FILTER BANK OPTIMIZATION FOR HIGH-DIMENSIONAL COMPRESSION OF PRE-STACK SEISMIC DATA

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ABSTRACT

A multi-dimensional variable-length subband coder for pre-stack seismic data was presented. A 2- and 3-D separable near perfect reconstruction filter bank was optimized to maximize the coding gain, assuming that the correlation properties of pre-stack seismic data can be modeled by directional dependent autoregressive processes. Identical quantization and entropy coder allocation strategies were utilized to isolate the compression efficiency of the different high-dimensional filter bank methods. An example, with compression ratios ranging from 160:1 to 320:1, showed that 3-D subband coding of common shot gathers performed 50 % better in terms of bit rate at a given signal-to-noise ratio compared to 2-D subband coding of common shot gathers.

1. INTRODUCTION

Acquisition of seismic data in marine exploration generates large datasets which today are managed by gigabyte tape storage units in combination with terabyte disk storage systems. Seismic data compression makes the data storage easier and has a potential of reducing the time for network transfer from hours to minutes.

Previous work on lossy seismic data compression [1, 2] based on discrete wavelet transform (DWT) coding, which is a compression technique strongly related to subband coding, show that the data organization (e.g., dimensionality and sorting) is important in order to achieve large compression ratios with an acceptable noise-level. Using a 2-D data organization, compression ratio of 50:1[1] [1] for pre-stack data is achieved without introduction of noticeable degradation. With 3- and 4-D data organizations compression ratios of 100:1 and 300:1 [2], respectively, are achieved for pre-stack data with small presented distortion due to increased redundancy. In this context, high-dimensional data organization does not mean a 3- or 4-D data survey in general, but rather datasets structured into one perpendicular direction (temporal or spatial) and at least two spatial dimensions at the sea surface (sensors and shots).

High-dimensional DWT coding of seismic data in particular has been investigated by Reiter [3] and by Villasenor et al. [4]. This topic is specially appealing because seismic data of nature are highly anisotropic and contain high amount of noise [5]. A full utilization of the dimensionality makes the seismic data compression easier. However, it becomes more involved to restore for instance single traces after high-dimensional coding. In addition, up to 10 % of a seismic survey is auxiliary data which is maximally compressed 10:1 by sophisticated lossless methods. Thus, compression ratios much greater than 100:1 provide less marginal gain [4]. Nevertheless, the distortion should be as small as possible for a given target compression ratio, say > 100:1, and using additional dimensions in seismic data compression is therefore attractive.

In a previous article [6], we proposed a subband coder for 2-D post-stack data which represents state-of-the-art among developed methods. In this paper, a high-dimensional compression scheme for 2- and 3-D[2] pre-stack data based on subband decomposition followed by uniform threshold quantization and entropy coder allocation is proposed. We basically address two issues: Firstly, a 3-D separable near perfect reconstruction filter bank for seismic cubes (i.e., 3-D pre-stack data) is optimized with respect to coding gain [7]. We keep the quantization and the entropy coder allocation constant, irrespective of non-optimality, to isolate the compression efficiency of the multi-dimensional subband decomposition techniques. Secondly, a proper seismic data organization is investigated. The distortion rate function of multi-dimensional seismic data compression is given in addition to quantitative comparisons between 2- and 3-D subband seismic data compression examples.

2. SUBBAND CODING

Subband coding is a popular algorithm for image compression [8]. The principles of subband image coding is based on the decomposition of the image into narrow spectral subbands by a separable analysis filter bank. Each subband is then decimated to keep the total number of samples unchanged compared to the original image representation, and finally coded. At the receiver side, the subband image is decoded and the image is reconstructed by interpolation in a separable synthesis filter bank. Likewise, subband coding of 3-D data requires a subband decomposition in the third direction. Unlike typical video sequences, the amount of "motion" is low in seismic cubes and the energy compaction will be high even without block motion estimation and compensation [4]. We therefore do not include block motion adaption [9] in our algorithm. In fact, such a block approach will perform very poorly when used on seismic data. Our compression scheme is a full-frame method which in principle can contain an arbitrary number of dimensions. This corresponds directly to seismic data acquisition methods since no partitioning of the seismic data will be required.

3. SEISMIC DATA

We consider 2-D pre-stack data that can be sorted to common shot gathers (CSGs, Figure 1 (a)) or common offset gathers (COGs, Figure 1 (b)) depending on the application [5]. In the case of a plane layer, the two-way travel time curve is non-flat for a CSG and flat for a COG. Hence, a COG image (see Figure 2 (b)) usually contains

[1] A compression ratio of c:1 corresponds to 32/c bits per sample.

[2] 3-D data organization.

(a) CSG arranging.

(b) COG arranging.

Figure 1: Illustration of CSG and COG sorting in the case of a single horizontal interface. The arrows indicate two-way travel times.

higher correlation and is simpler to compress in 2-D coding than a CSG image (see Figure 2 (a)).

A dataset consisting of 176 shots with 120 sensors per shot and 1000 samples per trace with 32 bits per sample was coded-decoded at very low bit rates ($\leqslant 0.2$ bits per sample) with different high-dimensional filter bank schemes. The dataset was preceded by 240 bytes per trace SEGY header information [5] which was stripped off and totally neglected.

4. SYSTEM DESCRIPTION

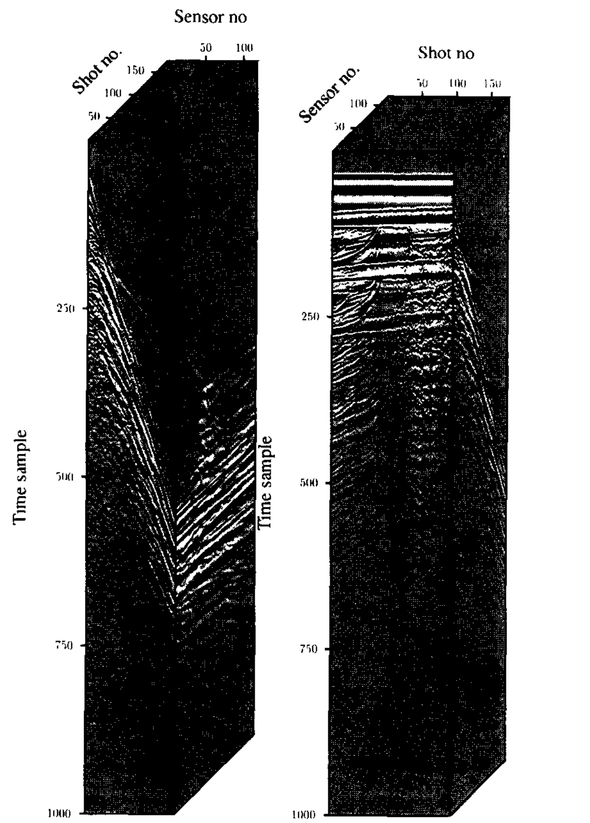
A multi-dimensional compression scheme based on subband decomposition followed by uniform threshold quantization and entropy coder allocation was applied to 2- and 3-D pre-stack data.

Four subband decomposition schemes were evaluated: Firstly, the sensor-time planes, e.g. CSG images, were filtered with a 2-D filter bank (method **Ia**) and in the 3-D case the shot-direction was additionally filtered by a transform as the third step (method **Ib**). Secondly, the shot-time planes, e.g. COG images, were filtered with a 2-D filter bank (method **IIa**) and in the 3-D case the sensor-direction was additionally filtered by a transform as the third step (method **IIb**). The difference in data organization between methods **Ia** to **IIb** is seen in Figure 2. Intuitively, method **IIa** performs better than method **Ia** in 2-D coding. The situation will most likely be reversed in 3-D coding, i.e. method **Ib** is better than method **IIb**, because CSG cubes have greater redundancy in the third direction than COG cubes.

4.1. Statistical Analysis

Efficient coder optimization calls for good statistical models of the signal to be compressed. In our case, two models are required. One for handling the statistics of the original signal to optimize the filter bank with respect to coding gain, and another statistical model of the subband signal to optimize the variable-length coder with respect to distortion rate performance. For the first model, we use a separable autoregressive (AR) process [10] fitted to the correlation of the seismic cubes shown in Figure 2. For the second model, on the other hand, we use a Gaussian mixture distribution representation [11].

In the case of CSG and COG data, the sample to sample correlation is much higher in the shot-direction as compared to the sensor- and time-direction. Figure 3 shows the normalized estimated autocorrelation functions (acfs) in the three directions for the seismic cubes in Figure 2. We choose a directional dependent separable statistical model with different AR processes to describe the acfs. A



(a) $120 \times 1000 \times 176$ CSG volume used in method **I**.

(b) $176 \times 1000 \times 120$ COG volume used in method **II**.

Figure 2: The seismic cubes where an image is a horizontal-vertical slide of the volume.

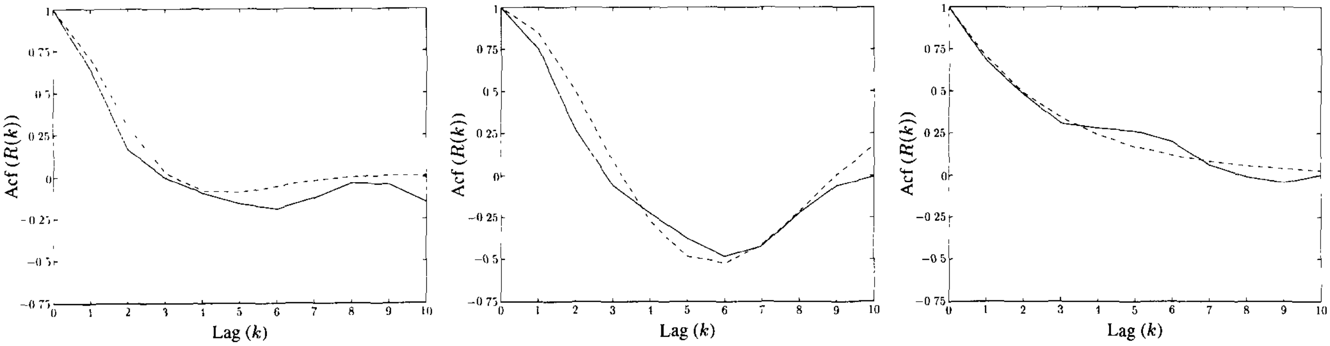
first-order AR (AR(1)) model is used to represent the correlation of the pre-stack data in the shot-direction, while AR(2) models are selected in the sensor- and time-direction. The correlation coefficients which provide a close fit to the normalized estimated acfs are given in Table 1, and their respective acfs are depicted in Figure 3.

Table 1: The selected AR processes and correlation coefficients.

Direction	Model	Coefficients	
Sensor	AR(2)	$\rho_1 = 0.70$	$\rho_2 = 0.30$
Time	AR(2)	$\rho_1 = 0.85$	$\rho_2 = 0.50$
Shot	AR(1)	$\rho_1 = 0.70$	

4.2. Filter Bank Optimization

The 2- and 3-D, separable, parallel, uniform, non-unitary, linear phase, FIR filter bank was optimized with respect to coding gain according to the AR processes described in the last section. We used a 4-, 8- and 8-channel filter bank in the sensor-, time- and shot-direction, respectively. Horizontally and vertically, the impulse re-



(a) Sensor-direction. (b) Time-direction. (c) Shot-direction.

Figure 3: The normalized estimated acfs (solid lines) and the acfs to the utilized AR processes (dashed lines).

sponses had lengths equal to four times the number of channels. In the third direction, however, the impulse responses had lengths equal to the number of channels (i.e., a square transform) because this provided us with a simple non-expansive subband decomposition method. A square transform may be regarded as a special case of a uniform filter bank, with the basis functions of a transform interpreted as the impulse responses of a filter bank [8]. We refer to Table 2 for a summary.

Table 2: Number of channels and taps.

	Number of channels/taps			
	Method **I**		Method **II**	
Direction	a	b	a	b
Sensor	4/16	4/16		4/4
Time	8/32	8/32	8/32	8/32
Shot		8/8	8/32	8/32

5. DISTORTION RATE FUNCTION

Obviously, it is advantageous with respect to compression efficiency to exploit all dimensions [4]. To motivate this, we calculate the distortion rate function (DRF) based on the assumptions of stationarity, separability and unit variance. In the high bit rate (i.e., $\gtrsim 1$ bit per sample) N-D-case the DRF can be written as [7]:

$$D(R) = \left(\prod_{n=1}^{N} \gamma_{x,n}^2 \right) 2^{-2R}, \quad N = 1, 2 \text{ and } 3, \tag{1}$$

where D is the mean-square error (MSE) as a function of the bit rate R, and $\gamma_{x,n}^2$ is the spectral flatness measure (sfm) of the AR process in the nth-direction. $n = 1, 2$ and 3 denote the time-, sensor- and shot-direction, respectively. For an AR(1) process the sfm is given by [7]

$$\gamma_x^2 = (1 - \rho_1^2). \tag{2}$$

Similarly, for an AR(2) process the sfm is given by [7]

$$\gamma_x^2 = \frac{(1+b_2)(1-b_1-b_2)(1+b_1-b_2)}{(1-b_2)}, \tag{3}$$

$$\text{with} \quad b_1 = \frac{\rho_1(1-\rho_2)}{1-\rho_1^2} \quad \text{and} \quad b_2 = \frac{\rho_2 - \rho_1^2}{1-\rho_1^2}.$$

D is normalized such that the signal-to-noise ratio (SNR)[3] can be written as $\text{SNR} = -10 \log_{10} D$. If we insert the AR processes and the correlation coefficients given in Table 1 into Equations 2 and 3, and finally use Equation 1, we can display the SNR as function of the bit rate (see Figure 4). For the sake of exemplification, at an SNR equal to 30 dB the bit rate is decreased approximately by 33 % and 20 % in the case of 3-D coding as compared to 1- and 2-D coding, respectively.

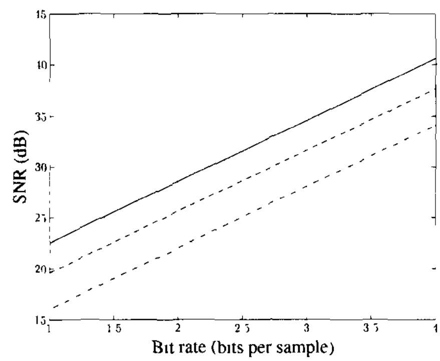


Figure 4: Distortion rate performance: 1-D coding of time traces (dashdotted line), 2-D coding of sensor-time images (dashed line) and 3-D coding of sensor-time-shot cubes (solid line).

In real simulations at low bit rates (i.e., $\ll 1$ bit per sample), we expect similar trends: 3-D coding performs better than 2-D coding, and 2-D coding performs better than 1-D coding. However,

[3]The SNR is defined as the ratio of the mean-square signal power to the mean-square error power and is usually given in dB.

in the high bit rate case we will not have an asymptotic match between the simulations and the distortion rate performance given in Figure 4. This is mainly due to the assumption that the overall statistical model is valid also when the local statistics vary. The assumption of separability, on the other hand, is more reasonable for seismic data than for most other types of data [4].

6. SIMULATIONS AND DISCUSSION

The purpose of the simulations was to evaluate the four subband decomposition methods described in Section 4.

The seismic cubes given in Figure 2 were coded-decoded at compression ratios ranging from 160:1 (0.2 bits per sample) to 320:1 (0.1 bits per sample). The SNR was used as a quality measure although the SNR is highly dependent on the bandwidth characteristic and the amplitude distribution (or balance) of the dataset used. A low bandwidth coded-decoded dataset normally has higher SNR than a high bandwidth dataset. A well balanced coded-decoded dataset (i.e., with approximately uniform amplitude distribution) tends to have lower SNR than a poorly balanced dataset despite that the actual quality on well balanced datasets are better [3]. Note that our dataset was not properly balanced. Nevertheless, the SNR is a simple and good scalar indicator when comparing 2- and 3-D subband filtering methods. The SNRs and the bit rates were averaged along the shot-direction for the CSG volume and along the sensor-direction for the COG volume.

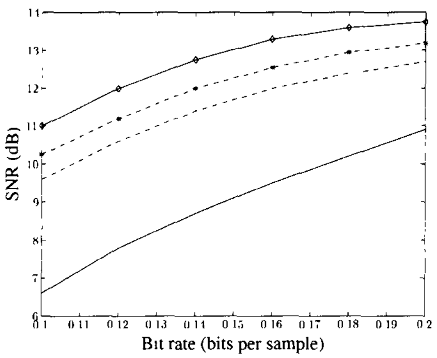


Figure 5: Simulation results for the four different subband decomposition methods: method **I**a (solid line), method **I**b (solid line with $\diamond$), method **II**a (dashed line) and method **II**b (dashed line with $*$).

The simulation results are shown in Figure 5. 3-D coding performs better than 2-D coding, and 3-D CSG coding is better than 3-D COG coding. This is consistent with our presumptions. For instance, at an SNR equal to 11 dB the bit rate is decreased as much as 50 % in the case of method **I**b as compared to method **I**a. Further improvements can be expected by optimizing the variable-length coder with respect to distortion rate performance.

7. SUMMARY

A high-dimensional subband compression scheme for pre-stack data was presented. The filter bank was optimized with respect to coding gain using a directional dependent separable statistical model with different autoregressive (AR) processes. A first order AR model was used to represent the correlation of the pre-stack data in the shot-direction, while second order AR models were used in the sensor- and time-direction. 3-D coding performed better than 2-D coding, and 3-D common shot gather coding was better than 3-D common offset gather coding.

Further work will include a study of how seismic data processing is affected by pre-stack data compression.

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