

JITTER IDENTIFICATION TECHNIQUES FOR A REGULAR EVENT-BASED PROCESS

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ABSTRACT

Two system identification techniques are proposed for discriminating between the type of timing jitter that perturbs the arrival time sequence recorded for an event-based process. A univariate point process is used to characterise the observed signal activity. The first jitter identification method requires a visual inspection of an estimate of the expectation density computed for the point process. The second method involves a statistical hypothesis test for a renewal process.

1. INTRODUCTION

Let $\{t_0, t_1, \dots, t_{N-1}\}$ denote a sequence of N event arrival times observed for an event-based process. The time delays or inter-event intervals (IEIs) that separate successive event occurrence times are assumed to be nominally regular, but will vary from one interval to the next due to timing jitter that exists on the individual event arrival times. Examples of this type of process may be found in a number of application areas, including the driven and spontaneous spike train activity recorded from a biological neuron [1, 2], a series of radar pulses collected by a passive radar intercept receiver [3–5], and a sequence of zero-crossing measurements recorded by a communications system [4]. The observed variation in IEIs is modelled here according to a Gaussian distribution with the n th IEI, $T_n = t_n - t_{n-1}$, distributed as $\mathcal{N}(T, \sigma_T^2)$.

Our analysis of the event-based process will focus on a study of the observed event arrival time sequence and this suggests a univariate *point process* representation for the signal activity such that the idealised events are distinguishable only by their occurrence times $\{t_0, t_1, \dots, t_{N-1}\}$. Two jitter models will be used to describe the displacement in event arrival times relative to a strictly periodic series of events. The first model involves a non-cumulative jitter (NCJ) variable whereby the arrival time of an event is independent of the occurrence times of the events that precede it. The second model incorporates a cumulative jitter (CJ) variable such that an event arrival time exhibits a dependency on the preceding event occurrence time.

The purpose of this paper is to investigate two novel approaches for discriminating between a NCJ and CJ process

for the signals of interest. The discrimination between the two jitter models is useful for signal identification and for revealing the nature of the underlying signal generating process. Optimal estimators have also been developed for the mean IEI and the timing jitter variance [5], but to yield optimal performance the jitter model must be known *a priori*. The jitter identification techniques described herein could therefore be used to pre-process the data and to identify the relevant jitter process before the parameter estimation techniques are applied. The first method that we consider for jitter discrimination involves an estimate of the *expectation density* for the point process associated with the observed event arrival time sequence. The second method is based on a statistical hypothesis test for a *renewal process*.

2. EVENT ARRIVAL TIME MODELS

Consider a regular event-based process with a set of N observed arrival times which also label the point events in time that we associate with some underlying point process. In the NCJ model, the n th event occurrence time is modelled as

$$t_n = t_\phi + nT + \alpha_n, \quad (1)$$

where $n = 0, 1, \dots, N-1$, T is the mean IEI, t_ϕ defines the start time for the process, and α_n is a NCJ variable with distribution $\mathcal{N}(0, \sigma_\alpha^2)$. The $\{\alpha_0, \alpha_1, \dots, \alpha_{N-1}\}$ represent a stationary sequence of independent and identically distributed (i.i.d.) random variables. In the NCJ model the n th IEI is given by

$$\begin{aligned} T_n &= t_n - t_{n-1}, \\ &= T + \alpha_n - \alpha_{n-1}, \end{aligned} \quad (2)$$

with $n = 1, 2, \dots, N-1$. In which case, T_n is distributed as $\mathcal{N}(T, 2\sigma_\alpha^2)$. Note also the dependence of successive IEIs on a common jitter variable which implies that the underlying process that generates the $\{T_1, T_2, \dots, T_{N-1}\}$ is of the *nonrenewal* type. For the CJ model, the n th event occurrence time is given by

$$t_n = t_{n-1} + T + \beta_n, \quad (3)$$

$$\text{where } t_0 = t_\phi + \beta_0, \quad (4)$$

$n = 1, 2, \dots, N - 1$, and t_ϕ and T are as above. The $\{\beta_0, \beta_1, \dots, \beta_{N-1}\}$ represent a stationary sequence of i.i.d. random variables with β_n a CJ variable distributed as $\mathcal{N}(0, \sigma_\beta^2)$. Observe that t_n may also be written as

$$t_n = t_\phi + nT + \sum_{m=0}^n \beta_m, \quad (5)$$

which demonstrates explicitly the accumulation of jitter from one event arrival time to the next. In the CJ model the n th IEI is given by

$$\begin{aligned} T_n &= t_n - t_{n-1}, \\ &= T + \beta_n, \end{aligned} \quad (6)$$

with $n = 1, 2, \dots, N - 1$. T_n is therefore distributed as $\mathcal{N}(T, \sigma_\beta^2)$. The IEIs $\{T_1, T_2, \dots, T_{N-1}\}$ are also i.i.d. in the CJ representation and, by definition, may therefore be associated with a *renewal process*. The amount of jitter on an event arrival time sequence will be quantified by the *coefficient of variation*, γ_T , defined for the random variable T_n , by

$$\gamma_T = \frac{[\text{Var}\{T_n\}]^{\frac{1}{2}}}{E\{T_n\}} = \frac{\sigma_T}{T}, \quad (7)$$

$$\text{where } \sigma_T = \begin{cases} \sqrt{2}\sigma_\alpha & \text{for NCJ,} \\ \sigma_\beta & \text{for CJ.} \end{cases} \quad (8)$$

3. JITTER IDENTIFICATION FOR AN EVENT-BASED PROCESS

3.1. Estimation of the Expectation Density

The expectation density [2] or intensity function [6], denoted by $m(\tau)$ for a point process series of events, provides a valuable analysis tool for investigating the properties of the process of interest and is formally defined by

$$m(\tau) = \lim_{\delta\tau \rightarrow 0} \frac{\text{Pr}\{\text{event in } (t + \tau, t + \tau + \delta\tau) \mid \text{event at } t\}}{\delta\tau}. \quad (9)$$

The expectation density therefore relates to the conditional probability, $m(\tau)\delta\tau$, that an event is observed in the small interval $(\tau, \tau + \delta\tau]$ sec after an event occurs at time t . From [1, 2], the expectation density for a point process may be constructed from the interval densities of the particular process under consideration. The probability density function of the k th-order interval is denoted by $m_k(\tau)$ and describes the distribution of the intervals between the n th and $(n + k)$ th events. The first-order density, $m_1(\tau)$, for example, specifies the distribution of the IEIs $\{T_n = t_n - t_{n-1}\}$. The expectation density may then be constructed as follows [1, 2]

$$m(\tau) = \sum_{k=1}^{\infty} m_k(\tau). \quad (10)$$

We remark also that if T denotes the mean time interval between events, from [1, 2]

$$\lim_{\tau \rightarrow \infty} m(\tau) = \frac{1}{T}. \quad (11)$$

The rate at which this limit is approached is dependent on the process under investigation [1, 2]. For a regular event-based process with NCJ the k th-order intervals are distributed as $\mathcal{N}(kT, 2\sigma_\alpha^2)$, whereas for CJ they are distributed as $\mathcal{N}(kT, k\sigma_\beta^2)$. From Equation (10) the theoretical expectation density for a point process series of events associated with each jitter model is given by

$$m(\tau) = \begin{cases} \sum_{k=1}^{\infty} \frac{1}{2\sigma_\alpha\sqrt{\pi}} \exp\left[\frac{-(\tau-kT)^2}{4\sigma_\alpha^2}\right] & \text{for NCJ,} \\ \sum_{k=1}^{\infty} \frac{1}{\sigma_\beta\sqrt{2k\pi}} \exp\left[\frac{-(\tau-kT)^2}{2k\sigma_\beta^2}\right] & \text{for CJ,} \end{cases} \quad (12)$$

where $\tau > 0$. The expectation densities for the processes of interest therefore take the form of a series of peaks centred on integer multiples of the mean IEI T . Inspection of (12) also reveals that for NCJ, the peaks in $m(\tau)$ are of the same width and independent of the time delay τ , whereas for CJ, the peaks become broader for increasing values of τ .

An estimate of $m(\tau)$ may be obtained in an experimental situation from a block of N recorded event arrival times, $\{t_0, t_1, \dots, t_{N-1}\}$, by considering the time intervals between pairs of events via

$$\hat{m}(\tau) = \frac{1}{N} \sum_{n=0}^{N-2} \sum_{k=1}^{N-n-1} \delta(t_{n+k} - t_n - \tau), \quad (13)$$

where $\delta(\cdot)$ denotes the Dirac delta function. In practice, a histogram is used to obtain a smoothed estimate of $m(\tau)$ by binning the $N(N - 1)/2$ event arrival time differences generated from the application of (13). If N_j corresponds to the number of time intervals between pairs of events that fall into the j th histogram bin based on

$$(j - 1)\Delta \leq (t_{n+k} - t_n) < j\Delta, \quad (14)$$

where Δ denotes histogram bin width, an estimator for the average value of $m(\tau)$ within that bin is

$$\hat{m}(\tau_j) = \frac{N_j}{N\Delta}, \quad j = 1, 2, \dots, J, \quad (15)$$

where $\tau_j = (j - 0.5)\Delta$ and J is the total number of histogram bins. This type of expectation density estimate finds application in neurobiological signal processing (e.g., see [1, 2]) and is analogous to the *time difference of arrival histogram* used extensively in radar signal analysis (e.g., see [3]). In Figure 1 we show estimates of $m(\tau)$ with $\tau > 0$ for regular point processes derived from an event arrival time sequence perturbed by NCJ and by CJ. In each case, $N = 200$ events, $T = 100$ ms, $t_\phi = 50$ ms, and $\gamma_T = 0.1$

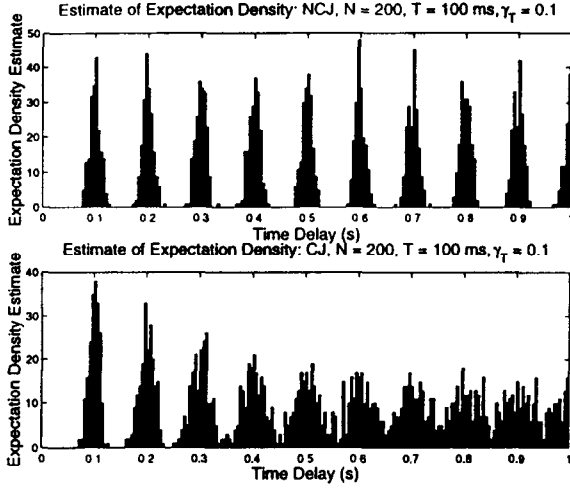


Figure 1: Histogram-based estimates of the expectation density for a point process event arrival time sequence with NCJ (upper plot) and with CJ (lower plot).

The shape of the peaks in each histogram are in general agreement with the above observations for a NCJ and CJ process. Unlike the case for NCJ, the estimate, $\hat{m}(\tau_j)$, for a CJ process can clearly be seen to approach the limit $T^{-1} = 10$ as defined by Equation (11). These distinctive properties of the expectation density provide visual cues as to the nature of the underlying jitter process and are particularly effective jitter discriminators for larger values of γ_T .

3.2. A Test for a Renewal Process

It was noted in Section 2 that for a regular event-based process with NCJ, the IEs are identically distributed as $\mathcal{N}(T, 2\sigma_\alpha^2)$, but exhibit a statistical dependence by virtue of a common jitter variable. In contrast, the IEs for a CJ process are *i.i.d.* as $\mathcal{N}(T, \sigma_\beta^2)$. This suggests that the discrimination between a NCJ and CJ process can be cast in terms of a test for a renewal process since a point process for which the IEs, $\{T_n\}$, are *i.i.d.* is said to be of the renewal type. In the specific case of two Gaussian distributed random variables, a lack of correlation between the random variables implies statistical independence. We can make use of this property to test for the independence of the IEs through an estimate of the *serial correlation coefficient*, ρ_j . The serial correlation coefficient of lag j for a process with mean IEI T and variance σ_T^2 is defined by [6]

$$\rho_j = \frac{E\{[T_n - T][T_{n+j} - T]\}}{\sigma_T^2}. \quad (16)$$

A set of serial correlation coefficients computed for a point process is often referred to as the *serial correlogram*. Given a sequence of $(N - 1)$ IEs $\{T_n\}_{n=1}^{N-1}$, an asymptotically

unbiased estimate of ρ_j may be obtained from [6]

$$\hat{\rho}_j = \frac{\sum_{n=1}^{N-j-1} (T_n - \bar{T}'_j)(T_{n+j} - \bar{T}''_j)}{\left[\sum_{n=1}^{N-j-1} (T_n - \bar{T}'_j)^2 \sum_{n=1}^{N-j-1} (T_{n+j} - \bar{T}''_j)^2 \right]^{\frac{1}{2}}}, \quad (17)$$

with

$$\bar{T}'_j = \frac{1}{N-j-1} \sum_{n=1}^{N-j-1} T_n \quad (18)$$

$$\text{and } \bar{T}''_j = \frac{1}{N-j-1} \sum_{n=1}^{N-j-1} T_{n+j}. \quad (19)$$

\bar{T}'_j and \bar{T}''_j are estimates of the mean IEI, T , calculated from the first and last $(N - j - 1)$ observations of T_n respectively. The sample serial correlogram has been used in neurobiological signal analysis, for instance, to investigate the serial dependence of the IEs generated by a spiking neuron [1, 2].

Computing the theoretical serial correlation coefficient for a lag of one for a regular event-based process with mean IEI T and NCJ, we obtain from (2) and (16) that

$$\begin{aligned} \rho_1 &= \frac{E\{[T + \alpha_n - \alpha_{n-1} - T][T + \alpha_{n+1} - \alpha_n - T]\}}{2\sigma_\alpha^2}, \\ &= -\frac{E\{\alpha_n^2\}}{2\sigma_\alpha^2} = -\frac{\sigma_\alpha^2}{2\sigma_\alpha^2} = -0.5. \end{aligned} \quad (20)$$

One may also show that $\rho_j = 0$ for $j > 1$. Similarly, for a CJ process, from (6) and (16) we have that

$$\begin{aligned} \rho_1 &= \frac{E\{[T + \beta_n - T][T + \beta_{n+1} - T]\}}{\sigma_\beta^2}, \\ &= \frac{E\{\beta_n\}E\{\beta_{n+1}\}}{\sigma_\beta^2} = 0. \end{aligned} \quad (21)$$

and $\rho_j = 0$ for $j > 1$. In Figure 2 we plot the sample serial correlogram using ten lags for a regular event arrival time sequence with NCJ and with CJ. In each case, $N = 200$ events, $T = 100$ ms, $t_\phi = 50$ ms and $\gamma_T = 0.1$. The values for $\{\hat{\rho}_j\}_{j=1}^{10}$ are consistent with our previous observations. The above analysis also indicates that in the context of the jitter identification problem, a test for a renewal process should be limited to an estimate of ρ_1 . Such a test can be formulated using the following simple binary hypotheses:

$$\begin{aligned} H_0 &: \rho_1 = 0, \\ H_1 &: \rho_1 = -0.5. \end{aligned}$$

For large sample observations of a renewal process, $\hat{\rho}_1(N - 2)^{0.5}$ is distributed as $\mathcal{N}(0, 1)$ [6]. Therefore, given N jittered arrival times from a regular event-based process, the null hypothesis, H_0 , is accepted or rejected based on

$$\hat{\rho}_1 \underset{H_0}{\overset{H_1}{\leq}} C_\nu(N - 2)^{-0.5}, \quad (22)$$

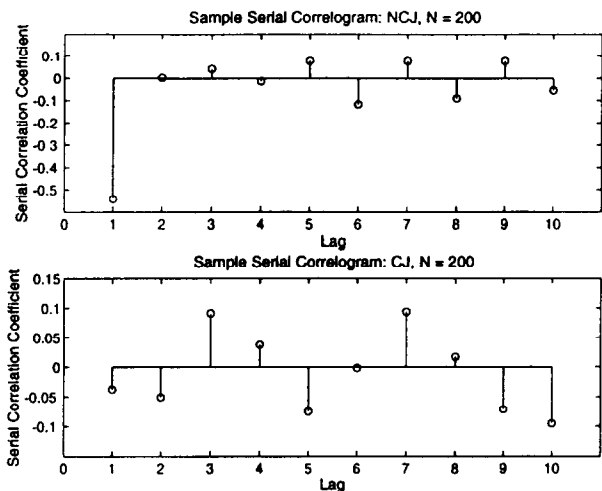


Figure 2: The sample serial correlogram for a point process with NCJ (upper plot) and with CJ (lower plot).

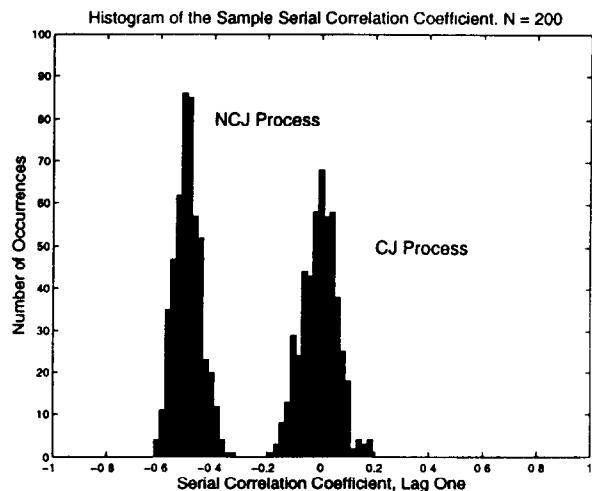


Figure 3: Histogram of the distribution of $\hat{\rho}_1$ obtained for a point process series of events with NCJ and with CJ.

where C_ν is a negative-valued threshold derived from the statistics of the unit Gaussian and a specified significance level ν . If H_0 is accepted the jitter is classified as CJ, otherwise it is classified as NCJ. In Figure 3 we plot a histogram of the number of occurrences of $\hat{\rho}_1$ obtained separately for a regular arrival time sequence with NCJ and with CJ. A total of 500 independent realisations of each type of sequence was generated with parameters: $N = 200$ events, $T = 100$ ms, $t_\phi = 50$ ms and $\gamma_T = 0.1$. The cluster of values centred on $\hat{\rho}_1 = 0$ correspond to CJ events, while those centred on $\hat{\rho}_1 = -0.5$ belong to NCJ events. The sample distributions are clearly well resolved for this value of N with no overlap and results in almost 100% correct jitter classification.

4. ROBUSTNESS CONSIDERATIONS

The results presented here relate to an event-based process that has been observed via some form of amplitude threshold detection scheme and under favourable conditions corresponding to a high signal-to-noise ratio (SNR). On occasion, however, the SNR might be low enough to prevent the registration of every event and lead to the detection of false events. Under these circumstances the expectation density estimation approach has an advantage, in terms of a robustness to contamination by missing and false events, over the test for a renewal process. Indeed the renewal test approach fails with only a modest amount of contamination. It is possible, however, that the performance of the renewal test could be improved by pre-filtering an arrival time sequence and this investigation is on-going. Despite these limitations, the renewal test is very effective for a wide range of γ_T 's, whereas the expectation density method was found to yield relatively poor discrimination results for small values of γ_T .

5. CONCLUSIONS

System identification techniques were examined for discriminating between two types of jitter model for an event-based process with a nominally regular IEI. The first method required a visual inspection of an estimate of the expectation density for the corresponding point process and, as such, is suitable for off-line analysis. In contrast, the second method involved a numerical calculation of the serial correlation coefficient for the observed process and is therefore amenable to on-line, automatic analysis of event-based data.

6. REFERENCES

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