

# FM INTERFERENCE SUPPRESSION IN SPREAD SPECTRUM COMMUNICATIONS USING TIME-VARYING AUTOREGRESSIVE MODEL BASED INSTANTANEOUS FREQUENCY ESTIMATION

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## ABSTRACT

In case of a strong frequency modulated interference in a spread spectrum communication system, the interference can be effectively suppressed by applying a time-varying notch filter with its zero(s) placed at an instantaneous frequency (IF) estimate of the interference. In this paper, we present the use of a time-varying autoregressive model based IF estimator in such a scenario. We model the received signal with an autoregressive model whose coefficients are time-varying and modeled as a combination of a set of known functions of time. The IF of the interference is estimated from the model. It is demonstrated that this method provides superior performance compared to using a time-frequency distribution. The comparison reveals that the Wigner-Ville Distribution peak based IF estimator suffers drawbacks such that its filtering gain is limited.

## 1. INTRODUCTION

The problem of frequency modulated (FM) interference suppression in direct-sequence spread spectrum (DSSS) communications has been attracting many researchers recently [1-5]. Amin and his colleagues, Lach and Lindsey, studied methods using time-frequency distributions (TFD): time-varying notch filtering using an instantaneous frequency (IF) estimated from a TFD [1], time-frequency filtering through signal synthesis after masking of the TFD, specifically the Wigner-Ville distribution (WVD) [2], and a comparison of the two approaches [3]. Bultan and Akansu [4] used the iterative matching pursuit algorithm with a chirplet dictionary to detect and excise chirp-like interference. Wei, Harding, and Bovik [5] used an iterative time-frequency filtering algorithm based on masking of the Gabor transform followed by signal synthesis.

All the aforementioned filtering procedures are highly non-causal, and require data-block based time-frequency analysis before signal synthesis or linear filtering. For practical system implementation, the associated time delay has to be controlled to an acceptable amount. This suggests that the entire filtering procedure employed be either implemented in a data-recursive form or can perform well with short data blocks. Similar to stationary cases, an appropriate parametric method with *a priori* information exploited would perform better than those non-parametric methods based on TFDs, such as WVD or other members of the Cohen class of time-frequency distributions. This is one of the reasons that, in this paper, we study FM interference suppression using parametric IF estimation.

Another reason is that WVD based IF estimation is not appropriate for non-linear FM laws due to the associated bias problem. In contrast, the parametric method we studied is not sensitive to non-linearity of IF laws, is suitable for short data blocks, and exhibits other features such as high resolution, is free of frequency quantization and cross-terms, and produces less of an end-of-data-record effect [10].

We model the received signal consisting of an FM interference, thermal noise and DSSS signal with a time-varying autoregressive (TVAR) model [6, 7]. The IF of the interference is estimated from the model and then a notch filter is formed according to the IF estimate.

The rest of the paper is organized as follows. In Section 2, for easy reference, we briefly review the TVAR model, TVAR based IF estimation, and compare the latter with WVD based IF estimation. In Section 3 we apply TVAR based IF estimation to FM interference suppression for DSSS communications. In Section 4 simulation experiments are presented.

## 2. TVAR BASED IF ESTIMATION

### 2.1 TVAR Modeling

First, we briefly review the time-varying AR model. For details of the TVAR model, please refer to Grenier [6] and Hall, Oppenheim and Willsky [7].

A discrete-time time-varying autoregressive (TVAR) process  $x(t)$  of order  $p$  is expressed as

$$x(t) = -\sum_{i=1}^p a_i(t)x(t-i) + e(t) \quad (1)$$

where  $e(t)$  is a stationary white noise process with zero mean and variance  $\sigma^2$ , and the TVAR coefficients  $\{a_i(t), i=1,2,\dots,p\}$  are modeled as linear combinations of a set of basis time functions  $\{u_k(t), k=0,1,\dots,q\}$ :

$$a_i(t) = \sum_{k=0}^q a_{i,k} u_k(t) \quad (2)$$

where  $\{u_k(t), k=0,1,\dots,q\}$  can be any appropriate set of basis functions. If  $\{u_k(t)\}$  are chosen as powers of time, then  $\{a_i(t)\}$  are polynomial functions of time  $t$ . If  $u_k(t)$  are trigonometric functions, then (2) is a finite order Fourier series expansion. In

any case, the TVAR model is described completely by the set of parameters  $\{ a_{i,k}, i=1,2,\dots,p; k=0,1,\dots,q; \sigma^2 \}$ .

The estimation of  $\{ a_{i,k} \}$  aims at minimizing the total squared prediction error in predicting the sequence  $x(t)$  :

$$J = \sum_t \left| x(t) + \sum_{i=1}^p \sum_{k=0}^q a_{i,k} u_k(t) x(t-i) \right|^2 \quad (3)$$

If we define the generalized covariance function as

$$c_{k,l}(i,j) = \frac{1}{N-p} \sum_{t=p}^{N-1} u_k(t) u_l(t) x(t-i) x(t-j) \quad (4)$$

then the solution  $\{ a_{i,k}, i=1,2,\dots,p, k=1,2,\dots,q \}$  that minimizes (3) can be solved for from the generalized covariance equations:

$$\sum_{i=1}^p \sum_{k=0}^q a_{i,k} c_{k,l}(i,j) = -c_{0,l}(0,j), \quad 1 \leq j \leq p, 0 \leq l \leq q \quad (5)$$

This is a system of  $p(q+1)$  linear equations, from which the coefficients are solved for.

## 2.2 TVAR Based IF Estimation

Time-varying autoregressive (TVAR) model based IF estimation, first proposed by Sharman and Friedlander in 1984 [8], was considered a poor estimator since being proposed [8, 9]. As a result, not much further investigation of this method was reported in the literature during the past decade. Our recent work [10] revealed that the TVAR based IF estimator is fairly good, and especially advantageous for those cases where a short data block is used and a nonlinear IF law applies.

For a signal consisting of  $M$  FM components in white noise with high to moderate signal-to-noise ratio (SNR), we use a TVAR signal model, with order  $p=M$  for complex exponential FM components and  $p=2M$  for real signals. The time-varying transfer function [6] corresponding to the TVAR model can be expressed as

$$H(z,t) = \frac{1}{1 + \sum_{i=1}^p a_i(t) z^{-i}} \quad (6)$$

By rooting the denominator polynomial formed by the TVAR coefficient estimates at each time instant  $t$ , we can obtain the  $p$  poles as functions of time:  $p_i(t), i=1,2,\dots,p$ . The trajectories of the poles associated with the FM components are on or close to the unit circle for moderate to high SNR. The rooting operation could be trivial for low order  $p$ . For example,  $p_1(t) = -a_1(t)$  for

$p=1$  and  $p_{1,2}(t) = \frac{1}{2} [-a_1(t) \pm \sqrt{a_1(t)^2 - 4a_2(t)}]$  for  $p=2$ . The instantaneous angles of the poles associated with the FM components can be used as an estimate of the instantaneous frequencies  $f_i(t)$  :

$$f_i(t) = \arg p_i(t) \quad \text{for } |p_i(t)| \approx 1 \quad (7)$$

The procedure for IF estimation based on the TVAR model consists of the following steps:

- Choose the basis functions  $u_k(t), k=1,2,\dots,q$  and the orders  $q$  and  $p$ .

- Calculate the generalized covariance function according to (4), solve for  $a_{i,k}$  from equation (5), and construct the TVAR coefficients  $a_i(t)$  by (2).
- Root the time-varying poles  $p_i(t), i=1,2,\dots,p$  at each instant  $t$ .
- Find their time-varying angles as the IF estimates of the FM components.

The set of basis functions and the orders  $p$  and  $q$  should be selected using *a priori* knowledge of the signal. For a signal consisting of continuous FM components it is appropriate to use powers of time as the basis function set.

## 2.3 IF Estimation: TVAR vs. WVD

WVD peak based IF estimation was shown to be optimal for linear FM signals with high to moderate SNR [11] and superior statistical performance was reported compared to most other IF estimation methods [9]. By contrast, the TVAR based IF estimator was considered a poor one [8, 9]. As a matter of fact, as we reported [10], the optimality of the WVD based method, which is not disputed, requires the following simultaneous conditions: 1) a linear FM law, 2) the time instances of the estimated IF are far from the ends of the data record, 3) generous zero-padding or frequency interpolation, and 4) high SNR. Violation of any of the conditions can make the WVD based method perform worse than the TVAR based method. The TVAR based method, though not optimal in any case, is more robust and performs satisfactorily in various situations.

For the scenario of FM interference in spread spectrum communications, it may not be appropriate to assume a linear FM law. Nonlinearity of FM law may lead to severe bias for WVD based IF estimation; hence, the resultant filter may not match the interference. This is harmful especially when the interference is strong. In addition, for the sake of time delay and computational cost, a small data block length is desired. Therefore, it may be highly advantageous to use TVAR based IF estimation rather than WVD - or other TFD - based IF estimation.

## 3. FM INTERFERENCE SUPPRESSION

At the receiver of a DSSS communications system, an interference suppression filter is applied before demodulation. We consider a received signal  $r(t)$  composed of the DSSS signal  $s(t)$ , thermal noise  $w(t)$  and interference  $i(t)$  :

$$r(t) = s(t) + w(t) + i(t) \quad (8)$$

Here the noise  $w(t)$  is assumed to be stationary and white. The DSSS signal  $s(t)$  is "slightly colored" in practice, which depends on the sampling rate (samples per chip) as well as the specific PN sequence. The FM interference  $i(t)$  could be an intentional or unintentional jammer, such as a strong FM communication emitter. The components in (8) are assumed to be mutually independent.

In the stationary case, tones in white noise can be modeled as an AR process and the frequencies of the tones can be estimated from AR modeling. We model the received signal  $r(t)$  with a

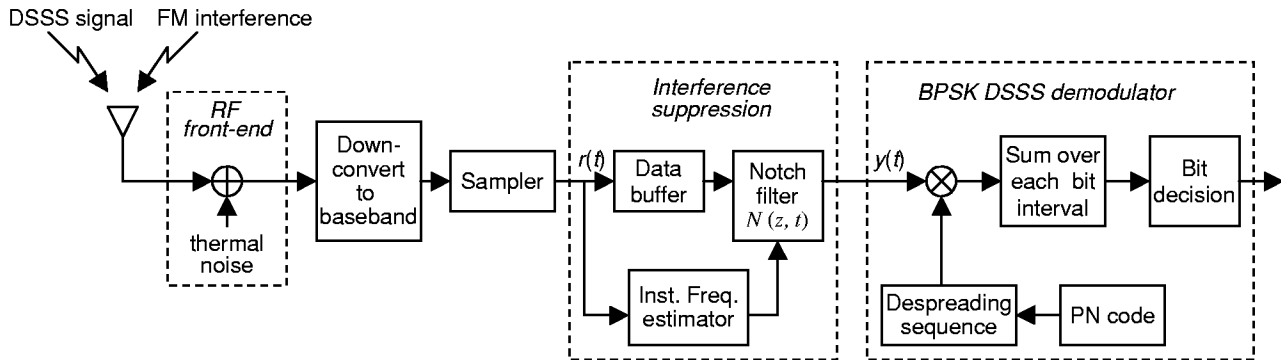


Figure 1. Block diagram of the DSSS receiver with IF estimation based FM interference suppression

time-varying AR model, viewed as an FM in white noise process, if we treat the DSSS signal as approximately white. The IF of the FM interference  $i(t)$  is estimated as described in Section 2. Then  $r(t)$  is fed into a time-varying filter  $N(z, t)$  to suppress the interference. The filter  $N(z, t)$  is a time-varying notch filter with its zeros placed at the IF estimate. A study of the design and performance issues of this filter can be found elsewhere [12]. The block diagram of the receiver (BPSK modulation assumed) with an IF based interference suppression filter is given in Figure 1.

#### 4. SIMULATION RESULTS

To observe the performance of the TVAR based interference suppression method, we compare it with the notch filtering method using the WVD peak based IF estimator [1, 3] through simulations. We consider a single-user code-on-pulse DSSS communication system with BPSK modulation and processing gain  $N=15$  chips per bit. To focus on the effect of interference cancellation on BER performance, it is assumed that perfect phase and code synchronization exist, and that the channel is ideal (that is, multi-path effects are ignored). To avoid the effects of a specific spreading PN code, we independently generate a random binary sequence as the PN code for each bit. The received signal is down-converted to baseband and sampled at the rate of 8 samples per chip. The interference suppression filtering procedure is then applied on data blocks. Each data block, of size 128 samples, consists of the current bit ( $8 \times 15 = 120$  samples) and the last chip of the previous bit (8 samples), which serves to cover the initial transient of the filter which is a time-varying short FIR filter. Bit decisions are made based on the current-bit portion of the filter output. A real signal is used in the modeling, filtering, and decision process. A length-5 symmetric (zero-phase) time-varying double-zero notch filter [1] is formed according to the TVAR IF estimate. The bit-error-rate (BER) is evaluated by estimating the mean and variance of the decision statistic, which is assumed Gaussian. BERs estimated in this way were verified via long simulations, by counting error events.

##### 4.1 Linear FM Interference

We first consider a system as described in Figure 1 with a jammer which chirps linearly from 0.1 to 0.4 Hz (normalized to unit sampling rate) according to  $f(t) = 0.1 + 0.3t/127, t = 0, 1, \dots, 127$ , inside each data block. The model used is a second-order

autoregressive model ( $p=2$ ) with the time-varying coefficient represented as a fifth-order polynomial function ( $q=5$ ). The BER vs. the jammer-to-signal ratio (JSR), defined as the power ratio before the filter, is compared among the filters using different IF estimates. These include the TVAR method, WVD peak based IF estimates using 128-point FFT (without zero-padding) and zero-padding to 512-point, and the known exact IF, as well as the plain receiver without filtering. Figure 2 shows a simulation result when the SNR is at  $E_b/N_0 = 10\text{dB}$  (where  $E_b$  stands for the signal bit energy and  $N_0$  is the single-sided noise power spectral density).

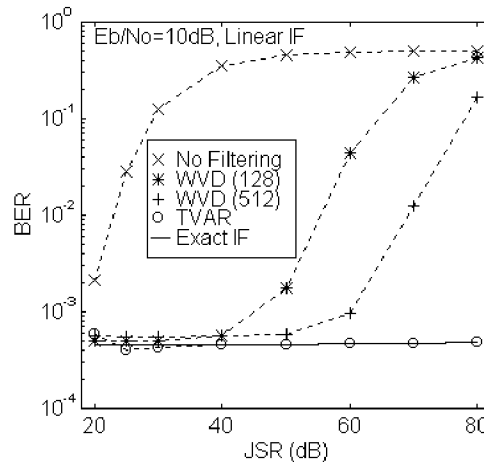


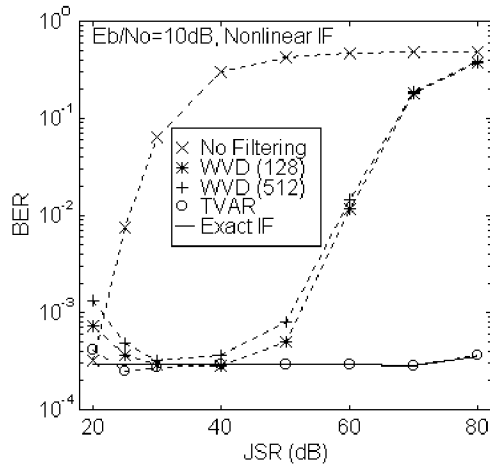
Figure 2. Linear IF law, BER vs. JSR for different IF estimates and exact IF.

In this case, it is observed from Figure 2 that the TVAR method significantly outperforms the WVD based method for high JSR. The interference remaining after WVD based filtering is substantial for JSR over 40dB. The WVD (without zero-padding) based filter reduces the apparent interference power by about 30dB. Zero-padding to 512 points, at a cost of computational effort, reduces the frequency quantization error and hence increases the filtering gain by another 15dB. By contrast, the BER curve of the TVAR based method overlaps well with the one using the exact IF and the associated filtering gain appears to be nearly without limit. This is attributed to the high precision of the TVAR based IF estimation.

## 4.2 Non-Linear FM Interference

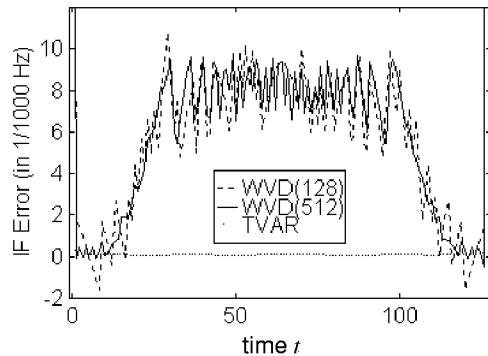
In the linear FM example, the BER performance of the WVD method can be improved by reducing the IF quantization error via zero-padding. For a nonlinear FM jammer this may not be the case since WVD based IF estimation is biased in this case. Consequently, the TVAR method is more advantageous for cases where FM interference linearity can not be assumed *a priori*.

We replace the linear FM jammer with a nonlinear FM law, which is  $f(t) = 0.1 + 0.3\sin(\pi t/128)$ ,  $t = 0, 1, \dots, 127$ . We run a similar simulation and the result is shown in Figure 3. Here the order  $q$  in the TVAR model is increased to 11 to accommodate more detailed frequency variation. Note that, in this case, zero-padding does not improve the performance of the WVD based method. The TVAR based method is not sensitive to the nonlinearity and performs as well as using the exact IF.



**Figure 3.** Non-linear IF law, BER vs. JSR for different IF estimates and exact IF.

Figure 4 shows IF estimation errors,  $f(t) - \hat{f}(t)$ , for a simulation realization at JSR=80dB, to offer an explanation of these results.



**Figure 4.** IF estimation error comparison for the nonlinear FM case with JSR=80dB.

The WVD based estimates are obviously biased toward the inner direction of the IF curve (positive error). Further calculation shows that the summed squared error in the center half of the

block, for  $t = 33, 34, \dots, 96$ , is 0.0040 for the 128-point WVD, 0.0042 for the 512-point WVD, and  $3.3 \times 10^{-8}$  for the TVAR method, respectively.

## 5. CONCLUSION AND DISCUSSION

We applied TVAR based IF estimation to notch filtering based interference suppression for DSSS communications. This method provides superior performance compared to using the WVD. This conclusion may also apply for other time-frequency distributions that can be viewed as smoothed versions of the WVD. We use a parametric model rather than a 2-D distribution. This also leads to much less computational cost, and thus it is more suitable for real time implementation. In addition, the TVAR method is able to identify multi-component jammers without first separating them.

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