# AN ADAPTIVE-GAIN ALPHA-BETA TRACKER COMBINED WITH THREE-DIMENTIONAL CIRCULAR PREDICTION USING ESTIMATION OF THE PLANE STATE

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## ABSTRACT

In tracking systems using phased array antenna, the adaptive-gain $\alpha$-$\beta$ tracker combined with circular prediction has been proposed for maneuvering targets. However, tracking quality of the circular prediction filter degrades for highly maneuvering targets that continue to change the turning plane, since the circular prediction is calculated on the assumption that a target flies on the same plane of previous three measured positions. In this paper, we extend the circular prediction to three-dimentional space and propose the adaptive-gain $\alpha$-$\beta$ tracker combined with three-dimentional circular prediction using estimation of the plane state to improve tracking quality.

## 1. INTRODUCTION

Tracking radar using phased array antenna is often used in air and sea surveillance. The Kalman filter or an $\alpha$-$\beta$ filter has been employed in single target tracking problem. The Kalman filter performs almost perfect tracking when the target trajectory is described as a state equation and the statistical characteristics of the target maneuver and measurement noise such as mean and variance are known[1]. In practice, it is difficult to know the statistical characteristics of the target in advance. Additionally, the Kalman filter requires growing computational requirements. On the other hand, an $\alpha$-$\beta$ filter realizes real-time tracking, since it omits the calculation of error covariance and filter gain[2]. When the target maneuvers, the quality of the position and velocity estimates provided by an $\alpha$-$\beta$ filter degrades significantly, since it uses only the linear prediction.

To track maneuvering targets, the adaptive-gain $\alpha$-$\beta$ tracker combined with circular prediction has been proposed[3]. However, tracking quality of the circular prediction filter degrades for a maneuvering target which changes the turning plane, since the circular prediction is calculated on the assumption that a target flies on the same plane of previous three measured positions. In addition, the algorithm of the circular prediction should be extended to three-dimentional space to evaluate the filter for a maneuvering target in three-dimentional space. In this paper, we extend the circular prediction to three-dimentional space and propose the adaptive-gain $\alpha$-$\beta$ tracker combined with three-dimentional circular prediction using estimation of the plane state. The output of the circular prediction filter is corrected by estimation of the target plane state to improve tracking quality. Simulation results for three target profiles are included for a comparison of the performances of our proposed scheme with that of conventional tracking filters.

## 2. PROPOSED TRACKER WITH ESTIMATION OF THE PLANE STATE

In this section, we extend the circular prediction to three-dimentional space and propose the adaptive-gain $\alpha$-$\beta$ tracker combined with three-dimentional circular prediction using estimation of the plane state. Our proposed scheme consists of the linear and circular prediction filters, and the maneuver detector as shown in Fig.1. The block diagram of the proposed scheme is same as the conventional adaptive-gain $\alpha$-$\beta$ tracker combined with circular prediction except the circular prediction filter calculates corrected circular prediction by estimation of the normal vector of the turning plane.

The concept of the circular prediction using estimation of the turning plane is shown in Fig.2. The circular prediction filter calculates the center and radius of an arc passing through previous measured positions, and the circular prediction is calculated on the assumption that a target flies on the same plane of previous three measured position[3]. However, when the target leaves the turning plane, the tracking quality of the circular prediction filter could degrade. In our proposed scheme, the corrected circular prediction is calculated on the predicted turning plane. By prediction of the

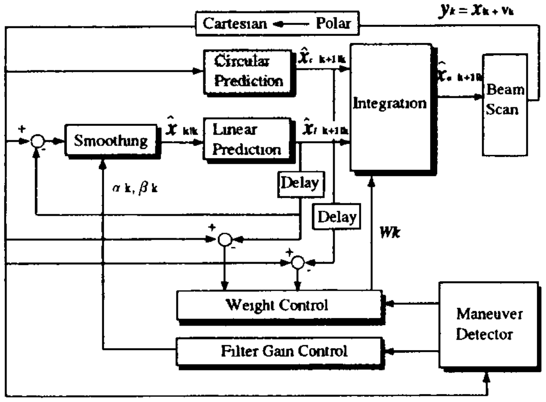


Fig. 1. Block diagram of the proposed method.

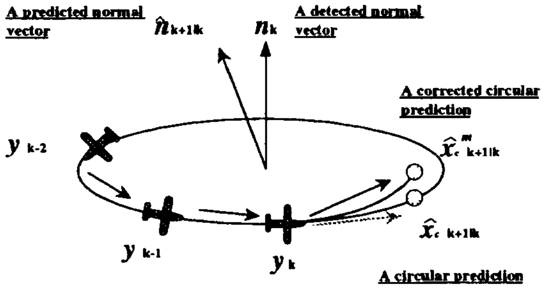


Fig. 2. Principle of the proposed circular prediction.

normal vector, the maneuver-following capability of the circular prediction filter could be improved. The sequence of calculating corrected circular prediction is as follows.

(1) A normal vector $\boldsymbol{n}_k$ of the turning plane that consists of previous three measured positions is detected.
(2) The center of an arc passing through three measured positions $\boldsymbol{x}_{0\ k}$ is calculated on the detected turning plane.
(3) An angle $\psi_k$ between the normal vector $\boldsymbol{n}_{k-1}$ and $\boldsymbol{n}_k$ is calculated, and the prediction of an angle $\psi_k$ at sample $k+1$ is calculated using an $\alpha$-$\beta$ filter.
(4) The predicted normal vector $\hat{\boldsymbol{n}}_{k+1|k}$ is calculated using the prediction of an angle $\psi_k$ and measured positions at sample $k-1$ and $k$.
(5) The center of an arc $\boldsymbol{x}_{0\ k}$ is transfered to the predicted turning plane.
(6) The corrected circular prediction $\hat{\boldsymbol{x}}^m_{c\ k+1|k}$ is calculated using the transfered center of an arc and measured positions at sample $k-1$ and $k$.

## 2.1. Three-dimentional Circular Prediction

We extend the algorithm of the conventional circular prediction to three-dimentional space. A normal vector detection is required to calculate the circular prediction, since the turning plane slopes away. A normal vector $\boldsymbol{n}_k$ of the turning plane which consists of previous three measured positions is calculated by solving simultaneous equations as follows.

$$a_k A + b_k B + c_k C = 0 \tag{1}$$

$$a_k D + b_k E + c_k F = 0 \tag{2}$$

where $(a_k, b_k, c_k)$ denote the coordinates of the measured normal vector of turning plane $\boldsymbol{n}_k$, and $(A, B, C)$, $(D, E, F)$ denote the coordinates of $\boldsymbol{y}_{k-2} - \boldsymbol{y}_{k-1}$ and $\boldsymbol{y}_{k-1} - \boldsymbol{y}_k$, respectively. The measured normal vector at sample k is

$$n_k = \begin{bmatrix} a_k \\ b_k \\ c_k \end{bmatrix} = \begin{bmatrix} BF - CE \\ CD - AF \\ AE - BD \end{bmatrix}. \tag{3}$$

The center of an arc is calculated on the turning plane that has the measured normal vector $\boldsymbol{n}_k$. The center of an arc $\boldsymbol{x}_{0\,k}$ and the radius $r_k$ is calculated as follows.

$$\begin{aligned} x_0 = & \frac{(AF - CD)(CH - FG)}{(BF - CE)^2 + (CD - AF)^2 + (AE - BD)^2} \\ & + \frac{(BD - AE)(EG - BH)}{(BF - CE)^2 + (CD - AF)^2 + (AE - BD)^2} \\ & + \frac{(BF - CE)^2 x_k}{(BF - CE)^2 + (CD - AF)^2 + (AE - BD)^2} \\ & + \frac{(BF - CE)(CD - AF) y_k}{(BF - CE)^2 + (CD - AF)^2 + (AE - BD)^2} \\ & + \frac{(BF - CE)(AE - BD) z_k}{(BF - CE)^2 + (CD - AF)^2 + (AE - BD)^2} \end{aligned} \tag{4}$$

$$y_0 = \frac{CD - AF}{BF - CE} x_0 + \frac{CH - FG}{BF - CE} \tag{5}$$

$$z_0 = \frac{BD - AE}{CE - BF} x_0 + \frac{BH - EG}{CE - BF} \tag{6}$$

$$r_k = \sqrt{(x_0 - x_k)^2 + (y_0 - y_k)^2 + (z_0 - z_k)^2} \tag{7}$$

where $(x_0, y_0, z_0)$ denote the coordinates of the center $\boldsymbol{x}_{0\,k}$ and $(x_k, y_k, z_k)$ denote the coordinates of measured position $\boldsymbol{y}_k$.

A circular prediction $\boldsymbol{x}_{c\ k+1|k}$ is calculated by solving simultaneous equations as follows.

$$a_k I + b_k J + c_k K = 0 \tag{8}$$

$$IL + JM + KN = {r_k}^2 \cos\theta_k \tag{9}$$

$$I^2 + J^2 + K^2 = {r_k}^2 \tag{10}$$

where $(I, J, K)$ denote the coordinates of $\hat{\boldsymbol{x}}_{c\ k+1|k} - \boldsymbol{x}_{0\ k}$, $(L, M, N)$ denote the coordinates of $\boldsymbol{y}_k - \boldsymbol{x}_{0\ k}$, and $\theta_k$ denotes the turning angle within one scan period. Eq.(8) is derived from the assumption that an circular prediction is on the detected turning plane, Eq.(9) is derived from the assumption that a target flies with a constant angular velocity, and Eq.(10) is derived from the assumption that a target flies on an arc.

### 2.2. Estimation and Prediction of Turning Plane State

Estimate and prediction of normal vector of the turning plane are performed to correct the output of the circular prediction. The angle $\psi_k$ between $\boldsymbol{n}_{k-1}$ and $\boldsymbol{n}_k$ is employed to a unique estimation parameter. Since the measured positions $\boldsymbol{y}_{k-1}$ and $\boldsymbol{y}_k$ are on the predicted turning plane, the prediction of $\psi_k$ is enough to calculate the predicted normal vector $\hat{\boldsymbol{n}}_{k+1|k}$. The estimate and prediction of the angle $\psi_k$ is calculated using an $\alpha$-$\beta$ filter. From the predicted angle $\boldsymbol{H}^{\psi}\hat{\boldsymbol{\psi}}_{k+1|k}$, the predicted normal vector $\hat{\boldsymbol{n}}_{k+1|k}$ is calculated by solving simultaneous equations as follows.

$$a_k\hat{a}_{k+1|k} + b_k\hat{b}_{k+1|k} + c_k\hat{c}_{k+1|k} = \cos(\boldsymbol{H}^{\psi}\hat{\boldsymbol{\psi}}_{k+1|k}) \quad (11)$$

$$D\hat{a}_{k+1|k} + E\hat{b}_{k+1|k} + F\hat{c}_{k+1|k} = 0 \quad (12)$$

$$\hat{a}^2_{k+1|k} + \hat{b}^2_{k+1|k} + \hat{c}^2_{k+1|k} = 1 \quad (13)$$

where $(\hat{a}_{k+1|k}, \hat{b}_{k+1|k}, \hat{c}_{k+1|k})$ is the coordinate of predicted normal vector $\hat{\boldsymbol{n}}_{k+1|k}$. Eq.(11) is derived from the assumption that the measured normal vector $\boldsymbol{n}_k$ and the predicted normal vector $\hat{\boldsymbol{n}}_{k+1|k}$ forms the angle $\boldsymbol{H}^{\psi}\hat{\boldsymbol{\psi}}_{k+1|k}$, Eq.(12) is derived from the assumption that measured positions $\boldsymbol{y}_{k-1}$, $\boldsymbol{y}_k$ exists on the turning plane that has the predicted normal vector $\hat{\boldsymbol{n}}_{k+1|k}$, Eq.(13) is derived from the assumption that $\hat{\boldsymbol{n}}_{k+1|k}$ is a unit vector.

### 2.3. Correction of circular prediction

A corrected circular prediction is calculated on the predicted turning plane. The center of an arc is calculated on the detected turning plane as shown in Eq.(4)~(6), and is transfered to the turning plane that has the predicted normal vector. The center of an arc on the predicted turning plane is calculated by solving the simultaneous equations as follows.

$$\hat{a}_{k+1|k}(x_0^m - x_k) + \hat{b}_{k+1|k}(y_0^m - y_k) + \hat{c}_{k+1|k}(z_0^m - z_k) = 0 \quad (14)$$

$$D(x_0^m - x_k) + E(y_0^m - y_k) + F(z_0^m - z_k) = r_k|\boldsymbol{y}_{k-1} - \boldsymbol{y}_k|\cos\xi \quad (15)$$

$$(x_0^m - x_k)^2 + (y_0^m - y_k)^2 + (z_0^m - z_k)^2 = r_k^2 \quad (16)$$

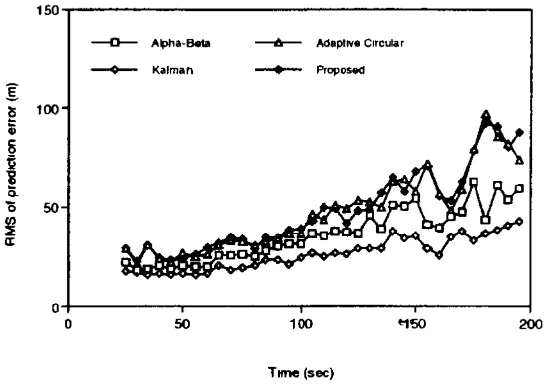


Fig. 3. RMS of prediction error versus time in the case of trajectory (a).

where $(x_0^m, y_0^m, z_0^m)$ is the coordinate of the transfered center $\boldsymbol{x}^m{}_{0\ k}$ on the predicted turning plane, and $\xi$ is the angle formed from the original center $\boldsymbol{x}_{0k}$, the measured positions $\boldsymbol{y}_k$ and $\boldsymbol{y}_{k-1}$. Eq.(14) is derived from the assumption that the transfered center of an arc is on the predicted turning plane, Eq.(15) is derived from the assumption that the angle between the vector $\boldsymbol{y}_{k-1} - \boldsymbol{y}_k$ and $\boldsymbol{x}^m{}_{0\ k} - \boldsymbol{y}_k$ is $\xi$, and Eq.(16) is derived from the assumption that the Euclid distance between the transfered center and the measured position $\boldsymbol{y}_k$ is the radius $r_k$. By applying the transfered center and the predicted normal vector to Eq.(8)~(10) as shown in Sec.2.1, the corrected circular prediction $\hat{\boldsymbol{x}}^m_{c\ k+1|k}$ is derived.

## 3. SIMULATION RESULTS

As an example a tracking system is considered to measure a target position at T=5s with errors that have 10m standard deviation in range and 1.0mil in azimuth and elevation direction. Measurement noise and maneuver are simulated by zero-mean independent white Gaussian random sequences. In the adaptive-gain $\alpha$-$\beta$ tracker combined with circular prediction and our proposed scheme, the filter gains are set to $\alpha = 0.88$ and $\beta = 0.85$ for a maneuvering target and $\alpha = 0.39$ and $\beta = 0.1$ for a nonmaneuvering target. The filter gains of an $\alpha$-$\beta$ filter are set to $\alpha$=0.66 and $\beta$=0.35. The Kalman has taken the standard deviation of system noise 0.5G[3]. This value is equivalent to the filter gain of an $\alpha$-$\beta$ filter.

A Monte-Carlo simulation of 20 runs was done. As the results of simulation, RMS(Root Mean Square) of prediction error versus time in case of the trajectories (a)~(c) are shown in Fig.3~5. Trajectory (a) is a straight trajectory with a constant velocity $150m/s$. Trajectory (b) is a sinusoid trajectory that a target con-

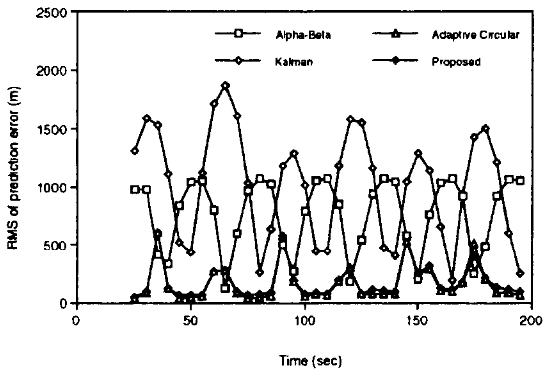


Fig. 4. RMS of prediction error versus time in the case of trajectory (b).

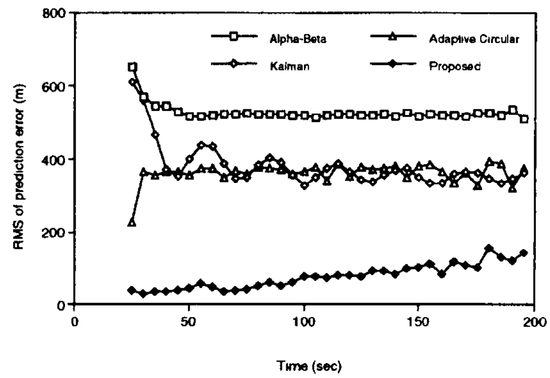


Fig. 5. RMS of prediction error versus time in the case of trajectory (c).

tinues to turn with turn gravity 2.0G. A target flies on the same plane in the trajectory (b). Trajectory (c) is a spiral trajectory that a target continues to turn from the radar site and goes up at an angle of elevation of 45°. A target continues to change the turning plane in the trajectory (c).

Fig.3 shows the relationship between RMS of prediction errors versus time in case of trajectory (a). Our proposed scheme shows relatively stable tracking and the prediction errors is almost same as the adaptive-gain $\alpha$-$\beta$ tracker combined with circular prediction. Since the turning plane is not formed in the straight trajectory, prediction errors of the angle $\psi_k$ becomes large. However, the tracking quality of our proposed scheme is not affected by increasing prediction errors of the angle $\psi_k$. This is because our proposed scheme can perform gain control and weighting to the linear prediction.

Fig.4 shows the relationship between RMS of prediction errors versus time in case of trajectory (b). Whenever the target continues to make turn, prediction errors of an $\alpha$-$\beta$ filter and the Kalman filter increase and decrease repeatedly. On the other hand, the adaptive-gain $\alpha$-$\beta$ tracker combined with circular prediction and our proposed scheme shows good maneuver-following capability. This is because these filters employ a circular prediction and the gain control by the use of a maneuver detection. As compared with the adaptive-gain $\alpha$-$\beta$ tracker combined with circular prediction, our proposed scheme shows almost same prediction errors, since a target in the trajectory (b) does not change the turning plane.

Fig.5 shows the relationship between RMS of prediction errors versus time in case of trajectory (c). Our proposed scheme shows good maneuver-following capability as compared with the adaptive-gain $\alpha$-$\beta$ tracker combined with circular prediction. Since a target of the trajectory (c) continues to change the turning plane, estimation of the plane state is effective for improvement of tracking quality of the circular prediction.

### 4. CONCLUSIONS

The adaptive-gain $\alpha$-$\beta$ tracker combined with three-dimentional circular prediction using estimation of the turning plane state has been proposed to improve maneuver-following capability. We also have extended a circular prediction to three-dimentional space. As the results of computer simulation, it was shown that our proposed scheme gives good maneuver-following capability for highly maneuvering targets in three-dimentional space and keep small prediction errors for nonmaneuvering targets. Further research is needed to develop this proposed scheme for multiple targets tracking.

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