

BER COMPUTATION OF AN MC-CDMA SYSTEM WITH CARRIER FREQUENCY OFFSET

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ABSTRACT

The effect of carrier frequency on the performance of an MC-CDMA (Multi-Carrier Code Division Multiple Access) system is considered. Bit error probability of the downlink of this system is analyzed taking into the effect of frequency offset. This paper will be focused on the derivation of the bit error probability mathematically.

Derived results show that the performance of MC-CDMA is very sensitive even to a relatively minute degree of frequency offset. Parameters such as degree of carrier offset, number of subcarriers, number of users, method of combining, and type of channel will be varied so that the influence of each parameter can be assessed.

1. INTRODUCTION

Narrowband communication systems are relatively immune to intersymbol interference as their symbol duration is much longer than the delay spread of the channel [1]. However, because of this a narrowband signal experiences the undesirable effect of flat fading.

To combat the adverse effect of flat fading, conventional DS-CDMA spread spectrum may be applied so that the bandwidth of the signal is spread much wider than the coherence bandwidth. By doing so, the possibility of the entire signal being lost in deep fades becomes much smaller [2]~[3]. The drawback in this scheme is that the signal is more susceptible to inter-chip interference. For reliable detection of the transmitted signal, complicated receiver structures such as a Rake receiver is necessary. Although the implementation of the Rake receiver is possible, there are complications and limitations that arise for certain applications.

Multi-carrier transmission schemes were conceived and are researched to overcome the limitations presented by narrowband and spread spectrum transmission schemes while still retaining their desirable qualities [3]. The advantage of using multi-carrier transmission schemes is it is possible to spread the signal bandwidth without the adverse effect of delay spread. This is possible because an MC-CDMA signal is composed of a number of narrowband subcarrier signals, each of which has a symbol duration much larger than the delay spread. As a result, an MC-CDMA signal does not experience an increase in susceptibility to delay spreads and intersymbol interference as does a DS-CDMA signal.

Another advantage of using multi-carrier transmission is that

channel equalization can be performed relatively easily. The bandwidth occupied by the signal carried by a single subcarrier is usually narrower than the coherence bandwidth of the system. Therefore, it is possible to assume that the subcarriers experience independent flat fading.

The disadvantages in using multi-carrier transmission schemes include difficulty in implementation of transmitter/receiver units and sensitivity to frequency synchronization errors and nonlinear amplifications [4]~[6]. This paper is focused on analyzing the effect of frequency synchronization errors mathematically

The paper is organized as follows. In Section 2, the basic MC-CDMA system under consideration will be described along with the channel model. In addition to this the carrier frequency offset considered throughout the paper will be explained. In Section 3, bit error probability of an MC-CDMA system will be derived in the presence of carrier frequency offset. Numerical results will be given in Section 4 followed by conclusions in Section 5.

2. MULTI-CARRIER CDMA SYSTEM AND CHANNEL MODEL

2.1 MC-CDMA System Configuration

The block diagram of a downlink MC-CDMA system using M subcarriers is shown in Figure 1. For simplicity, only one of the total K users is taken into account. The k^{th} user's i^{th} binary data symbol, $a_k[i]$ having a symbol duration of T is multiplied with a chip of the user specific Walsh Hadamard code $c_k[l]$ ($l=1 \sim M-1$) and serial/parallel converted. Each parallel branch is then binary phase-shift keying modulated to a corresponding subcarrier.

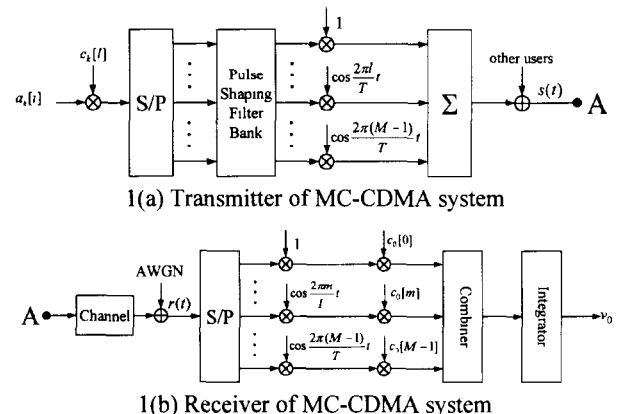


Figure 1. Simplified diagram of a downlink MC-CDMA system detecting for user 0.

2.2 Multi-Carrier Channel Model

The channel of the MC-CDMA under consideration has been modeled as one that consists of a number of independent narrowband fading subchannels. The l^{th} subcarrier is assumed to experience an independent fading subchannel which can be expressed as follows:

$$h_l = \rho_l e^{j\phi_l}, \quad (1)$$

where ρ_l and ϕ_l represent the amplitude and phase response of the fading channel, respectively. In the following analysis, ρ_l is either a Rayleigh or a Rician random variable and ϕ_l is a uniformly distributed random variable distributed in $[0, 2\pi]$.

2.3 Carrier Frequency Offset

The carrier frequency offset considered throughout this paper is the error made during the carrier tracking process of communication and is a fraction of the minimum subcarrier spacing $1/T$, as shown in Figure 2.

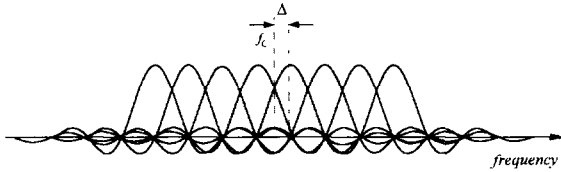


Figure 2. Carrier frequency offset of Δ from carrier frequency f_c

3. ANALYSIS

3.1 Transmitted and Received Signals

The transmitted signal $s(t)$ in Figure 1 can be expressed as

$$s(t) = \sum_{k=0}^{K-1} \sum_{l=0}^{M-1} a_k[i] c_k[l] p(t) e^{\frac{j2\pi l t}{T}}, \quad (2)$$

where $p(t)$ is the shape of the transmitted pulse. After passing through a fading channel of equation (1), Additive White Gaussian Noise (AWGN) is added and the following signal $r(t)$ is received.

$$r(t) = \sum_{k=0}^{K-1} \sum_{l=0}^{M-1} \rho_l a_k[i] c_k[l] p(t) e^{\frac{j2\pi l t}{T}} e^{j\phi_l} + n(t), \quad (3)$$

where $n(t)$ is AWGN having a one sided power spectrum density of N_0 . Signal carried in each carrier is extracted from the received signal and multiplied with a chip of the user' designated code. The resulting signals are combined using specific combining algorithms. In this paper only Equal Gain Combining (EGC) and Maximal Ratio Combining (MRC) will be considered. Assuming that there is a carrier frequency offset of Δ ($-0.5 \leq \Delta \leq 0.5$), the decision variable v_0 can be derived as follows.

$$v_0 = \frac{1}{\pi} \sum_{m=0}^{M-1} \sum_{k=0}^{K-1} \sum_{l=0}^{M-1} \rho_l \chi_m a_k[i] c_k[l] \frac{\sin \pi \Delta}{\Delta + m - l} e^{j(\phi_l - \phi_m)} + \eta, \quad (4)$$

where χ_m is the gain multiplied to the signal components extracted from each subcarrier. χ_m equals 1 when EGC is used and channel amplitude, ρ_m .

3.2 Signal Decomposition

The decision variable in equation (4) can be decomposed into the following signal components.

$$D: \frac{\sin \pi \Delta}{\pi \Delta} a_0[i] \sum_{m=0}^{M-1} \rho_m \chi_m, \quad (5)$$

$$MAI: \frac{\sin \pi \Delta}{\pi \Delta} \sum_{m=0}^{M-1} \sum_{k=1}^{K-1} \rho_m \chi_m a_k[i] c_k[m] c_0[m], \quad (6)$$

$$ICI_1: \frac{\sin \pi \Delta}{\pi} \sum_{m=0}^{M-1} \sum_{l=0}^{M-1} \rho_l \chi_m a_0[i] c_0[l] c_0[m] \frac{1}{\Delta + m - l} e^{j(\phi_l - \phi_m)}, \quad (7)$$

$$ICI_2: \frac{\sin \pi \Delta}{\pi} \sum_{m=0}^{M-1} \sum_{k=1}^{K-1} \sum_{l=0}^{M-1} \rho_l \chi_m a_0[i] c_k[l] c_0[m] \frac{1}{\Delta + m - l} e^{j(\phi_l - \phi_m)}, \quad (8)$$

$$AWGN: \eta, \quad (9)$$

where the terms D , MAI , ICI_1 , and ICI_2 denote the desired signal, the multiple access interference, the intercarrier interference generated from the chips within user 0's code, and the intercarrier interference generated from the chips in user 0's code and the chips in the other $K-1$ users' codes, respectively.

From equations (5)~(9), following observations can be made concerning the effect of carrier frequency offset. The desired signal has been attenuated by a factor that is a function of Δ . Although multiple access interference has also been reduced, because of Δ , there are terms that were not present when frequency offset was zero, namely, the terms ICI_1 and ICI_2 . These terms will act as additional interference to the multiple access interference considered in conventional MC-CDMA analysis.

In equation (6), the average multiple access interference for each subcarrier is dependent only on the ratio of K to M . Therefore, for two systems having equal K/M ratios, their average multiple access interferences for each subcarrier would also be equal. However, unlike the MAI term, the ICI terms in an MC-CDMA system is a direct function of the number of subcarriers and the number of users. Therefore, if the total numbers of subcarriers two system have are different, the ICI term in each system will differ even if their K/M ratios are the same. In short, a system having more subcarriers will suffer more performance degradation from carrier frequency offset even if their K/M ratios are equal.

The degree of degradation caused by carrier frequency offset may be observed by finding the BER. In order to find the BER which takes into account the effect of carrier frequency offset, we need to find the statistics of the interference and noise components. The statistical values of the terms MAI , ICI_1 , and ICI_2 have different values depending on what kind of fading channel the transmitted signal passes through.

In case of EGC and MRC, the MAI terms can be approximated as zero mean Gaussian random variables having the following variances:

$$\begin{aligned} \text{var}[MAI]_{EGC}^{Rayleigh} &= \left(\frac{\sin \pi \Delta}{\pi \Delta} \right)^2 (K-1)M \text{var}[\rho_l] \\ &= 2 \cdot \left(\frac{\sin \pi \Delta}{\pi \Delta} \right)^2 (K-1) \left[1 - \frac{\pi}{4} \right] \bar{p}, \end{aligned} \quad (10)$$

$$\begin{aligned} \text{var}[MAI]_{EGC}^{Rician} &= \left(\frac{\sin \pi \Delta}{\pi \Delta} \right)^2 (K-1)M \text{var}[\rho_l] \\ &= \left(\frac{\sin \pi \Delta}{\pi \Delta} \right)^2 (K-1) \left[1 - \frac{\pi}{4} \left(\frac{e^{-J}}{J+1} \right) \left[(1+J)I_0 \left(\frac{J}{2} \right) + JI_1 \left(\frac{J}{2} \right) \right]^2 \right] \bar{p}, \end{aligned} \quad (11)$$

$$\begin{aligned} \text{var}[MAI]_{MRC}^{Rayleigh} &= \left(\frac{\sin \pi \Delta}{\pi \Delta} \right)^2 (K-1)M \text{var}[\rho_l^2] \\ &= 4 \cdot \left(\frac{\sin \pi \Delta}{\pi \Delta} \right)^2 \left(\frac{K-1}{M} \right) \bar{p}^2, \end{aligned} \quad (12)$$

$$\begin{aligned} \text{var}[MAI]_{MRC}^{Rician} &= \left(\frac{\sin \pi \Delta}{\pi \Delta} \right)^2 (K-1)M \text{var}[\rho_l^2] \\ &= \left(\frac{\sin \pi \Delta}{\pi \Delta} \right)^2 \left(\frac{K-1}{M} \right) \frac{8J+4}{(J+1)^2} \bar{p}^2, \end{aligned} \quad (13)$$

where \bar{p} denotes the total mean power of each user defined by

$$\bar{p} = \frac{M}{2} E[\rho_l^2], \quad (14)$$

while $I_0(\cdot)$ and $I_1(\cdot)$ are modified Bessel functions of zeroth and first order, respectively. J is the Rician factor defining the ratio of the power of the LOS (Line Of Sight) component to the power of the scattered component. $E[\cdot]$ and $\text{Var}[\cdot]$ denote the mean and variance of the term inside the brackets.

The ICI terms are also approximated as zero mean Gaussian random variables having the following variances.

$$\text{var}[ICI_1] = E[ICI_1^2], \quad (15)$$

$$\text{var}[ICI_2] = (K-1)E[ICI_1^2], \quad (16)$$

Detailed derivations are given in [6].

The $AWGN$ terms are zero mean Gaussian random variables having the following variances for EGC and MRC, respectively.

$$\text{var}[AWGN]_{EGC} = M \frac{N_0}{T}, \quad (17)$$

$$\text{var}[AWGN]_{MRC} = 2 \frac{N_0}{T} \bar{p}. \quad (18)$$

3.3 Bit Error Probability

Assuming that the transmitted bit for user l was '-1' and Δ_0 is the frequency offset experienced by user l , the bit error probability of an MC-CDMA system conditioned on the amplitudes of the channel defined by equation (2) can be expressed in a mathematical form as the following

$$\begin{aligned} p(\text{error}|\Delta_0, \{\rho_l\}) &= p(-D < MAI + ICI_1 + ICI_2 + AWGN) \\ &= \int_{-D}^{\infty} \frac{1}{\sqrt{2\pi(\sigma_{\text{int}}^2 + \sigma_{\eta}^2)}} e^{-\frac{y^2}{2(\sigma_{\text{interference}}^2 + \sigma_{\eta}^2)}} dy \\ &= \frac{1}{2} \text{erfc} \left(\frac{\frac{1}{2} D^2}{\sigma_{\text{interference}}^2 + \sigma_{\eta}^2} \right) \\ &= \frac{1}{2} \text{erfc} \left(\frac{\frac{1}{2} \left(\frac{\sin \pi \Delta}{\pi \Delta} \sum \rho_l \right)^2}{\sigma_{\text{interference}}^2 + \sigma_{\eta}^2} \right) \quad \text{for EGC} \\ &= \frac{1}{2} \text{erfc} \left(\frac{\frac{1}{2} \left(\frac{\sin \pi \Delta}{\pi \Delta} \sum \rho_l^2 \right)^2}{\sigma_{\text{interference}}^2 + \sigma_{\eta}^2} \right) \quad \text{for MRC} \end{aligned} \quad (19)$$

where

$$\sigma_{\text{interference}}^2 = \text{var}[MAI] + \text{var}[ICI_1] + \text{var}[ICI_2]. \quad (20)$$

Averaging equation (20) over the distribution of instantaneous amplitude ($\rho_l, l=0, M-1$), the following BER can be obtained for a Rayleigh and a Rician fading channel, respectively.

$$\text{BER}_{EGC} \cong \frac{1}{2} \text{erfc} \left(\frac{\frac{1}{2} E \left[\frac{\sin \pi \Delta}{\pi \Delta} \sum \rho_l \right]^2}{\text{Var} \left[\frac{\sin \pi \Delta}{\pi \Delta} \sum \rho_l \right]^2 + \sigma_{\text{interference}}^2 + \sigma_{\eta}^2} \right), \quad (21)$$

$$\text{BER}_{MRC} \cong \frac{1}{2} \text{erfc} \left(\frac{\frac{1}{2} E \left[\frac{\sin \pi \Delta}{\pi \Delta} \sum \rho_l^2 \right]^2}{\text{Var} \left[\frac{\sin \pi \Delta}{\pi \Delta} \sum \rho_l^2 \right]^2 + \sigma_{\text{interference}}^2 + \sigma_{\eta}^2} \right). \quad (22)$$

4. BER COMPARISON

Figure 3 shows the BER curves of an MC-CDMA system with 64 subcarriers experiencing different carrier frequency offsets. EGC is used at the receiver end and a Rayleigh/Rician fading channel is assumed. As the degree of carrier frequency offset increases, so does the performance deterioration. At a carrier frequency offset of 4%, a BER of 10^{-3} is no longer possible when transmitting through a Rayleigh channel. In case of transmitting through a Rician channel, a BER of 10^{-4} is no longer possible if the carrier frequency offset is 4%.

Figure 4 show the BER curves of an MC-CDMA system with 64 subcarriers experiencing different carrier frequency offsets. MRC is used at the receiver end and a Rayleigh/Rician fading

channel is assumed. Observations similar to those made for Figure 3 can be made. Note that the overall performance of MRC is inferior to that of EGC. This is a result of the squaring operation done in MRC which significantly increases the orthogonality gap between the codes.

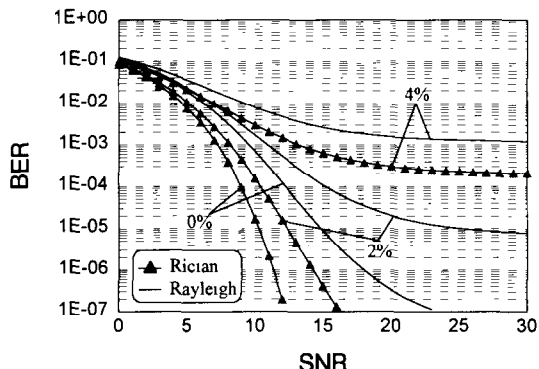


Figure 3. BER of an MC-CDMA with 64 subcarriers and 8 users experiencing 0, 2, 4% of carrier frequency offset. (EGC, channel : Rayleigh and $J=10$ Rician)

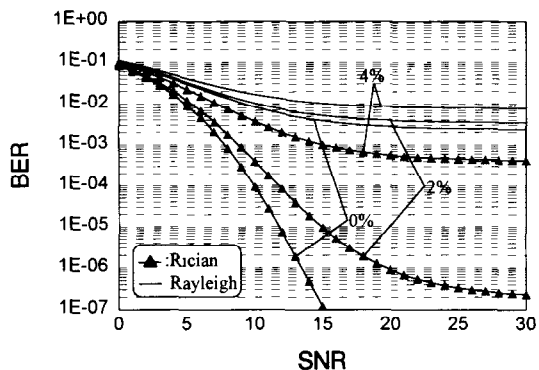


Figure 4. BER of an MC-CDMA with 64 subcarriers and 8 users experiencing 0, 2, 4% of carrier frequency offset. (MRC, channel : Rayleigh and $J=10$ Rician)

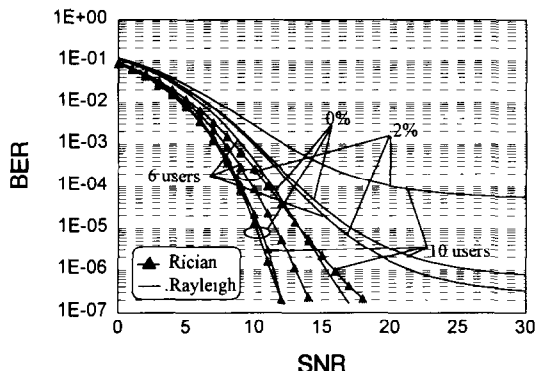


Figure 5. BER of an MC-CDMA with 64 subcarriers and 6, 10 users experiencing 3% of carrier frequency offset. (EGC, channel : Rayleigh and $J=10$ Rician)

Figure 5 and 6 show the BER curves of an MC-CDMA system with 64 subcarriers experiencing 0 and 2% of carrier frequency offset in a Rayleigh/Rician channel. The Rician factor J is set at

10. The number of users is varied in both figures. As the number of users increases, the degradation in performance increases as well because of the fact that the ICI_2 term in equation (8) is proportional to the number of users. Therefore, for two systems having the same number of subcarriers and experiencing the same degree of carrier frequency offset, the system having a larger number of users will experience more performance deterioration than the system having a smaller number of users

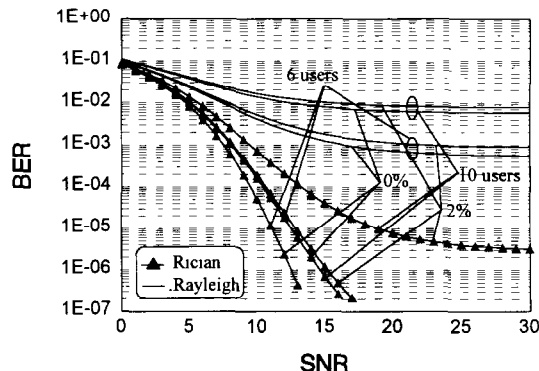


Figure 6. BER of an MC-CDMA with 64 subcarriers and 6, 10 users experiencing 3% of carrier frequency offset. (MRC, channel : Rayleigh and $J=10$ Rician)

5. CONCLUSIONS

The performance of an MC-CDMA system has been analyzed taking into account the effect of carrier frequency offset. The derived BER shows that the performance of an MC-CDMA system is sensitive even to a relatively small degree of carrier frequency offset. In addition, results show that the degree of degradation in an MC-CDMA system due to carrier frequency offset is a function of not only the degree of frequency offset but also the number of subcarriers and the number of users. Numerical results have verified the validity of the derivations and the assessments made.

6. REFERENCES

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