LEAST SQUARES BASED DECODING FOR BCH CODES IN IMAGE APPLICATIONS

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ABSTRACT

BCH codes in the frequency domain provide robust channel coding for image channel coding applications. The underlying problem of estimation of real/complex sinusoids in white additive noise may be formulated and solved in different ways. The standard approach is based on the least squares method and Berlekamp-Massey algorithm (BMA). In this paper we compare the performance of the BMA with other LS based algorithms including: minimum norm solution based algorithm (MNS), forward-backward linear prediction based algorithm (FBLP) and singular-value decomposition based minimum norm algorithm (SVD-MNA). Results of computer experiments show that the introduction of minimum norm solution, forward-backward prediction and the SVD decomposition may significantly improve the performance of the decoder in the case of the relatively low SNR. In selecting between the proposed algorithms a performance/complexity trade-off has to be considered.

Keywords*: image coding, BCH codes, least squares method*

1. INTRODUCTION

The majority of image compression coding standards (JPEG, MPEG, H.261, H.263) are designed for communication channels in which average bit error rates (BERs) are better than 10^{-6}. But for example, wireless channels are characterized by long bursts of bit errors, and average BERs of 10^{-3} are common in cellular telephony. Therefore, a powerful and robust channel coding needs to be applied to achieve acceptable image quality. As regards the required transmission or storage quality, relatively low noise is tolerable, but relatively high noise amplitude (impulse noise) must be corrected. Error detecting and correcting techniques are typically based on BCH and RS codes in combination with the interleaving techniques. These codes allow for correction of a certain number of erroneous samples (bytes). BCH codes in the frequency domain can be defined over the finite field, or over the real/complex field. Finite field codes over GF(q) are based on Galois Fourier Transform (GFT) [1], real/complex number codes are based on Discrete Cosine Transform (DCT) [2] and on Discrete Fourier Transform (DFT) [3].

Codes defined over the real/complex field such as in [4] have several advantages since these codes can correct errors and reduce data rate simultaneously, there is no restriction against certain block lengths, decoding algorithms are tolerant to small errors on every sample (pixel), and finally error control coding problem is turned to a signal processing problem. A syndrome in BCH codes defined in the frequency domain is a sum of complex sinusoids, which corresponds to a nearly nonstationary autoregressive models.

However, the underlying problem of estimation of real/complex sinusoids in white additive noise may be formulated and solved in different ways. The standard approach to decoding of BCH codes is based on using the Berlekamp-Massey algorithm (BMA) [5]. In this paper we compare the performance of the BMA with other LS based methods: minimum norm solution based algorithm (MNS), forward-backward linear prediction based algorithm (FBLP) and singular value decomposition based minimum norm algorithm (SVD-MNA) [6]. Results of computer experiments show how the introduction of minimum norm solution, forward-backward prediction and SVD decomposition contribute to the performance improvement.

In section 2 a short review of BCH codes in the frequency domain followed by the introduction of the nonstationary autoregressive models is given. LS method based decoding algorithms including BMA, MNS, FBLP and SVD-MNA are introduced in the section 3. Computer simulation results showing the performance of these algorithms as well as their analysis are presented in the section 4.

2. THE PROPOSED CHANNEL CODING METHOD

If vector **x** contains c zeros at predefined successive locations in the sequence and the rest of N-c=M values contain data information, the vector **X**=DFT(x) represents a code vector of an (N,M) BCH code. Such real or complex-number (N,K) BCH codes are, like Reed-Solomon codes(RS), the minimum distance separable (MDS) codes and exist for every $N \geq 2$, $0<M<N$ [1]. Due to disturbances during transmission or storage of a sequence of coefficients, the received vector is **Y**=**X**+**E**, where **E** is an error vector. An inverse transform of the received vector **Y** is a vector consisting of the first M information data disturbed by noise and the last N-M=c values representing the syndrome. Thus $\mathbf{y}=\mathrm{IDFT}(\mathbf{Y})=[\mathbf{y}_M,\mathbf{s}]$, where $\mathbf{y}_M=\mathbf{x}+\mathbf{e}$, and $\mathbf{e}=\mathrm{IDFT}(\mathbf{E})$. In image coding applications, vector **x** can represent an inverse discrete Fourier transform (IDFT) of non-zero DCT coefficients computed for example in a common JPEG coder [7].

Since errors appear as impulses in the frequency domain, the corresponding error sequence in the time domain is always a periodic sequence. Therefore AR(p) models describing such periodic sequences have parameters located at the boundary of the so called nearly nonstationary AR models [8]. An autoregressive model of order p in a polynomial form is

$$\mathbf{a(L)}\, \mathbf{x}_n = \mathbf{0}, \qquad (1)$$

with $\mathbf{a(L)} = \prod_{i=1}^{p}(1-\mathbf{G}_i\mathbf{L})$ where G_i^{-1} are the roots of the characteristic equation a(L)=0. In order to obtain a nearly nonstationarity we require that $|G_i|=1$. Equation (1) represents the well known key equation [1]. For a complex AR(1), a single complex root is $G_1=e^{j2\pi f}{}_1$ and $a_1=G_1$.

For example, let us consider spectra with N=20 components so that with c=p=2 we have (20,18) BCH code. The number of different AR(1) models is N/2-1=9. Figure 1 illustrates model locations on the triangle which defines the stationarity region for an equivalent real AR(2) model [7].

3. DECODING OF REAL/COMPLEX BCH CODES: AR PARAMETER ESTIMATION IN ADDITIVE NOISE

Consider a received signal {x(f)} that consists of L complex sinusoids whose complex amplitudes are $\alpha_1, \alpha_2, \ldots, \alpha_L$ and angular frequencies are $f_1, f_2, \ldots, f_L$ respectively:

$$x_n = s_n + n_n = \sum_{i=1}^{L} \mathbf{x}_i \mathbf{e}^{j2\pi f_i i} + \mathbf{n}_n \text{ , n=0,1,...,N-1} \qquad (2)$$

where $E[n_n]=0$, $E[n_n n_{n-j}]=\delta_j \sigma_n^2$, $E[s_n n_n]=0$ and $\mathbf{E}\left[\alpha_i \alpha_j^*\right] = 0$ for $j \neq i$ since the white noise is assumed to be uncorrelated with the signal and the complex sinusoidal components of the received signal are uncorrelated with each other

In this paper we take that the data vector length N is minimal one, i.e. data vector consists of only N=2L complex elements. We choose a minimal number of samples such that it is enough to make it possible to obtain correct spectrum estimate in noiseless case. In addition frequency and phase of sinusoids are discrete i.e. with L=1: $f_i=\{f_1,f_2,\ldots, f_1\}$; $\varphi_i=\{0,\pm\pi/2,\pi\}$, where K is codeword length. However, amplitudes of sinusoids are continuous variables.

3.1 Common Least Squares Solution

The key equation (1) can be directly solved by matrix inversion or, more efficiently, by using the Berlekamp-Massey algorithm (BMA) [5]. In matrix form we have

$$\mathbf{A}\,\mathbf{a} = \mathbf{b} \qquad (3)$$

where **A** is LxL data matrix, **a** and **b** are L component vectors.

Sensitiveness of the parameter estimation can be analyzed by exploring covariances of estimates of the AR parameters. The error covariance matrix **V**(a) for the estimation of the AR parameters for the case of noisy observation is given by [9]

$$\mathbf{V}(\mathbf{a}) = E[(\mathbf{a} - \mathbf{a})(\mathbf{a} - \mathbf{a})^T] = \Gamma_p^{-1}\ \tilde{\Gamma}\ (\Gamma_p^{-1})^{\mathbf{T}}$$

with $$\tilde{\Gamma} = \mathbf{c}_0\Gamma_0 + \sum \mathbf{c}_i(\Gamma_i + \Gamma_i^{\mathbf{T}}) \qquad (4)$$

where Γ is covariance matrix of data and $c_i = \sum_{j=0}^{q-i} \mathbf{b}_j \mathbf{b}_{j+i}$,and $b_j = a_j$.

Berlekamp-Massey algorithm represents a standard method to decoding of BCH codes

3.2 Minimum Norm Solution for Undetermined Case

Assuming that signal plus noise is an ARMA(p,p) stochastic process, the matrix equation (3) represents a linear system of p simultaneous equations which can be considered as undetermined system since ARMA(p,p) model is equivalent to an AR(∞) model. Here we can use the minimum norm approach directly by solving an undetermined system to exploit its super resolution feature. The problem can be solved by finding the vector **a** of dimension K>p that satisfies (3), subject to the condition that its Euclidean norm is minimised. Writing equation (3) directly in terms of data matrix as $\hat{\mathbf{a}} = \mathbf{A}^{\mathbf{H}}(\mathbf{A}\mathbf{A}^{\mathbf{H}})^{-1}\mathbf{b}$. The term $\mathbf{A}^{+} = \mathbf{A}^{\mathbf{H}}(\mathbf{A}\mathbf{A}^{\mathbf{H}})^{-1}$ represents a Moore-Penrose pseudoinverse of matrix **A**. We write

$$\hat{\mathbf{a}} = \mathbf{A}^{+}\,\mathbf{d} \qquad (5)$$

The two forms given in (3) and (5) are mathematically equivalent but they lead to different estimates of the least-squares estimate $\hat{\mathbf{a}}$. for underdetermined case The later is preferable since it offer a super-resolution approach to estimate sinusoids in noise [10].

3.3 Forward Backward Linear Prediction (FBLP)

Given the time series of length N and the transversal filtering structure of length M+1 we may define an augmented data matrix **A** as [10].

$$A^H = \begin{bmatrix} x(M) & \cdots & x(N-1) & x^*(0) & \cdots & x^*(N-M+1) \\ x(M-1) & \cdots & x(N-1) & x^*(1) & \cdots & x^*(N-M+2) \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ x(0) & \cdots & x(N-M+1) & x^*(M) & \cdots & x^*(N-1) \end{bmatrix} \qquad (6)$$

The left half of the data matrix $\mathbf{A}^{\mathbf{H}}$ represents sets of tap inputs used for forward filtering, while the right half of the data matrix $\mathbf{A}^{\mathbf{H}}$ represents sets of tap inputs used for backward filtering. Computing of a coefficient vector **a** is based on FBLP LS method of equation (3) where data matrix **A** is defined by (6). The solution may be obtained by using the Moore-Penrose pseudoinverse as in (5). Using FBLP model makes it possible to use the available data samples more efficiently in order to improve the quality of the estimate.

4.4 Least Squares Estimate Based on Singular Value Decomposition

In real systems we have to base our analysis on the estimate of the ensemble-averaged correlation matrix $\mathbf{R}=\mathbf{A}^{\mathbf{H}}\mathbf{A}$, where A is defined by (6) and the scaling factor is neglected.

The eigenvectors of the correlation matrix estimate $\hat{\mathbf{R}}$ are divided into two sets: the set $\mathbf{v}_1, \ldots, \mathbf{v}_L$ that spans the signal subspace and defines the matrix $\mathbf{V}_S$ and the set $\mathbf{v}_{L+1}, \ldots, \mathbf{v}_{M+1}$ that spans the noise subspace and defines the matrix $\mathbf{V}_N$. These two matrices may be partitioned as

$\mathbf{V}_S = \begin{bmatrix} \mathbf{g}_S^T \\ \mathbf{G}_S \end{bmatrix}$, $\mathbf{V}_N = \begin{bmatrix} \mathbf{g}_N^T \\ \mathbf{G}_N \end{bmatrix}$, where the (1xL) row vector $\mathbf{g}_s^{\mathbf{T}}$ and the 1x(Mx1-L) row vector $\mathbf{g}_n^{\mathbf{T}}$ have the first elements of $\mathbf{V}_S$ and $\mathbf{V}_N$ respectively, and the MxL matrix $\mathbf{G}_S$ and the Mx(M+1-L) matrix $\mathbf{G}_N$ have the remaining elements [6].

A single vector **a** that spans the noise subspace (i.e. a linear combination of the noise-subspace eigenvectors $\mathbf{v}_{L+1}, \ldots, \mathbf{v}_M$) is computed with the constraint that its first element a(0)=1 and its norm $\sum_{k=1}^{M-1}\left|\mathbf{a}(\mathbf{k})\right|^2$ is minimal.

Vector **a** may be viewed as the tap-weight vector of a transversal filter operating as a prediction-error filter of order M, and it can be partitioned in the form $\mathbf{a} = \left[1 \ \ \text{-}\mathbf{w}^{T}\right]^{T}$

Vector **a** (or **w**) may be computed from the relation which is based on the fact that it is orthogonal to the eigenvectors spanning the signal subspace $\mathbf{v}_S^H\mathbf{a} = 0$ or equivalently $\mathbf{G}_S^T\hat{\mathbf{w}} = \mathbf{g}_S$

With $\mathbf{L} \leq \mathbf{M}$ this system of equations is undeterminated, and has no unique solutions. Using the minimum norm constraint enables us to obtain the unique solution to this LS problem as follows [6]:

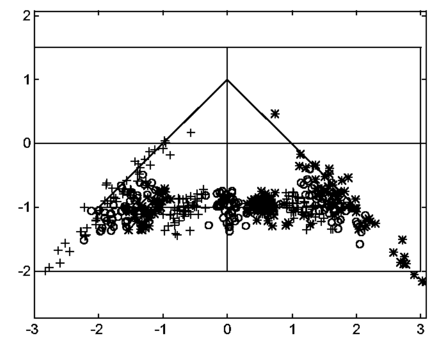
$$\mathbf{w} = (\mathbf{g}_N^H\mathbf{g}_N)^{-1}\mathbf{G}_N^{*}\mathbf{g}_N \qquad (7)$$

The eigenvectors of the matrix $\hat{\mathbf{R}}$ can be obtained applying singular value decomposition (SVD) directly to data matrix **A**.

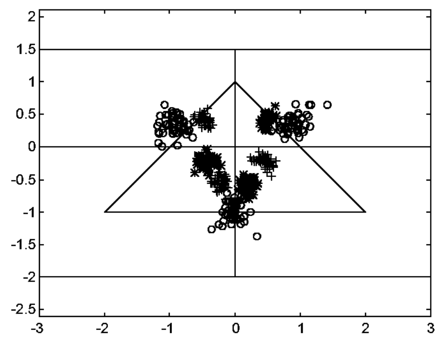
4. COMPUTER SIMULATIONS

For example, let us to consider an example of (20,18) BCH code defined in the frequency domain as in Section 2. Let a single complex impulse noise of amplitude 10 be added to spectral components separately so that syndrome signals are complex sinusoids of the unit amplitude. Additive noise is a zero-mean white process of variance 0.01 i.e. SNR=17dB.

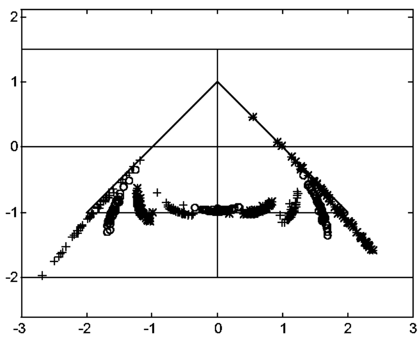
Fig. 1a illustrates the parameters scattering graph obtained by simulation based on approach defined in 3.1 (equation (3) or BMA algorithm). The agreement with equation 4 is excellent. Note that parameter variances are not the same and are strongly dependent on the component frequency. For SNR=17dB scattering patterns are not quite separated so we can expect a low decoder output quality.



a)



b)



c)

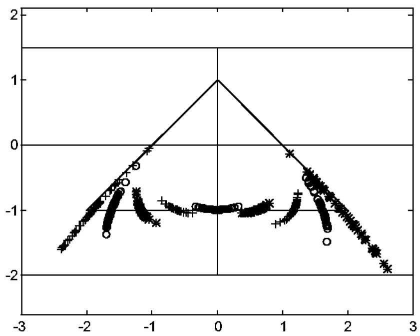


Figure 1 *AR(2) parameters distribution (white noise variance of 0.0025 for a) BMA b) MNS c) FBLP d) SVD-MNA.*

Consider next the application of the minimum-norm algorithm defined in 3.2. to the same example (Fig 1b). Note that, compared with Figure 1a, the scattering patterns are located at greater Euclidean distances with lesser sensitiveness to noise. It could be concluded that minimum-norm algorithm based on pseudoinverse of data matrix for underdetermined case assures about twice resolution than method based on BMA. This corresponds to more than 3 dB better SNR for the minimum-norm approach. Note that Fig 1b gives only two dimensional presentation. Since we have computed 3 parameters, 3-D analysis should give better resolution.

Figure 1c illustrates the parameter scattering for the model described in 4.3. which is based on FBLP LS algorithm. Here the scattering parameters are located in the similar directions as in Fig. 1a. and with similar variation of a_1 and a_2 parameters, but not in an ellipsoidal form. Parameters scattering are almost of a line form, so that parameter resolution is better. It can be calculated that FBLP LS approach gives further improvement compared even to minimum norm solution approach. Finally, Fig.1d depicts the parameter scattering for the model based on FBLP and singular value decomposition. Scattering patterns are generally the same as in Fig.1c but without any fluctuation from line form. It confirms that eigenvectors in the signal space are less sensitive to noise than other estimates.

Figure 2 shows an example of a test gray-scale image with 640x480 pixels and an 8bit/pixel. Suppose that the picture should be compressed to a very good JPEG quality, this image has more than 75 per cent zero-valued spectral coefficients so that, on average, there are only 16 nonzero spectral coefficients per each 8x8 block. Adding extra 4 zero samples to the corresponding time sequence we have, in the average, 20-sample time series with a redundancy of 0.2. DFT transform of this sequence is the codeword of a (20,16) real-valued BCH code capable to correct up to r=2 impulse errors. To illustrate its capability, each sequence of N spectral components (N/2 real and N/2 imaginary) is corrupted with a single impulse noise of fixed-valued amplitude 75 (with random sign) in the way that the impulse noise is added to the randomly chosen component. In addition, all components are corrupted with a Gaussian zero-mean white noise with σ_n^2 =2.5.

Figure 2b shows the corrupted images by single impulse and white noise both. Figure 2c shows a decoded picture based on BMA and Fig. 2d shows the decoded picture based on minimum-norm algorithm (MNA-SVD). As it can been seen, residual disturbance in the decoded picture by MNA-SVD is practically caused only by the white noise. It should be noted that an amplitude estimate has been based on a least squares for overdetermined case.

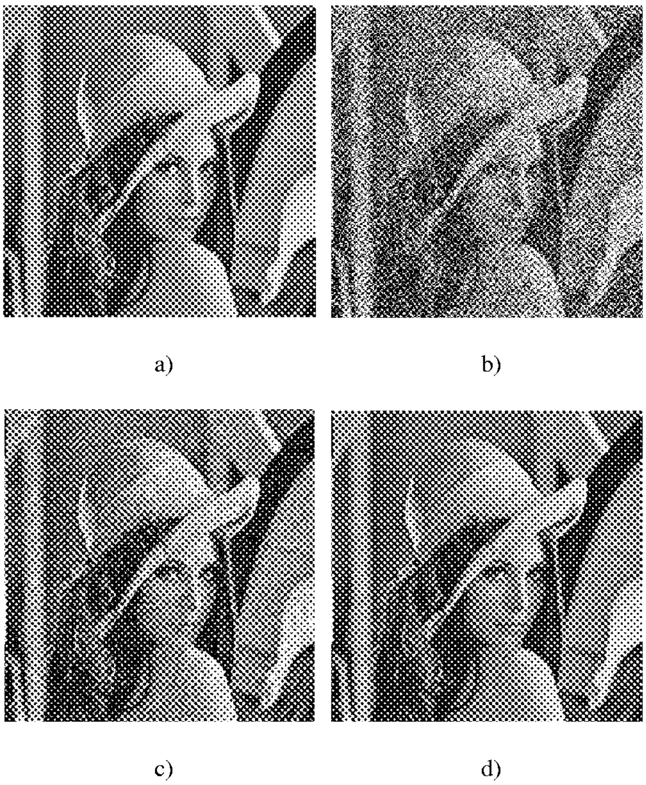


Figure 2 A two-dimensional example of a real-valued DFT code
a) original image
b) image corrupted by white noise and impulse noise
c) decoded image by BMA
d) decoded image by MNA-SVD

5. CONCLUSION

In this paper we have considered the problem of decoding BCH codes in the frequency domain for image channel coding applications by using different LS method based algorithms. The performance of the standard BMA algorithm is compared to the performance of other LS based algorithms: minimum norm solution based algorithm (MNS), forward-backward linear prediction based algorithm (FBLP) and singular-value decomposition based minimum norm algorithm (SVD-MNA). Results of computer experiments show that the introduction of minimum norm solution, forward-backward prediction and the SVD decomposition may significantly improve the performance of the decoder in the case of low SNR. In selecting between the proposed algorithms there is a performance/complexity trade-off to be considered. Our further work will focus on statistical analysis of the proposed algorithms as well as on their improvement.

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REFERENCES

[1] R.E. Blahut: *Transform Techniques for Error Control Codes*, IBM J.Res.Develop., vol.32, May 1979, pp 299-315.

[2] J.L. Wu, J. Shin: *Discrete Cosine Transform in Error Control Coding*, IEEE Trans. on Comm., vol.43, May 1995, pp 1857-1861.

[3] J.K. Wolf: *Redundancy, the Discrete Fourier Transform, and Impulse Noise Cancellation*, IEEE Trans. on Comm., COM-31, March 1983, pp 458-461.

[4] T.G. Marshall Jr.: *Coding of Real-Number Sequences for Error Correction: A Digital Signal Processing Problem*, IEEE Journal on Select. Area in Commun., Vol. SAC-2, No.2, March 1984, pp 381-392.

[5] J.L. Massey: *Shift-Register Synthesis and BCH Decoding*, IEEE Trans. on IT, vol. IT-15, January 1969, pp 122-127.

[6] S. Haykin: *Adaptive Filter Theory*, Prentice-Hall, Inc., Englewood Cliffs, NJ 07632, 1991.

[7] N.Rozic, J. Ursic, H. Dujmic: *Joint Source/Channel Coding of Images for Wireless Applications*, submitted for publication on Vitel'98.

[8] N. Rozic: *Image Coding over Real/Complex Fields: Parameters Estimation of Nearly Nonstationary AR(p) Models*, Int. Symp. on Inf. Theory ISIT 97, Ulm, Germany, June 1997.

[9] D.F. Gingras: *Asymptotic Properties of High-Order Yule-Walker Estimates of the AR Parameters of an ARMA Time Series*, IEEE Trans. on Acoustics, Speech and Signal Processing, Vol. ASSP-33, N0.4, October 1985, pp 1095-1101.