

BAYESIAN ESTIMATION OF NON-MINIMUM PHASE WAVELETS APPLIED TO MARINE REFLECTION SEISMIC DATA

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ABSTRACT

In this paper, the problem of wavelet estimation for marine seismology is investigated with a bayesian approach applied to a Bernoulli-Gaussian model. We specify proper prior distributions for all unknown quantities including the seismic wavelet, the parameters of the reflectivity sequence and noise. To solve this estimation problem, an algorithm close to a stochastic version of the EM algorithm is used. The random variables are generated iteratively by a simple Monte-Carlo method namely the Gibbs sampler. But the direct application of this procedure often leads to a local minimum of the likelihood function resulting in a shifted and distorted wavelet. We propose a general method to obtain the true solution which systematically uses different shifted wavelets to reinitialize the algorithm. Then we rerun the procedure on each initialization and retain the wavelet which minimizes the noise variance.

1. INTRODUCTION

An important preliminary step in linear deconvolution problems is the estimation of the convolution filter often referred to as the wavelet in geophysics. As this seismic wavelet is generally non-minimum phase, its estimation from the second-order statistics of the observed data fails. Higher order methods have received considerable attention since last decade, but they often lack robustness because of the small amount of observation we have [1] [4]. Moreover reflections at the sea surface induce correlation in the reflectivity sequence which therefore cannot be considered as white. This makes all methods based on parametric modelling insufficient. With the well-known EM algorithm, the problem of model estimation can be viewed in a different manner. Lavielle proposed different stochastic approaches to solve the problem in a maximum likelihood framework[5]. Recently, Doucet has adopted a bayesian strategy for state-space estimation in the case of AR signals [2] but does not take the phase problem into account. We adapt this technique to seismic signals with an MA representation of the

source wavelet and apply it to seismic traces.

2. PROBLEM STATEMENT

A seismic trace can be modelled as the convolution of the unknown wavelet with the reflectivity sequence:

$$z(n) = \sum_{i=0}^L h(i)r(n-i) + w(n) \quad (1)$$

where z , h and r respectively denote the observation, the wavelet and the reflectivity. The noise will be assumed white gaussian with variance σ_w^2 . The order L of the MA wavelet will be assumed known. The reflectivity obeys a Bernoulli-Gaussian model:

$$r(n) \sim \lambda \mathcal{N}(0, \sigma_1^2) + (1 - \lambda) \mathcal{N}(0, \sigma_0^2). \quad (2)$$

λ is the density of the reflectivity, σ_1^2 its variance and σ_0^2 is commonly used in geophysics to model the inhomogeneity of the medium. At each $r(n)$ is associated a Bernoulli indicator variable $q(n)$ with:

$$P(q(n) = 1) = 1 - P(q(n) = 0) = \lambda \quad (3)$$

and the missing variables $y = (q, r)$ are introduced. Our aim is to estimate the parameter vector $\theta = (h, \lambda, \sigma_1^2, \sigma_w^2)$ when just the seismic data are available. As σ_0^2 is an artificial term it will not be estimated but we will choose $\sigma_0^2 = \alpha^2 \sigma_1^2$ with $\alpha \ll 1$.

3. THE BAYESIAN APPROACH

In bayesian estimation, the hyperparameter is supposed to admit a prior density $p(\theta)$. When no prior distribution is available, it is always possible to specify non-informative or flat prior distributions easy to incorporate. Then the objective is to estimate the a posteriori joint distribution $p(y, \theta|z)$ which can be expressed by Bayes rule:

$$p(y, \theta|z) = \frac{p(z|y, \theta)p(y|\theta)p(\theta)}{p(z)} \quad (4)$$

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with

$$p(z) = \int p(z|y, \theta) p(y|\theta) p(\theta) dy d\theta. \quad (5)$$

This requires integrations that are generally impossible to perform. To overcome this difficulty we can use Monte-Carlo methods which estimate a complex distribution by samples drawn from it. The aim is to build a Markov chain whose equilibrium distribution coincides with the desired joint a posteriori distribution of the unknown parameters.

3.1. The Gibbs sampler

The Gibbs sampler is one of the most popular algorithm used in bayesian estimation. It can be applied when the variables have conditional distributions that can easily be sampled from. For the problem we are interested in, the algorithm proceeds as follows:

1. Initialization: choose arbitrary values $\theta^{(0)} = (h^{(0)}, \lambda^{(0)}, \sigma_1^{2(0)}, \sigma_w^{2(0)})$ and set $k=1$.
2. Simulate y from $p(y|z, \theta^{(k-1)})$.
3. Simulate $\theta^{(k)}$ from $p(\theta|z, y)$.
4. Replace k by $k+1$ and go to step 2.

3.2. Simulation of the missing variables

The simulation of the missing variables is a difficult problem because it is impossible in practice to sample directly from $p(r, q|z, \theta^{(k-1)})$. We use a detection-estimation procedure similar to the one described in [5]:

- Detection: generate $q(n)$ according to $P(q(n) = 1|z, q_{-n}, r_{-n})$.
- Estimation: simulate $r(n)$ from $p(r(n)|z, r_{-n}, q)$.

We will note for any vector v :

$$v_{-n} = [v(1), \dots, v(n-1), v(n+1), \dots, v(N)]^T.$$

We adopt a random permutation of $\{1, 2, \dots, N\}$ for the visiting schedule at each iteration of the Gibbs sampler.

3.3. Simulation of the hyperparameters

We choose conjugate priors classically used in bayesian estimation [2] [3] [7]:

$$h \sim \mathcal{N}(m_0, \Sigma_0) \quad (6)$$

$$\lambda \sim \mathcal{Be}(\zeta, \tau) \quad (7)$$

$$\sigma_1^2 \sim \mathcal{IG}\left(\frac{\nu_1}{2}, \frac{\gamma_1}{2}\right) \quad (8)$$

$$\sigma_w^2 \sim \mathcal{IG}\left(\frac{\nu_w}{2}, \frac{\gamma_w}{2}\right). \quad (9)$$

This leads to simple conditional posterior distribution. Let $R(n) = [r(n), \dots, r(n-L)]^T$; then we have:

$$h|r, z, \sigma_w^2 \sim \mathcal{N}(m, \Sigma) \quad (10)$$

where

$$\Sigma^{-1} = \frac{1}{\sigma_w^2} \sum_{n=L+1}^{N-L} R(n)R(n)^T + \Sigma_0^{-1} \quad (11)$$

and

$$m = \Sigma \left[\frac{1}{\sigma_w^2} \sum_{n=L+1}^{N-L} z(n)R(n) \right]. \quad (12)$$

The posterior distribution of the noise variance follows:

$$\sigma_w^2|z, r, h^{(k)} \sim \mathcal{IG}\left(\frac{\nu_w + N}{2}, \frac{\gamma_w + s_w^2}{2}\right) \quad (13)$$

with

$$s_w^2 = \|z - h * r\|^2. \quad (14)$$

For the variance of the reflectivity we have:

$$\sigma_1^2|r, q \sim \mathcal{IG}\left(\frac{\nu_1 + n_1}{2}, \frac{\gamma_1 + s_1^2}{2}\right) \quad (15)$$

with

$$s_1^2 = \sum_{n=1}^N r^2(n)q(n). \quad (16)$$

For the density of the reflectivity:

$$\lambda|q \sim \mathcal{Be}(\zeta + n_1, \tau + N - n_1) \quad (17)$$

with

$$n_1 = \sum_{k=1}^N q(k). \quad (18)$$

4. SIMULATION RESULTS

4.1. Synthetic data examples

We start with a simple example of synthetic data where the reflectivity obeys a white Bernoulli-Gaussian model (Figure 1). Then we convolve the obtained sequence with the wavelet depicted in Figure 2 and add noise to generate the data plotted on Figure 3. The variance of the white gaussian additive noise is adjusted so that the signal to noise ratio is equal to 15 dB. We initialize the algorithm with a null sequence for r and q and a Dirac for the wavelet. In all the examples given in the article, the missing variables are sampled five times more often than the hyperparameters. Direct application of the algorithm can result in a bad wavelet estimate as shown in Figure 4. The wavelet appears shifted and distorted. Lavielle analysed this effect as a phase problem in [6]. The main problem is that the missing variables

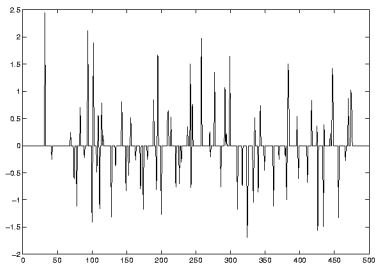


Figure 1: White reflectivity example

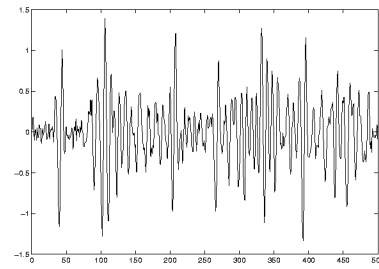


Figure 3: White reflectivity example: noisy seismic data

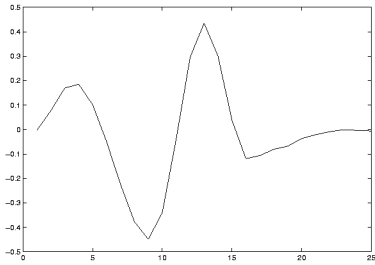


Figure 2: Seismic wavelet

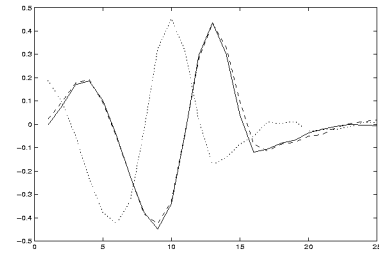


Figure 4: White reflectivity example: true wavelet (—), estimation before (- -) and after (· · ·) phase correction

simulated by the Gibbs sampler are often correlated from an iteration to another. So the algorithm can be trapped in local maxima and cannot escape from them in spite of the stochastic nature of the simulation procedure. Thus the algorithm is sensitive to the initialization. A simulated annealing version of the EM algorithm has been given in [6] to alleviate this problem. We adopt a totally different strategy. As there is a phase shift in the wavelet, an idea is to use different versions of the shifted wavelet to reinitialize the Gibbs sampler. The procedure is the following:

1. Choose a shifted wavelet h_s .
2. Reinitialize the missing variables and the hyperparameters by running the Gibbs sampler for several iterations without changing the wavelet parameters.
3. Run the complete estimation procedure.
4. Store the hyperparameters that minimize the noise variance estimate.

The second step is useful for a correct initialization of the missing variables. We have used 20 iterations in step 2 and then 100 in step 3. The parameters are estimated by averaging the 50 last iterations. As shown in Figure 4, this procedure greatly improves the results and we obtain a very satisfactory estimated wavelet.

We now present another example where the reflectivity is not white anymore. The reflectivity shown in Figure 5 has

been obtained by first generating a Bernoulli-Gaussian variable and then adding pulses to mimic the “ghosts” present in marine seismic surveys. These “ghosts” are due to reflec-

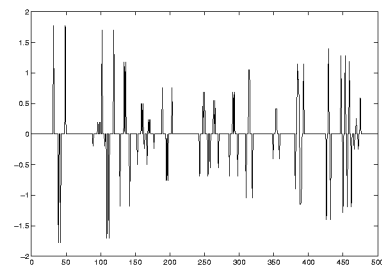


Figure 5: Non-white reflectivity example

tion at the sea surface which follow each primary reflection. They induce correlation in the reflectivity process which makes the estimation procedure very difficult. In particular, parametric modelling algorithm are unable to retrieve the correct seismic wavelet. However, even in this case, our algorithm gives good results as shown in Figure 6.

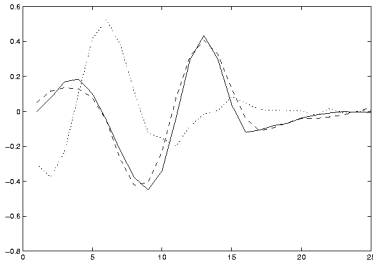


Figure 6: Non-white reflectivity example: true wavelet (—), estimation before (---) and after (···) phase correction

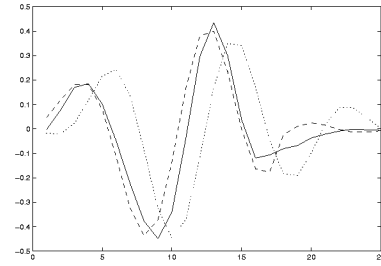


Figure 8: Marmousi data: true wavelet (—), estimation before (---) and after (···) phase correction

4.2. Simulation on seismic data

We present an application of our algorithm on the Marmousi dataset. These data are considered as a reference in seismic data processing and inversion problems because they approach quite well real seismic data. In this model a near field signature of the source wavelet is given and is the same as in Figure 2. The seismic trace appears in Figure 7. In this case our algorithm still improves the estimation of the seismic wavelet (cf Figure 8). In particular, the estimated noise variance which was $\sigma_w^2 = 68$ without phase correction is reduced to $\sigma_w^2 = 17$ when applying our procedure.

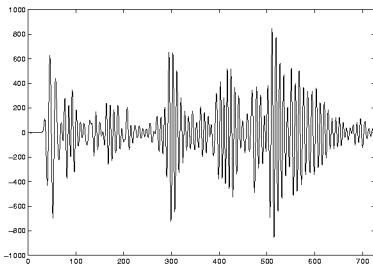


Figure 7: A seismic trace of the Marmousi data

5. CONCLUSION

We have presented a powerful algorithm for estimating an MA signal in the case of a convolution with a Bernoulli-Gaussian process. The Gibbs sampler was used to simulate both the missing variables and the model parameters. However we have shown that the algorithm hardly escaped from local maxima of the likelihood function. As those local maxima were generally shifted and distorted versions of the true wavelet, we have decided to carefully reinitialize the estimation procedure by different phase shiftings. The retained solution is the one that minimizes the noise vari-

ance estimate. This method has been applied to synthetic data as well as to more realistic seismic data and has exhibited satisfactory behavior.

6. ACKNOWLEDGMENTS

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7. REFERENCES

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