ON THE CONVERGENCE PROPERTIES OF MULTIDELAY FREQUENCY DOMAIN ADAPTIVE FILTER

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ABSTRACT

Frequency domain adaptive filters have gained much attention recently. Although some work on performance analysis has been reported, there is still much to be done. This paper presents a convergence analysis of the multidelay frequency domain adaptive filter. We show, for the first time, the relationship between the convergence step-size and the convergence rate. The effect of step-size on adaptation accuracy is presented also. Extensive simulation results are provided to support the analysis. Surprisingly, all block processing algorithms run well even though the step-sizes utilized are much bigger than the convergence bounds currently available in the literature.

1. INTRODUCTION

Adaptive digital filters have become very popular in many applications. In recent years, there has been popular attention in applications that require filters with very long impulse response [5]. For example, in acoustic echo cancellation, we may need thousands of filter coefficients to achieve the desired level of performance [4]. An attractive approach to reducing the prohibitive computational complexity associated with large filter coefficients is to use frequency domain adaptive filters [1],[4]-[6].

Several frequency domain adaptive filtering algorithms have been proposed in the past. For an application that requires L filter coefficients, the FLMS (fast least mean square) algorithm requires five $2L$-point FFT's for processing each L point block of data [1]. There are practical implementation problems of the FLMS such as long block delay and inefficient use of a hardware. By segmenting the filter into several partitions and using as many adaptive filters, [6] proposed the MDF (multidelay FLMS filter) that allows one to choose the size of an FFT. The MDF selects the desired block size N and the number of filters K in a way that $L=NK$. For simplicity, we can assume that L and N are power of 2 integers. Note that the MDF employs $2N$-point FFT's. A feature of the MDF is that the transform size and block delay all depend on N. A more general structure that allows one to select transform size and the resulting block delay independently is the GMDF (generalized MDF) [4].

In this paper, a simple performance analysis is given of the (multidelay) frequency domain adaptive filter. The properties of interest are convergence rate and adaptation accuracy. While [4] presented a comprehensive performance analysis, we show new results that are different from that in [4]. Extensive simulation results presented in the paper indicate that bounds of convergence step-size could be much higher than that given by [2] and [3]. The rest of the paper is organized as follows. Section 2 briefly reviews the time domain LMS and the multidelay frequency domain LMS. Section 3 presents a performance analysis of the MDF. Examples that demonstrate convergence properties are provided in Section 4. The concluding remarks are made in Section 5.

2. LMS ADAPTIVE FILTERS

2.1 Time Domain LMS

Let $x(n)$ and $d(n)$ represent the reference input and the desired output signal, respectively, to the adaptive filter. Let L denote the total number of filter coefficients. Define the $L \times 1$ coefficient vector $H(n)$ and the input vector $X(n)$ as

$$H(n) = [h_0(n), h_1(n), \cdots, h_{L-1}(n)]^T, \tag{1}$$

$$X(n) = [x(n), x(n-1), \cdots, x(n-L+1)]^T. \tag{2}$$

The LMS is described as

$$e(n) = d(n) - H^T(n)X(n), \tag{3}$$

$$H(n+1) = H(n) + \mu_S X(n)e(n). \tag{4}$$

In practice, we may replace (4) by

$$H(n+1) = H(n) + \frac{\mu}{X^T(n)X(n)+\sigma} X(n)e(n) \tag{5}$$

or

$$H(n+1) = H(n) + \frac{\mu}{Lr(0)} X(n)e(n) \tag{6}$$

where the positive step-size μ is bounded by 2, σ is a small positive number and $r(0)$ is the estimated autocorrelation function value of $x(n)$ for lag 0.

2.2 Frequency Domain Block LMS

We review block LMS implemented in the frequency domain. The idea is to carry out the time domain convolution of block LMS by overlap-save fast convolution. The FLMS developed by [1] processes and updates filter coefficients for each L samples of data. The MDF is another implementation of the FLMS. It segments the filter into K blocks in a way that $L=NK$. As a result, the MDF updates the weights for each N data it receives. Denote the vectors formed by the N new samples of x and d during the jth block iteration as

$$X_j = [x(jN), x(jN+1), \cdots, x(jN+N-1)]^T, \tag{7}$$

and

$$D_j = [d(jN), d(jN+1), \cdots, d(jN+N-1)]^T. \tag{8}$$

The coefficient vector H can be expressed as

$$H_j = [H_{0,j}^T, H_{1,j}^T, \cdots, H_{K-1,j}^T]^T, \tag{9}$$

where $H_{m,j}$ is the coefficient vector of the mth filter during the jth iteration. In equation (9), $H_{m,j}$ is an $N \times 1$ vector.

The frequency domain coefficient vector is obtained by taking FFT of an augmented $2N \times 1$ vector,

$$W_{m,j} = FFT[H_{m,j}^T, \underbrace{0,0,\cdots,0}_{N}]^T,\ m = 0, 1, \ldots, K\text{-}1, \tag{10}$$

and the frequency domain input vector is calculated as

$$U_j = FFT[X_{j-1}^T, X_j^T]^T. \tag{11}$$

The filter output vector Y_j is obtained as

$$Y_j = \text{ last } N \text{ elements of } FFT^{-1}[\sum_{m=0}^{K-1} W_{m,j} \bullet\!* U_{j-m}], \tag{12}$$

where $\bullet *$ represents element-to-element multiplication.

The $N \times 1$ time domain error vector is then given by

$$E_j = D_j - Y_j, \tag{13}$$

and the $2N \times 1$ frequency domain error vector is formed as

$$\Omega_j = FFT[\underbrace{0,0,\cdots,0}_{N}, E_j^T]^T \tag{14}$$

Frequency domain coefficient vectors are updated as

$$W_{m,j+1} = W_{m,j} + \frac{\mu_b}{N}\Psi_{m,j},\ m = 0, 1, \ldots, K\text{-}1 \tag{15}$$

where μ_b is the block step-size. Frequency gradient $\Psi_{m,j}$ is calculated as

$$\phi_{m,j} = \text{ first } N \text{ elements of } FFT^{-1}[\overline{U}_{j-m} \bullet\!* \Omega_j], \tag{16}$$

$$\Psi_{m,j} = FFT[\phi_{m,j}^T, \underbrace{0,0,\cdots,0}_{N}]^T, \tag{17}$$

where $\overline{U}_{j-m}$ is the complex conjugate of U_{j-m}.

3. CONVERGENCE PROPERTIES

Because the FLMS is simply a fast implementation of the block LMS algorithm, both algorithms have the same convergence property [5]. Note that the MDF is another implementation of the FLMS, it has the same convergence properties as well. Therefore, by employing the convergence results presented in [2], we conclude that the MDF and LMS algorithms converge at the same rate and achieve the same adaptation accuracy if $\mu_b = N\mu_S$.

We consider adaptive filters with normalization in the following. If we use (6) or (5) for the LMS, the associated frequency coefficients update equation of the MDF is

$$W_{m,j+1} = W_{m,j} + \frac{\mu_B}{N}\frac{1}{NKr(0)}\Psi_{m,j},\ m = 0, 1, \ldots, K\text{-}1 \tag{18}$$

It is straightforward to see that the MDF with equation (18) and the LMS with equation (6) have the same converge rate and adaptation accuracy if $\mu_B = N\mu$.

Assuming that $x(n)$ is white, then from Parseval's relation, we can approximate the power of input signal in each frequency bin by $2Nr(0)$. Let the $2N \times 1$ vector Z_j denote the estimated frequency domain power of the jth block iteration, we can rewrite equation (18) as

$$\begin{aligned} W_{m,j+1} &= W_{m,j} + \frac{2\mu_B}{NK}\left(\Psi_{m,j} \bullet / Z_j\right) \\ &= W_{m,j} + \frac{2\mu_B}{L}\left(\Psi_{m,j} \bullet / Z_j\right),\ m = 0, 1, \ldots, K\text{-}1 \end{aligned} \tag{19}$$

In equation (19), $\bullet /$ represents element-to-element division. In practice, Z_j can be obtained as

$$Z_j = \beta Z_{j-1} + (1-\beta)(\overline{U}_j \bullet\!* U_j), \tag{20}$$

where β is a weighting factor. The MDF with equations (19) and (20) for coefficients updating is referred to as the self-orthogonalization implementation [3],[4],[6].

With the assumption that $x(n)$ is white, equation (19) is an accurate implementation of equation (18). Therefore, we conclude that the self-orthogonalized MDF and the normalized LMS have the same convergence property provided that $\mu_B = N\mu$.

Based on [2] and [3], μ_B and μ have the same convergence bounds. However, results of all simulation presented in the paper indicate that convergence bound of μ_B could be much higher.

4. EXAMPLES DEMONSTRATING MDF CONVERGENCE PROPERTIES

Computer simulation was conducted to verify the analysis for the convergence rate and the adaptation accuracy. We consider the problem of system identification here. The system to be identified has an impulse response of 512 taps obtained by truncating an acoustic impulse response measured in a small office with 8000 Hz sampling rate. The excitation signal was white Gaussian with zero mean and variance 0.05. This setup gave an almost unit power of the system. White Gaussian noise of zero mean and variance 0.01 was added.

We have employed LMS algorithm with equation (6), the self-orthogonalized MDF, and the following BLMS (block LMS)

$$e(jN+l) = d(jN+l) - H_j^T X(jN+l),\ l = 0, 1, \ldots, N\text{-}1 \tag{21}$$

$$H_{j+1} = H_j + \frac{\mu_B}{N}\frac{1}{Lr(0)}\sum_{l=0}^{N-1} X(jN+l)e(jN+l). \tag{22}$$

Several cases were studied: three convergence step-sizes (μ =0.05, 0.1, and 0.2) were used for the LMS; four block sizes (N=512, 256, 128, and 64) were practiced for the MDF and block LMS. The convergence step-size μ_B for the block adaptive filters was selected as $\mu_B = N\mu$ (μ =0.05, 0.1, and 0.2) so that each algorithm should have the same performance properties. It is obvious to see that such setup violate the convergence condition of [2] and [3] for block adaptive filters.

We have conducted 10 independent runs for each case. Simulation results validate our analysis made in Section 3. Due to page limitation, only part of the results was presented. For the purpose of smoothing the curves, mean squared error samples are averaged over 32 points.

Learning curves for $\mu = 0.2$ are shown in Figure 1. Figure (1a) shows result for the LMS with $\mu = 0.2$; Figure (1b) for the BLMS with $\mu_B = \mu N = 51.2$ (N=256); Figure (1c) for the self-orthogonalized MDF with $\mu_B = \mu N = 102.4$ (N=512); and Figure (1d) for the self-orthogonalized MDF with $\mu_B = \mu N = 12.8$ (N=64). Similarly, learning curves for $\mu = 0.05$ are shown in Figure 2. It is easy to observe close agreement between analytical and experimental results.

5. CONCLUDING REMARKS

We have investigated the convergence properties of multidelay frequency domain adaptive filter. Extensive simulation results were provided to verify our analysis. Surprisingly, all block processing algorithms run well even though the step-sizes utilized are much bigger than the convergence bounds. We are currently investigating this issue.

6. REFERENCES

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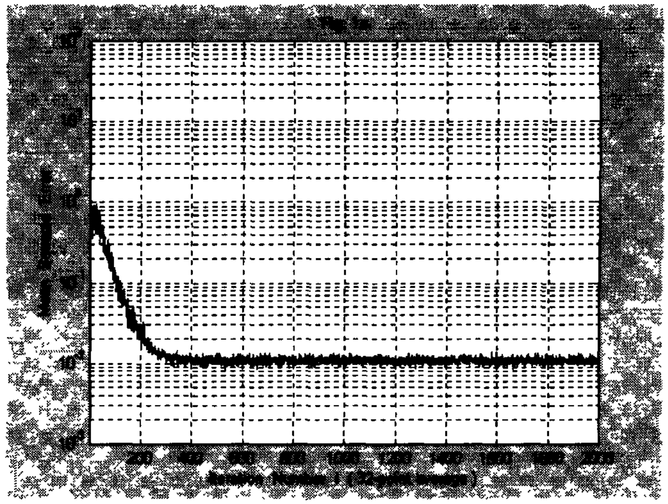
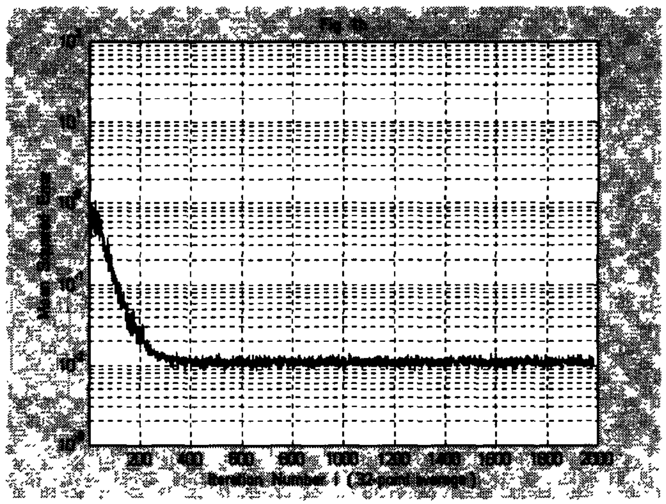
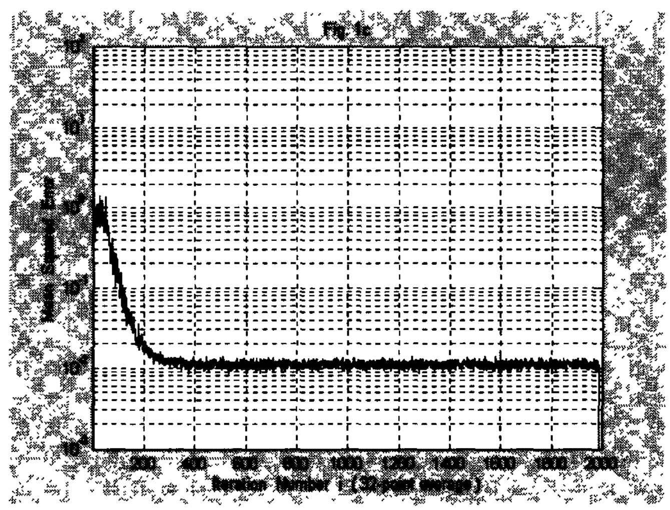
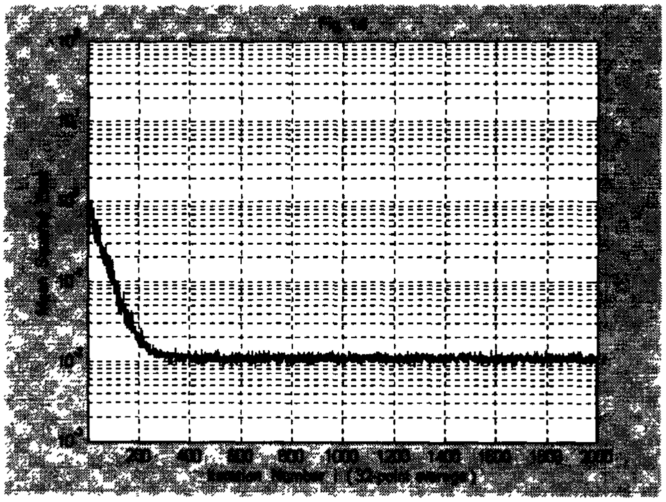








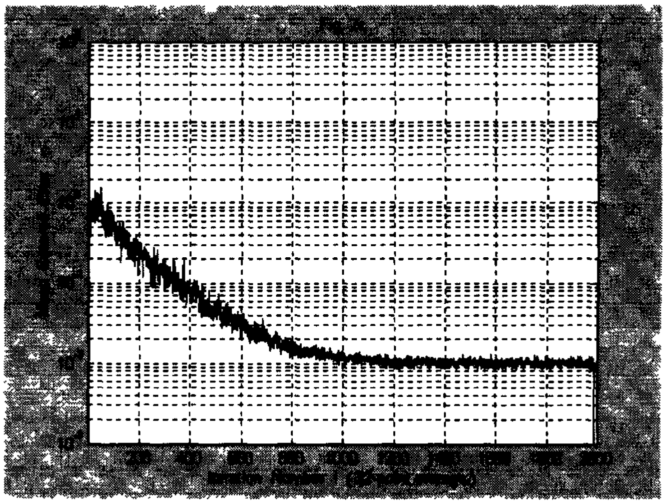
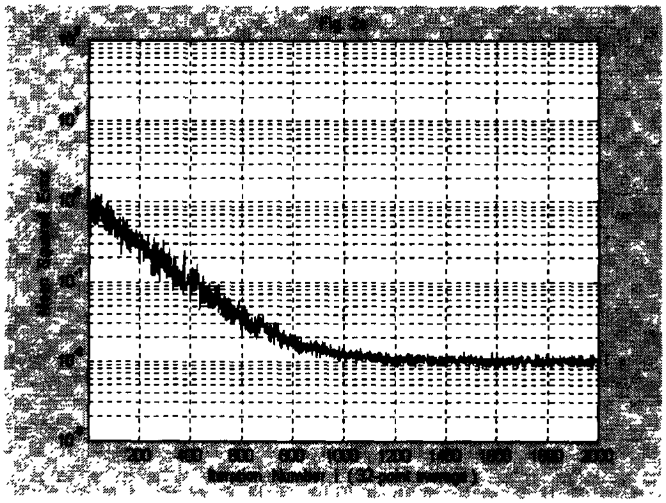


Figure1. (1a) LMS, μ=0.2; (1b) BLMS, μ_B=51.2, N=256;

(1c) MDF, μ_B=102.4, N=512; (1d) MDF, μ_B=12.8, N=64



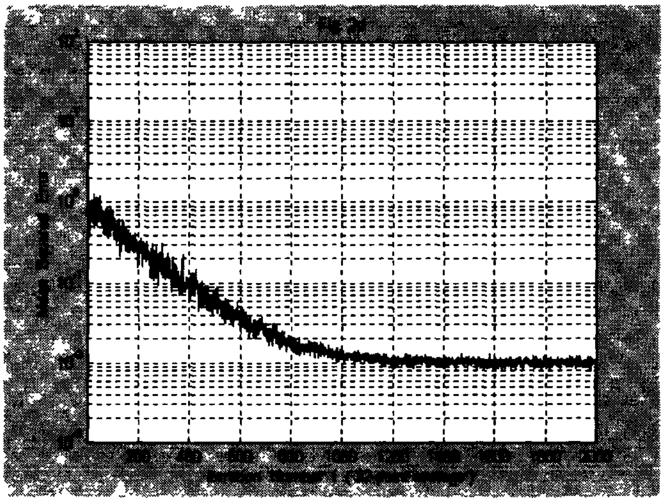


Figure2. (2a)LMS, μ=0.05; (2b)BLMS, μ_B=12.8, N=256;

(2c) MDF, μ_B=25.6, N=512; (1d) MDF, μ_B=3.2, N=64