost function is independent of the length of $\mathbf{c}$, it is sufficient to consider minimizing $P(\mathbf{c})$ on the set

$$I = \{\mathbf{c} : \ ||\mathbf{c}|| = 1, \ \mathbf{c} \text{ satisfies } (5)\} \quad (6)$$

Let the global minimum point of the above constrained minimization problem be $\mathbf{c}_1 \in I$ and assume that there exists another local minimum point $\mathbf{c}_2 \in I$ such that

$$P(\mathbf{c}_1) < P(\mathbf{c}_2) \quad (7)$$

Let $\alpha < 1$ be a positive constant and

$$\mathbf{c} = \frac{\alpha\mathbf{c}_1 + (1-\alpha)\mathbf{c}_2}{||\alpha\mathbf{c}_1 + (1-\alpha)\mathbf{c}_2||} \quad (8)$$

It follows from (8) that $\mathbf{c} \in I$. Since $||\alpha\mathbf{c}_1+(1-\alpha)\mathbf{c}_2|| < 1$, we have

$$\mathbf{c}^T\mathbf{v}_i > \alpha\mathbf{c}_1^T\mathbf{v}_i + (1-\alpha)\mathbf{c}_2^T\mathbf{v}_i \quad (9)$$

Hence

$$Q\left(\frac{\mathbf{c}^T\mathbf{v}_i}{\sigma}\right) < Q\left[\frac{\alpha\mathbf{c}_1^T\mathbf{v}_i + (1-\alpha)\mathbf{c}_2^T\mathbf{v}_i}{\sigma}\right] \quad (10)$$

Using the fact that $Q(x)$ for $x \geq 0$ is a convex function, from (4) and (10), we have

$$\begin{aligned} P(\mathbf{c}) &< \alpha P(\mathbf{c}_1) + (1-\alpha)P(\mathbf{c}_2) \\ &< P(\mathbf{c}_2) \qquad \text{for all } \alpha \in (0,1) \end{aligned} \quad (11)$$

This contradicts the fact that $\mathbf{c}_2$ is a local minimum point and hence the proposition follows. △

Note that in the above proposition, we have assumed that the set I defined by (6) is not empty. This assumption is true for most practical systems as stated in the following proposition.

Proposition 2: If the (imaginary) signature signals $\{\mathbf{s}_i, 1 \leq i \leq K\}$ are linearly independent of each other, then there always exists an infinite number of elements in I.

Proof: It is easy to show that if $\{\mathbf{s}_i, 1 \leq i \leq K\}$ are linearly independent, then the cross-correlation matrix $\mathbf{R}$ is positive definite and the ZF solution can be achieved. Consequently, we have

$$\frac{\mathbf{c}_d^T\mathbf{v}_i}{||\mathbf{c}_d||} = \frac{1}{||\mathbf{c}_d||} \quad \text{for } 1 \leq i \leq 2^{K-1} \quad (12)$$

The validity of the proposition follows immediately. △

The CMBER multiuser detector is defined as the detector whose coefficient vector is the global minimum point of $P(\mathbf{c})$ in (4) subject to constraints in (5). From Proposition 2, once the ZF solution can be achieved, the CMBER detector exists and outperforms the decorrelating detector. Several remarks are now in order.

a) Since the constraints given by (5) are convex, efficient constrained optimization algorithms, such as *sequential quadratic programming* [7], can be used to solve this problem and global convergence is guaranteed.

b) In our simulations, it was found that for systems where the signal-to-noise ratio (SNR) at the output of the decorrelating detector is not very low (e.g., the SNR is greater than about 0 dB), the CMBER multiuser detector is always optimal among linear detectors. An intuitive explanation is as follows: Since a detector with coefficient vector $\mathbf{c}$ that does not satisfy (5) would usually yield a poorer BER than that of the decorrelating detector, the global minimum point most likely satisfies (5). If so, from Proposition 1, this global minimum point is exactly the coefficient vector of the CMBER detector.

c) The above results can be readily extended to the case of channel equalization for single-user communication systems. Even though an exact ZF solution is not always achievable with a finite-length linear equalizer, the channel is still *equalizable* for most cases. In other words, set I is most likely not empty and a constrained minimum-BER equalizer exists.

IV. Numerical Examples

We first compare the performance of the CMBER multiuser detector and that of the optimal linear multiuser detector. Shown in Fig. 2 are the BER curves of different detectors for a system with 10 equal-power users. The optimal linear detector was taken as the best solution of 40 runs of a quasi-Newton optimization algorithm. In order to make it more likely that the CMBER detector differs from the optimal linear multiuser detector, the user signature signals were randomly selected. As can be seen in Fig. 2, the BER curve of the CMBER detector and that of the optimal linear detector are indistinguishable. In fact, these two detectors are exactly equivalent when the SNR is greater than −3 dB. It is also clear that the MMSE detector also achieves very similar performance to that of the optimal linear detector, while the decorrelating detector has a significantly poorer performance.

We now consider some more practical situations where the SNR is greater than 0 dB. Figs. 3 and 4 show the BER curves for 10-user systems with randomly generated signature signals and the 31-chip Gold codes, respectively. The cross-correlation properties among signature signals for Fig. 3 are poorer than those for Fig. 4. The power of the desired user was set to 0 dB and the powers of interfering users were randomly generated in the interval $(0, 10)$ dB. As can be observed, the CMBER detector always outperforms the decorrelating detector and the MMSE detector. In the case of relatively poor cross-correlation properties, the performance gain of the CMBER detector over those of the decorrelating detector and the MMSE detector is significant; while in the case of good cross-correlation properties, the three BER curves are relatively close to each other. Note that when the SNR is greater than 0 dB, the CMBER detector is optimal.

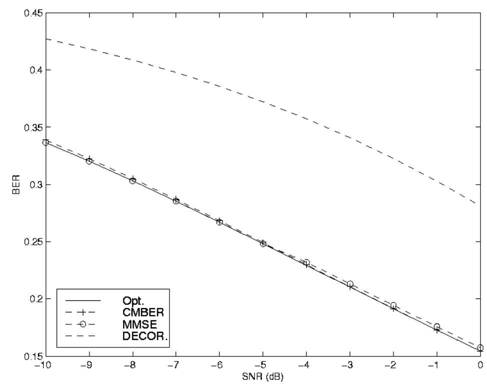

Figure 2: Performance comparison of the linear multiuser detectors: 10 equal-power users.

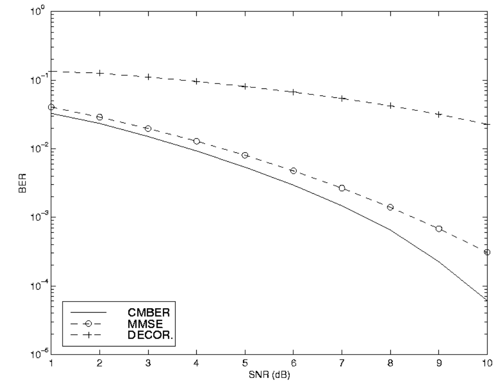

Figure 3: Performance comparison of linear multiuser detectors: randomly generated signature signals.

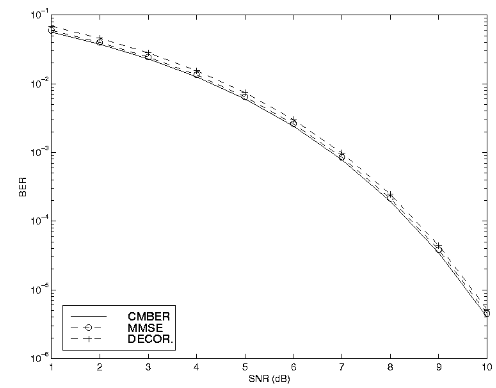

Figure 4: Performance comparison of linear multiuser detectors: 31-chip Gold codes.

V. Conclusions

We have studied the minimum BER criterion as applied to multiuser detection, and proposed a constrained minimum-BER multiuser detector. In contrast to the case of the optimum linear multiuser detector, global convergence is guaranteed when a robust constrained optimization algorithm such as the SQP is used. Analysis and numerical examples have shown that the proposed detector is optimal or nearly optimal for most realistic systems and always achieves better performance than the decorrelating detector. The proposed multiuser detector can be readily extended to equalization for ISI single-user channels.

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VI. References

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