AN ENHANCED TEA ALGORITHM FOR MODAL ANALYSIS

Gabriella Olmo, Letizia Lo Presti and Paolo Severico

Dipartimento di Elettronica, Politecnico di Torino
Corso Duca degli Abruzzi 24, 10129 Torino, Italy
Tel. +39 011 5644033, fax. +39 011 5644099
e@mail olmo@polito.it, lopresti@polito.it

ABSTRACT

Turbo Estimation Algorithms (TEAs) for non random parameters are able to yield high accuracy estimates by means of an iterative process. At each iteration, a noise reduction is performed by means of an *External Denoising System* (EDS), which exploits the estimation results obtained at the previous step; the enhanced data are then input to the master *Estimation Algorithm* (EA) for next iteration. Recently, a basic TEA scheme has been proposed in the context of modal analysis, which makes use of the Tufts and Kumaresan (TK) algorithm as the master EA, and of a multiband IIR filter as the EDS. In this paper, two improvements of this basic scheme are proposed; the former implies a different design of the EDS, able to achieve better estimation accuracy while reducing the outlier probability; the latter permits the autodetermination of the number of modes making up the signal.

1. INTRODUCTION

A new class of parameter estimation algorithms (EA), called *Turbo Estimation Algorithms* (TEA), has been recently introduced [1]. The basic idea is that each EA must perform a sort of intrinsic denoising of the input data in order to achieve reliable estimates. Optimum algorithms implement the best possible noise reduction, compatible with the problem definition, and reach the lower bound of the estimation error variance; however, their computational complexity is often overwhelming, so that in real life one must frequently resort to suboptimal algorithms; in this case, some amount of residual noise which impairs the estimation process could be still eliminated. The TEA methods reduce the residual noise by means of a closed loop configuration, reported in Fig. 1, in which an external denoising system (EDS), fed by the master EA output, processes the input data so as to reduce the residual noise generating an enhanced signal to be input to the EA for next iteration. The *Convergence Check* (CC) module controls the iterative procedure termination on the basis of a predetermined convergence criterion. The working principle of such schemes can be described in terms of a more general *turbo principle* [2], well known in an information theory context.

In [1], a TEA for modal analysis is described, which employs the Tufts and Kumaresan (TK) method [3, 4, 5] as a master EA. The described algorithm, called *TK Algorithm with Iterative Filtering* (TKIF), is able either to achieve the same performance of the simple TK, with reduced computational complexity, or to improve the TK performance, also in quite severe noise conditions, maintaining the same computational complexity. In this paper, two improvements of the basic TKIF scheme are proposed. The former implies a different design of the EDS, able to achieve better estimation accuracy while reducing the outlier probability so making possible the realization of high resolution frequency estimation; the latter permits the autodetermination of the number of modes making up the signal.

2. THE MODAL ANALYSIS PROBLEM AND THE TK ALGORITHM WITH ITERATIVE FILTERING (TKIF)

The modal analysis deals with signals represented by the sum of undamped sinusoidal components or *modes*, whose parameters are deterministic but unknown, and a random noise. The algorithms must estimate (some or all) the mode parameters, based on a set of measured data, which can be modeled as

$$y[n] = \sum_{m=1}^{M} A_m \cos(2\pi f_m n + \varphi_m) + w[n] \qquad (1)$$

where M is the number of modes which make up the signal (often considered as known *a priori*), A_m, f_m, and φ_m are respectively the amplitude, digital normalized frequency and initial phase of the m-th sinusoidal mode, and $w[n]$ is a realization of a noise process $W[n]$, which is generally assumed to be white Gaussian with zero mean and variance σ_w^2.

The TKIF algorithm is conceived as a development of the basic Tufts and Kumaresan method [3, 4, 5] for the mode frequency estimation. In this section, the TKIF algorithm is briefly described; more details and the theoretical principle of its operation are reported in [1]. The TKIF scheme is shown in Fig. 2.

The block labeled $H_k(z)$, where $k \geq 0$ is the index of an iterative process, is a digital multiband filter performing a noise suppression which, for each iteration $k \geq 1$, benefits from the knowledge of the frequency vector estimated at the previous step, $\hat{\mathbf{f}}_{k-1}$.

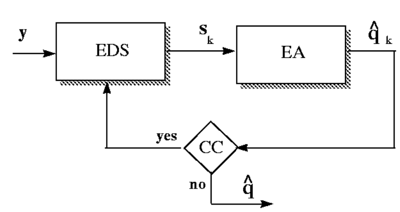

Figure 1: General scheme of a Turbo Estimation Algorithm

In the general TEA context, this filter acts as the EDS block. The system inside the dotted box represents the standard TK scheme, which acts as the master EA; it encompasses the *Singual Value Decomposition* (SVD) module, the largest singular values selection block, which can be interpreted as an intrinsic denoising process [1], and the mininum norm and polynomial root selection modules, which yield the k-th iteration vector frequency estimate $\hat{\mathbf{f}}_k$. The convergence criterion is based on a threshold comparison of the *difference vector* $\hat{\mathbf{d}}_k = |\hat{\mathbf{f}}_k - \hat{\mathbf{f}}_{k-1}|$.

At the initialization step $k = 0$, the filtering is omitted ($H_0(z) = 1, \mathbf{s}_0 = \mathbf{y}$) and a rough pre-estimate of $\mathbf{f}$, $\hat{\mathbf{f}}_0$, is obtained using the TK algorithm with relatively small values of N and L, where N is the number of points used by the TK basic algoritms, and L is the Prony filter order [3]. Then, at each step of subsequent iteration, the IIR multiband filter is designed, exploiting the knowledge of the previous cycle frequency estimates $\hat{\mathbf{f}}_{k-1}$, and $\mathbf{y}$ is passed through the filter $H_k(z)$, giving rise to $\mathbf{s}_k$. At this point, $\mathbf{s}_k$ is processed by the TK algorithm and a new frequency estimate $\hat{\mathbf{f}}_k$ is worked out. The procedure stops at the K-th iteration according to the preassigned convergence criterion, and the last iteration yields the final estimated frequency vector $\hat{\mathbf{f}}_K = \hat{\mathbf{f}}$. The key feature of TKIF lies in its capability of progressively *refining* the estimates $\hat{\mathbf{f}}_k$, thanks to the noise reduction performed on the measured data $\mathbf{y}$ by the filtering operation.

In [1], $H_k(z)$ is implemented as an IIR filter

$$H_k(z) = \frac{\prod_{i=1}^{N_w}(z - w_i)}{\prod_{m=1}^{N_p}(z - p_m^{(k)})} \tag{2}$$

where N_w and N_p are respectively the numbers of zeros w_i and poles $p_m^{(k)}$ of $H_k(z)$. For the sake of simplicity, only the pole locations $p_m^{(k)}$ are updated at each iteration. In the actual implementation, this filter is also subject to an amplitude equalization in order not to privilege any frequency with respect to the others.

The design of the EDS boils down in the selection of parameters N_p, and N_w, of the zeros w_i and initial poles $p_m^{(0)}$ (all these operations are made once and for all) and in the definition of a procedure of pole location updating. In the simplest solution addressed in [1], the filter $H_k(z)$ exhibits M pairs of complex conjugate poles, where the number M of modes is assumed to be known. The pole locations are defined as $p_m^{(k)} = m_p \cdot \mathrm{e}^{j2\pi \hat{f}_{m,k-1}}$, $1 \leq i \leq M$; their magnitude m_p is independent of k, and $\hat{f}_{m,k-1}$ are the estimated frequencies at iteration $(k-1)$ (i.e. the components of the estimated vector $\hat{\mathbf{f}}_{k-1}$). As for the numerator of $H_k(z)$, it has been experimentally verified that a proper choice is to introduce two transmission zeros at the extreme frequencies of the domain, that is at $f = 0$ ($w_1 = 1$) and $f = 0.5$ ($w_2 = -1$). The main reason for this choice lies in the fact that the noise components close to $f = 0$ and $f = \frac{1}{2}$ are *a priori* eliminated without affecting the significant signal components; moreover, in the presence of these transmission zeros, the overall probability of outlier gets sensibly reduced [1].

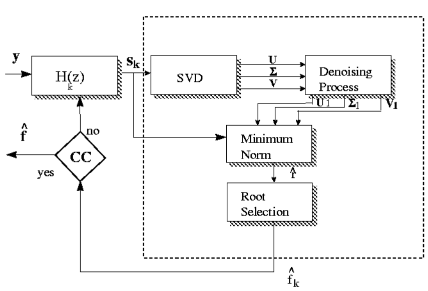

Figure 2: Scheme of the TKIF algorithm

The value m_p of the pole magnitude could be arbitrarily chosen in the range $[0, 1[$, excluding the upper extreme to guarantee stability. However, the choice of this parameter is crucial as for the TKIF method performance, as it affects the filter frequency selectivity, and consequently its effectiveness in rejecting the residual noise. A natural choice would be to select values of m_p very close to unity. However this choice presents two serious drawbacks:

- A high frequency selectivity involves that the filter impulse response time decay is slow; this induces a long transient in the filtered signal $\mathbf{s}_k$. As the TK estimation algorithm must be applied only to the steady-state signal, a long transient leads to the need for extra measured data $y[n]$, in order to accommodate both the transient (to be skipped) and the steady-state data to be processed by the EA. However, it must be remarked that this number of extra data little affect the computational complexity, as they are only used in the filtering process and do not enter the SVD module, which virtually determines the TEA algorithm complexity [1];
- it has been experimentally verified that the filter selectivity is correlated with the overall outlier probability. This can be intuitively explained noticing that, when the pre-estimated frequencies are imprecise (due either to an outlier occurrence in the initialization EA or to simple estimation variance), a very selective filtering around the pre-estimated frequencies could completely cut off some of the true signal components, forcing an outlier in the overall estimation process. On the other hand, if less selective filters are employed, an initial outlier can be recovered by means of the noise suppression mechanism.

As a consequence of these facts, a moderately high value of m_p must be selected. Simulation results have shown that the probability of outlier steeply increases when m_p exceeds 0.99; therefore, values in the range $[0.97, 0.99]$ have been considered.

3. MODIFIED TKIF ALGORITHM FOR HIGH ACCURACY ESTIMATION

In this section a modification of the TKIF algorithm is proposed, able to appreciably improve the estimation accuracy and control the probability of outlier, with a negligible increase of the complexity. It is based on the following considerations:

- When the value of m_p is high, say in the range $[0.97, 0.99]$, the EDS filter $H_k(z)$ is very selective, and the estimation accuracy is very high; however, the TKIF is subject to a non negligible probability of outlier if the SNR is low and the EA yielding the pre-estimate operates below or close to the outlier threshold.
- When the value of m_p is low, say in the range $[0.90, 0.96]$, the denoising filter is less selective, and the estimation accuracy can be worse, but the probability of outlier is reduced.

Table 1: Comparison between TKIF and modified TKIF in terms of the individual estimation error variance; three mode signal

	$\sigma^2_{1,K}$	$\sigma^2_{2,K}$	$\sigma^2_{3,K}$
TKIF	$6.64 \cdot 10^{-8}$	$6.92 \cdot 10^{-6}$	$2.50 \cdot 10^{-6}$
Modified TKIF	$6.39 \cdot 10^{-8}$	$6.96 \cdot 10^{-8}$	$5.56 \cdot 10^{-8}$

Therefore, a straightforward solution to improve the TKIF performance is to modify (namely, to augment) the value of m_p at each iteration. In this way, the first iterations with a relatively low value of m_p reduces the risk that an outlier occurs, while the last iterations with high values of m_p refine the estimation, appreciably reducing the absolute error and the variance. The convergence towards a new settling point could be slower than in the original TKIF algorithm, requiring a larger number of iterations, but an improved accuracy is reached.

3.1. Experimental results

The proposed method has been tested with m_p values linearly increasing in the range $[0.9, 0.99]$ with steps of 0.03. The test signal is made of three sinusoids with frequencies $\mathbf{f} = [0.1,\ 0.2,\ 0.4]$, initial phases $\varphi = [0.1,\ 0.2,\ 0.1]$, amplitudes $\mathbf{A} = [1,\ 1,\ 1]$, and with SNR=5 dB.

In Table 1 the performance of TKIF and modified TKIF are compared in terms of the individual estimation error variances. The algorithm parameters are $N = 50$ and $L = 9$. The simple TKIF algorithm is processed with the fixed value $m = 0.98$; for both algorithms, the number of iterations is $K = 4$.

It can be noticed that the modified TKIF method achieves an appreciable reduction of the estimation error variances (which approach the Cramér-Rao bound, of the order of 10^{-9} for all the three modes) without affecting the computational load. In fact, this latter practically depends only on the number of data points entering the SVD module, and therefore is the same for the two algorithms. The only drawback of the modified TKIF algorithm is the need for extra data necessary to cope with the long transient of the more selective filters; however, the filtering process little affects the algorithm complexity [1]do not influence the algorithm complexity.

4. TKIF WITH AUTODETERMINATION OF THE NUMBER OF MODES

A limitation in the methods deriving from the Prony family is that the number of modes, M, must be known *a priori*. Many algorithms have been developed to determine M and some of these are based on the singular value decomposition [6]. However, these methods do not work properly if the modes are characterized by different amplitudes, and their performance is heavily affected by a critical threshold parameter [7]. In this section we discuss an improvement of the TKIF method, which is able to recognize automatically the number of modes and overcomes the mentioned drawbacks.

Let us consider a signal made of the superimposition of M modes, with M unknown, plus additive noise, and let us assume that the TKIF algorithm for frequency estimation is applied introducing and evaluating the frequencies of a redundant number of modes $M_1 > M$. If the algorithm parameters are properly chosen, the set of M_1 estimated frequencies will certainly contain M values very close to the true mode frequencies, and $(M_1 - M)$ spurious values or *false frequencies*, due to the noise and randomly distributed along the whole frequency axis. A possible way to identify the correct modes within the redundant set of estimated frequencies is to partition the input sequence into a number N_b of disjointed blocks, and to run the TKIF algorithm on each block, with redundant number of modes M_1. The segmentation guarantees that, under the assumption of white gaussian noise, the estimates obtained from each block are statistically independent of each other. From the comparison among the N_b vectors of M_1 estimated frequencies, it is possible to identify M components of each vector (corresponding to the true frequency values) which are correlated with the corresponding components of the other vectors, while the remaining $(M_1 - M)$ components should be uncorrelated because are due to the noise. This procedure allows one to estimate both the number of modes M and their corresponding frequencies at the same time. Different criteria can be devised to measure the similarity between pairs of components belonging to two different vectors of estimated frequencies. In this paper, we assign a *tolerance* E_t, which can be identified with the estimation accuracy; two frequencies belonging to different vectors are considered correlated if their absolute difference does not exceed the tolerance threshold. Then, when the set of correlated frequencies is worked out, the final estimates can be obtained as the average of the estimates on each block.

The main problems which could arise from the application of this method are:

1. some false frequencies related to the noisy components can be so close to each other to be selected as true frequencies by the comparison procedure. This event is extremely unlikely, as the spurious frequencies are statistically independent of each other; its probability can be virtually reduced to zero by processing more than two data blocks (three or four blocks are generally sufficient);
2. some true frequencies are not correctly estimated in one or more blocks, and therefore are not selected by the comparison procedure. In order to limit the probability of this event (which can be considered as an outlier), a very precise estimation procedure, such as the modified TKIF described in Sect. 3, must be employed.

The method has been tested on many different signals, in several SNR conditions. As a first example, we report the frequency estimation results obtained on a synthetic signal made of $M = 4$ modes with digital frequencies $\mathbf{f} = [0.1, 0.2, 0.3, 0.4]$ and amplitudes $\mathbf{A} = [1, 1, 1, 1]$; the SNR is 10 dB. The signal has been split into $N_b = 4$ blocks of $N = 80$ points. The parameter M_1 has been fixed to 10, so that 10 frequencies per block are estimated by means of the TKIF algorithm; these frequencies are reported in Table 2 with the correct ones in boldface. Then, the frequency selection criterion is applied with a tolerance $E_t = 0.001$, yielding a final estimate

$$\hat{\mathbf{f}} = [0.1001, 0.2000, 0.3001, 0.3999]$$

It can be noticed that the number of modes is estimated correctly. As another example, the estimation procedure has been run on a signal made of $M = 3$ modes with digital frequencies $\mathbf{f} = [0.1, 0.2, 0.4]$ and amplitudes $\mathbf{A} = [1, 1, 0.3]$; the SNR is

Table 2: Estimated frequencies for a four mode signal and $M_1 = 10$

Frequency	Block 1	Block 2	Block 3	Block 4
f_1	0.0706	0.0608	0.0468	0.0148
f_2	**0.0999**	**0.1002**	**0.1002**	0.0456
f_3	0.1468	0.1564	0.1657	**0.1001**
f_4	**0.1999**	0.1909	**0.2003**	0.1511
f_5	0.2218	**0.2001**	0.2472	**0.1998**
f_6	**0.3000**	0.2508	**0.2996**	0.2322
f_7	0.3095	**0.3004**	0.3368	**0.3002**
f_8	0.3534	0.3523	**0.3997**	0.3603
f_9	**0.4001**	**0.4001**	0.4390	**0.3996**
f_{10}	0.4445	0.4170	0.4519	0.4391

10 dB, and the other algorithm parameters are $N_b = 4$, $N = 80$, $M_10 = 10$, $E_t = 0.005$. The obtained final estimate

$$\hat{\mathbf{f}} = [0.1, 0.20005, 0.4]$$

shows that the method is able to work properly also when the mode amplitudes are not equal.

The complexity of the proposed algorithm can seem larger than that of the SVD-based methods proposed in [6], as the TKIF algorithm must be run on N_b data blocks. However, as discussed, three or four blocks are generally sufficient to reduce the false alarm probability; therefore, the complexity increase can be neutralized if the TKIF algorithm is employed instead of the basic TK with the same estimation accuracy level. Moreover, the method is sensibly more reliable and flexible than the SVD-based one, and can be applied also when the number of modes is changing in time.

5. CONCLUSIONS

In this paper, a turbo estimation algorithm has been described in the context of modal analysis, which makes use of the Tufts and Kumaresan algorithm as the master EA, and of a multiband IIR filter as the external denoising system. Two improvements of the basic algorithm are proposed; the former implies a different design of the EDS, able to achieve better estimation accuracy while reducing the outlier probability without affecting the computational complexity; the latter permits the autodetermination of the number of modes making up the signal. The performance of the algorithm is very encouraging from both the accuracy and the complexity point of view.

6. REFERENCES

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